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# Traveling in spatiality, in spatial sense

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*What attention is given to spatial sense in Geometry? The outcome (2015) of a special ZDM issue, Geometry in the Primary School and the CERME conference is a good opportunity to think about and compare different approaches or frameworks regarding this topic.*

*Keywords: Spatial sense, spatial knowledge, geometrical thinking.*

## Introduction

A travel abroad enriches our way of thinking and furthers our understanding of the values and behaviours of his/her own culture. For us just the same an intellectual trip outside the country (or the language) helps deepen our knowledge and analysis of our way of thinking. The study initially aims to compare the perspectives of educational researchers regarding the notion of *spatial sense*. Along the way, it documents the conceptual frameworks which support research.

In ZDM issue 47, *Geometry in the Primary School*, the theme of *spatial sense* is common to all papers (Mulligan, 2015). Could it be that the study involves primary school or that the imagery in thought is a topical subject in recent neuroscience research? In the 36 papers of the ICMI Study *Perspectives on Teaching Geometry for the 21st Century* (1998) the word *spatial* was mentioned 353 times but never in a title. Our ongoing study is based on (not yet completely extensive) reading of ZDM 47 and CERME texts on Geometry since the first time (2003) that a specific group about Geometry exists. Such a group has persisted for all the CERME under the name *Geometrical Thinking* (except 2011 *Geometrical teaching and learning*).

In short, this paper provides an overview on spatial sense the difference theoretical frameworks regarding it, based on literature published in ZDM issue 47 and CERME papers of the last decade (2003-2015).

## First definitions and motivations for studies about spatial sense

Many expressions (*spatial reasoning, spatial sense.....*) are related to *spatial* when considering English papers from cross- and interdisciplinary fields of mathematics education, psychology, child development and neuroscience: In a first approach we consider these expressions as equivalent and agree the large definition quoted by Mulligan (2015), from Spatial Reasoning Group (2015)

“*Spatial reasoning* (or *spatial ability, spatial intelligence, or spatiality*) refers to the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects. Examples of spatial reasoning include: locating, orienting, decomposing / recomposing, balancing, diagramming, symmetry, navigating, comparing, scaling, and visualizing.” (Mulligan, 2015, p. 513).

It should be noted that this definition considers *spatial* in a broader sense than 3D- situations or 3D-geometrical thinking. **In this paper we are interested in *spatial skills*, which are not associated with objects 3D.**

Many researchers stress the utility of *spatial reasoning* for mathematical learning and problem solving (Owens & Outhred, 2006; Sinclair & Bruce, 2015), and to Science, Technology, Engineering and Mathematics Education (Mulligan, 2015). Tahta (1980) and others note the ability to mobilize wisely spatial skills in mathematical and scientific thought, which Battista (1999, quoted by Mulligan, 2015, p. 514) named *spatial structuring* and defined as: “the mental operation of constructing an organization or form for an object or set of objects. It determines the object’s nature, shape, or composition by identifying its spatial components, relating and combining these components, and establishing interrelationships between components and the new objects” (Battista, 1999).

*Spatial structuring* is also an important component of the early learning of numbers, of measurement units, and well of geometrical thought (Van den Heuvel-Panhuizen et al., 2015; Mulligan, 2015). Van den Heuvel-Panhuizen et al. (2015, p. 347) outline the strong relationship between spatial and mathematical abilities. Mathematical performance and spatial abilities are positively correlated, not only in mathematical domains that are ostensibly spatial.

French scholars as Berthelot & Salin – with a long story of research on geometrical education – agree on the place of *spatial abilities* in geometry learning, also for geometrical proof. But they have always emphasised (since the 1990s) their interest for everyday life: “the treatment of spatial abilities is the main source not only of many of the further learning difficulties met by secondary school pupils, but also of some of the main deficiencies in spatial representation needs in everyday life” (Berthelot & Salin, 1998, p. 71). This point of view has encouraged them to define and elaborate, unconstrained by geometry (or dissociated by classical geometry), what they called *spatial knowledge*.

In conclusion it appears that spatial abilities are critical for learning mathematics and beyond. As these abilities are naturally associated with geometry, geometry seems the ideal niche for their teaching. It could even be read between the lines that it would be, nowadays, the main reason for teaching geometry in compulsory school education. Indeed, the present trend in some countries is to marginalize geometry in curricula in favor of probability and statistics (Maschietto et al., 2013).

Obviously *spatial reasoning* nourishes *geometric reasoning* (Mithalal-Le Doze, 2015), but *geometric reasoning* needs other abilities, like defining and classifying (in the sense of Brunheira & Ponte, 2015), axiomatic reasoning, and doesn’t take in account non-mathematical forms of deduction... Furthermore spatial problems, like finding one’s way in an unknown town, cannot be assimilated to geometrical tasks. Studying *spatial reasoning* for itself seems to be interesting.

### **Spatial sense in CERME papers (2003 to 2015)**

Let us first examine how the successive CERME (2003 to 2015) working groups named *Geometrical Thinking* deal with spatial sense.

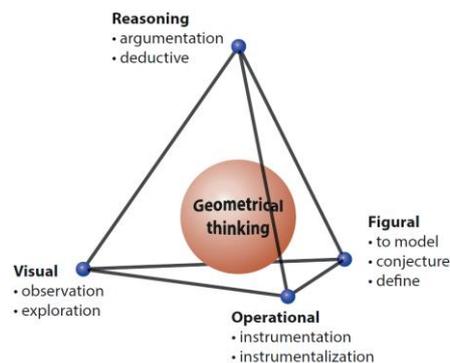
The Group *Geometrical Thinking* worked within the continuity in the CERME3, 4 and 5 (2003, 2005, 2007). In CERME5 the topic *Spatial abilities and Geometrical tasks* was considered; it is

noted: if it seems possible to agree about Geometrical tasks, it is necessary to precise what *spatial abilities* mean. We try to do it in the next section.

In CERME6 (2009) *spatial abilities* is not a specific topic of the Group even though it could be present in the sub-theme *Teaching, thinking and learning 3D Geometry*.

In CERME7 (2011) *spatial abilities* is connected to *diagrammatic reasoning*. Deliyanni et al. (2011) explore spatial abilities in relation with 2D-geometrical figure understanding and consider the influence on reasoning of the different diagram's apprehensions (Duval, 1995): perceptual, sequential and operative. Braconne-Michoux (2011) proposes to intertwine the Geometrical Paradigms (Houdement & Kuzniak, 2003; Houdement, 2005) and the Van Hiele levels which integrates visualization, a spatial ability.

In CERME8 (2013) the Group introduction proposed four competencies (see Figure 1) to support geometrical thinking: reasoning, figural, operational and visual (Maschietto et al., 2013) and assume that the links between these competencies are more important for geometrical work. *Spatial abilities*, *spatial sense* are not explicitly mentioned but it seems (see above in Mulligan, 2015) to “have a place” in each competency. Two papers assume a *spatial* entry (other than 3D activities): in Sevil & Aslan-Tutak (2013) and particularly de Freitas & Mc Carthy (2013) emerges a new face of *spatial abilities*, the gestural / haptic ones.



**Figure 1: The geometrical competencies (Maschietto et al., 2013)**

CERME9 (2015) in the continuity of CERME8 is supported by the same model (Figure 1). The topic quoted in relation with spatial abilities is *visualization*. The authors (Ceretkova et al., 2015) stress the influence of geometric knowledge on visualization, beyond perceptive and psychological aspects.

How do educational researches deal with the issue of *spatial abilities*? More precisely what kind of theoretical frames do they use or construct for their research?

## Theoretical frameworks for spatial abilities

Specifying the frame the authors use in order to analyse spatial abilities is quite rare: only once in ZDM, only a few times in CERME papers before CERME9. Let us give some examples of such frames.

### Example 1

In Panaoura et al. (2007) *spatial abilities* are commonly addressed by three major dimensions **spatial visualization, spatial orientation and spatial relations**. These researchers use an analysis model for *spatial abilities* (Demetriou & Kyriakides, 2006) with three components, namely **image manipulation, mental rotation and coordination of perspectives** to investigate whether or not and to what extent primary and secondary school students' spatial abilities are related to their performance on geometry tasks involving 2D figures, 3D figures, or nets of solids.

### Example 2

In Berthelot & Salin's research (quoted in Douaire & Emprin, 2015) *spatial knowledge* is knowledge which enables to control one's relations to the surrounding space, the sensible world. This control may consist in recognizing, describing, manufacturing or transforming objects; moving, finding, communicating objects' position; recognizing, describing, constructing or transforming a route (Berthelot & Salin, 1999, p. 38). Children begin to integrate *spatial knowledge* before going to school while experimenting, and sharing with adults about their actions. Spatial knowledge cannot be reduced to geometrical knowledge but can be necessary to solve a geometrical problem.

It should be noted that this definition relies on problems (what Berthelot & Salin name spatial problems). In Brousseau's theory (the theoretical frame of Berthelot & Salin's research) knowledge is what enables to solve problems, and problems solving is a condition for learning. For instance how to define knowledge to be taught to use efficiently a map when lost in an unknown town? First identifying situations in which using plans and maps are necessary; second analysing the spatial interactions to solve them and thus indentifying the necessary knowledge.

Daily life interactions take place in space of different sizes which exert different constraints on the actions. *Microspace* is very close to the subject, like a sheet of paper, a computer screen, a touch screen; in this space objects can be moved, touched, turned; it corresponds to the usual grip relations. *Mesospace* is the surrounding space, inside a room, a building; the subject can move inside it, mesospace is the space of usual domestic spatial interactions. *Macrospace* is the broader space, unknown city, rural or maritime spaces; the subject has only local views, he had to conceptualize (Berthelot & Salin, 1998, p. 72; Douaire & Emprin, 2015, p. 532). Thus *spatial knowledge* is structured into three main conceptions, microspatial conception, mesospacial conception, macrospatial conception. For instance, following this frame, a straight line can be conceived as a print trace produced with a ruler, the edge of a door, or a set of trees properly aligned in an orchard.

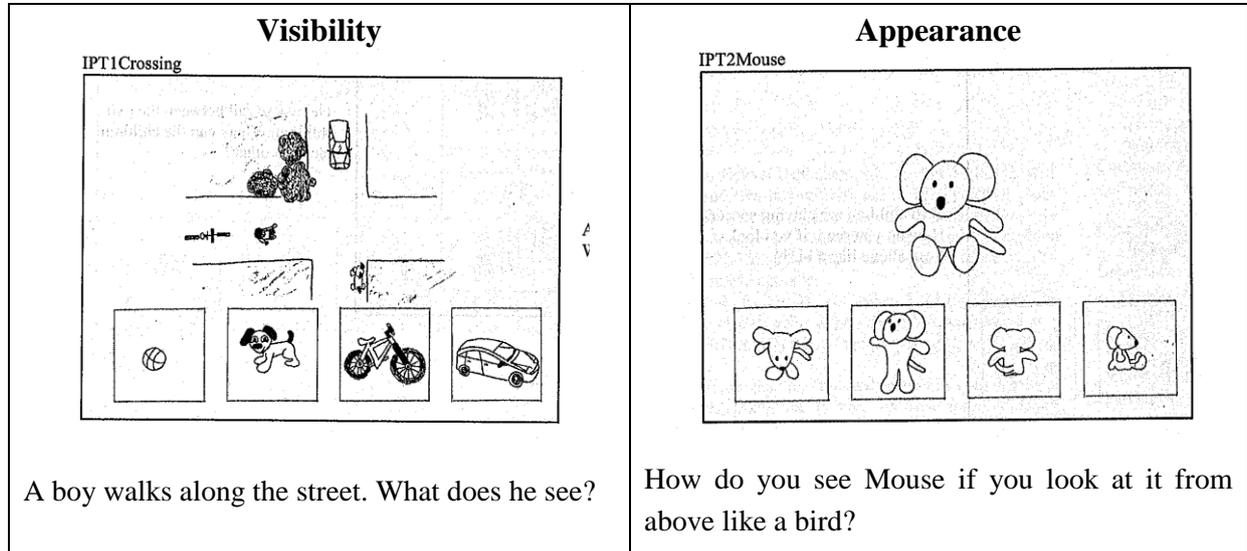
This framework has different functions: in Berthelot & Salin, as in Douaire & Emprin (2015) the framework allows them to construct situations as means to teach students spatial knowledge (as to alignment and straightness, through the good use of a map to navigate). In other described cases, the frame allows to evaluate and compare performances of students, or to map spatial abilities.

### Example 3

Following Newcombe et al. (2013), Van den Heuvel-Panhuizen et al. (2015, p. 346) distinguish between two kinds of spatial skills: **between**-objects representation and transformation skills (for

example in Perspective-Taking tasks –PT–, like the Three Mountains of Piaget & Inhelder, 1956) and **within**-objects representation and transformation skills (for example a mental rotation).

The aim of their research is to assess children’s PT-skills focusing on the difference between two components of what they named IPT (Imaginary Perspective-Taking): **visibility** and **appearance**. These two competencies are highlighted by the items they proposed to the children.



**Figure 2: Examples of drawings and questions in the test (Van den Heuvel-Panhuizen et al., 2015)**

It could be noted that the 3D situation (meso- or macro-spatial) is communicated to the 2D representation (micro-spatial).

The analysis of their tests with more than 300 children of Netherlands and Cyprus (age 4-5) shows that kindergartners of the two countries can answer correctly: on average respectively 70% and 55% of the visibility items, and 40% and 30% of the appearance items; that the development of the IPT competence visibility precedes the development of the IPT competence appearance; that specific item characteristics of the evoked context could also influence the difficulty level of the item.

#### Example 4

In the 1990s Duval (1995, 2006) has brought an important contribution to necessary visualization of the drawing (implicitly in a microspace) for geometric reasoning. Some authors rely particularly on Duval’s research, e.g., Mithalal-LeDoze (2009, 2015), Papadaki (2015) and Swoboda (2015).

With iconic visualization “the drawing is a true physical object, and its shape is a graphic icon that cannot be modified. All its properties are related to this shape (...)” (Mithalal-LeDoze, 2009, p. 797). With non iconic visualization “the figure is analysed as a theoretical object represented by the drawing, using three main processes: *Instrumental deconstruction*: in order to find how to build the representation with given instruments), *Heuristic breaking down of the shapes*: the shape is split up into subparts, as if it was a puzzle), *Dimensional deconstruction*: the figure is broken down into figural units — lower dimension units that figures are composed of —, and the links between these units are the geometrical properties (...)” (Mithalal-LeDoze, 2009, p. 797; Papadaki, 2015). Duval quoted two other processes *Change of Scale and Change of Orientation*.

Studying visually impaired students moving 2D objects to imagine 3D objects, Papadaki (2015) introduces a new source for mental images, the kinesthetic one. For these students visualization integrated many repetitions of the same gesture, cross-checking it with one's everyday life tactile experience and geometrical knowledge as objects' definitions. Visualization is clearly more than vision, what Duval wrote for a long time describing different ways (iconic and non iconic) of visualizing a figure. But Papadaki (2015) introduces a new dimension in visualization, a dynamic one; conceiving a figure 3D as the result of a reproducible movement of a figure 2D.

Swoboda (2015) stresses the rotation as a natural transformation for young students; mental rotation is a fundamental component of the frame of Demetriou & Kyriakides (2006), and of non iconic apprehension of Duval too. Maybe the rotary motion and the rotation as transformation could be studied sooner (than the line symmetry) in compulsory school to enrich visualization skills.

## Conclusion

Spatial sense and the different frames regarding it is now better mapped. We will not go back on what was discussed above, but just stress some difference between the frames regarding it.

1) The framework of Berthelot & Salin in relation with the size of space enables to realize that almost all the mentioned situations or items of the ZDM and CERME papers (except Douaire & Emprin, 2015) are located in the microspace; a priori they only request microspatial knowledge<sup>1</sup>. The existence and importance of mesospacial and macrospatial knowledge seem underestimated. For example in the paper of Van den Heuvel-Panhuizen et al. (2015) spatial apprehension of a picture (or a drawing, microspace) is considered as an apprehension of the evoked real world (meso- or macro-space).

2) Some papers try to isolate "basic" skills or "basic" items which could be predictive of spatial sense and serve for students assessment, analysing spatial situations (for example Van den Heuvel-Panhuizen et al., 2015) or geometrical problems (Duval 1999, 2006). On the other hand Berthelot & Salin, in coherence with their support framework, don't attempt to describe finely "the" spatial skills but ask students to solve real spatial problems in which spatial skills are at work.

3) Papadaki (2015) and Swoboda (2015) allow us to realize that gestures can be a powerful help to mentally construct geometrical images and facilitate visualization. Among the gesture the rotary motion (rotation) could play an important role.

4) Visualization is enriched with new entries as gestures and motion; it has gained increasing importance on the spatial skills. But what is meant by this term? Another inquiry to lead...

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<sup>1</sup> It is not always the case: situations with use of DGS often incite students to use mesospacial conception.

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