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Multi-directional local search for the leximax-VRP

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Most vehicle routing problems (VRPs) studied in the Operations Research literature deal with the design of a set of routes of minimal cost to serve a set of customers. In many practical cases, companies seek cost minimization as well as the optimization of criteria related to vehicles and drivers. In particular, preserving equity among drivers through a good balance of their workload is often sought. Historically, two main *equity measures*, have been proposed in the VRP literature. First, *min-max approaches*, such as proposed by Golden et al. [1], propose to minimize the length of the longest route. An unfortunate consequence of this approach is that all solutions with the same longest route length have the same equity measure. The second equity measure, recently designated by the term *range* [4], considers the difference between the longest route and the shortest route [3]. An important drawback of this measure is its non-monotonicity. As recently exposed by Halvorsen-Weare and Savelsbergh [2] or Matl et al [4] among other drawbacks: the *range* of a solution can be improved by increasing, even in an inconsistent way, the length of its shortest route of a solution without decreasing the length of the others.

As recently summarized by Ogryczak et al. [5] in a survey on *fair optimization and networks*, the lexicographic minimax approach has been used in domains such as network optimization, facility location and network optimization to produce equitable or fair solutions. The lexicographic minimax, based on the *leximax* operator, refines the min-max approach: informally speaking, when a minimal value has been found for the longest route, the lexicographic minimax considers the second longest route, the third longest route, and so on, until all ties have been broken.

In this presentation, we introduce the *leximax-VRP* as a refinement of the CVRP with

two objective functions: the sum of routing costs and the lexicographic minimax over routes durations.

1 Solution method

We propose to integrate the lexicographic minimax approach in a multi-objective optimization framework called Multi-Directional Local Search (MDLS) [7].

1.1 MDLS principle

MDLS offers a very simple local search framework but it still competes with state-of-the-art methods when solving multi-objective optimization problems. In MDLS, a local search LS_j is defined for each objective j . This local search is later performed in order to improve solutions with respect to objective j . A set of non-dominated solutions is kept in an archive and returned at the end of the algorithm. An iteration consists in (i) selecting a solution from the archive, (ii) performing local search on this solution for each objective/direction, thus producing a new feasible solution in each direction and (iii) updating the archive using newly produced solutions.

1.2 Local search components

In our algorithm, we consider that local search consists of one Large Neighborhood Search (LNS) iteration. Several ruin and recreate operators are defined for each objective. Hence, at each iteration, for each objective (i) a ruin and a recreate operator are randomly selected in the set of operators for that objective and (ii) a new solution is produced using the selected operators.

The *cost operators* are defined according to the classical LNS operators for the VRP [6]. The set of ruin operators that we use are: *random removal*, *worst removal*, *related removal* and *route removal*. The recreate operators for the cost objective are the *cheapest insertion heuristic* and the *k-regret heuristic* for $k = 2, 3, 4$.

We introduce *lexicographic minimax operators* which extend the classical operators to the lexicographical minimax approach. The ruin operators include the *random removal* and the *related removal* as well as the following two operators:

- *worst max removal*: at each iteration this operator removes, from the longest route, the customer that decreases the most the length of this route.
- *longest route removal*: at each iteration, all customers from the longest route are removed.

Two sets of recreate operators have been designed to guide the search towards lexicographic minimax efficient solutions:

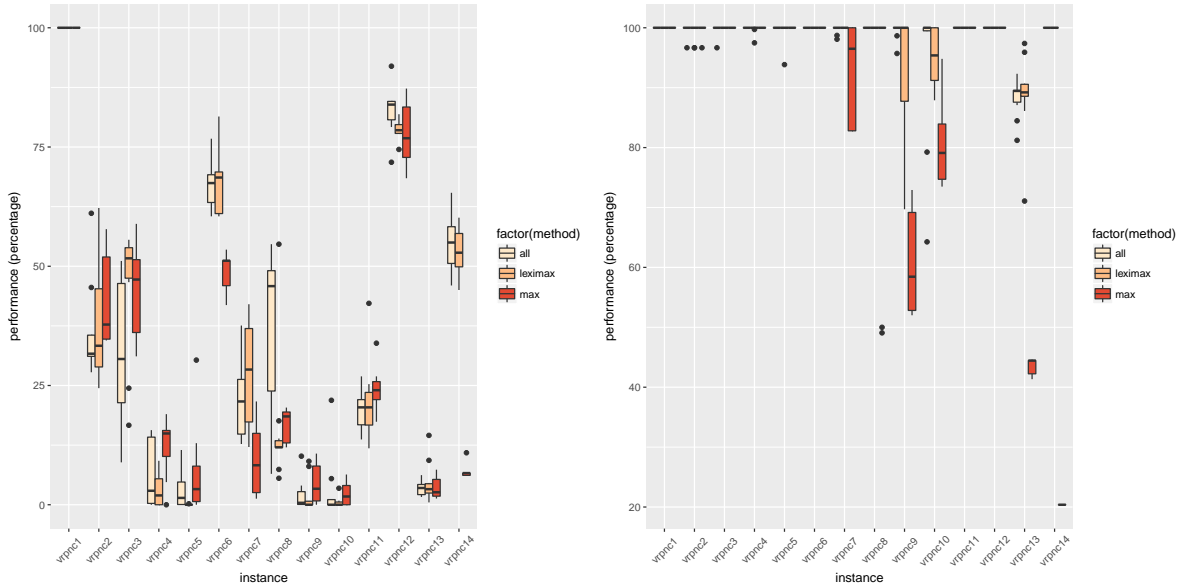


Figure 1: Boxplots charts for the number of solution of the reference front that have been found and within 2% distance for each run and each instance in one hour.

- The *leximax cheapest insertion* and *leximax k-regret* extend the cheapest insertion and k-regret heuristics to the lexicographic minimax approach.
- The *min-max cheapest insertion* and *min-max k-regret* extend cheapest insertion and k-regret, but using only the duration of the longest route as a criterion to guide the search and the solution cost increase to break ties.

As the lexicographic minimax heuristics involve sorting route vectors to compare solutions, our objective is to assess whether the faster min-max heuristics can be more efficient to guide the search towards lexicographic minimax solutions.

2 Experiments

To illustrate the experimental study in this abstract, we present the comparison of three configurations, which all include cost operators and lexicographic minimax ruin operators. Configuration *leximax* integrates the *leximax cheapest insertion* and *leximax k-regret*. Configuration *max* integrates *min-max cheapest insertion* and *min-max k-regret*. Configuration *all* includes all of these recreate operators for the lexicographic minimax objective.

Our MDLS is evaluated on the Christofides CVRP instances, which are traditionally used to benchmark the VRP with load balancing. 10 runs of 60 minutes are performed for each configuration and each instance and a reference set is constructed by taking the non-dominated union of the sets returned by each run for each configuration over all experiments. To evaluate configurations, we consider two indicators: the percentage of

solutions from the reference front found in a given run, and the percentage of solutions found which are within 2% of a solution from the reference front. A solution x_1 is within 2% of another solution x_2 if, when the cost and all route lengths of x_2 are multiplied by 1.02, then x_1 dominates this transformed solution. This information is summarized in the two plots from Figure 1. On the left plot, for each instance and each configuration, we represent the percentage of solutions of the reference front that are found on each run. The distribution of the performance evaluation of each run is displayed with a Box Plot. For the majority of instances, the percentage of solutions of the reference front found for each run remains quite low. Comparing the three configurations, it is not possible to clearly state that one dominates the other. The right plot shows the percentage of solutions of the reference front that lie within a 2% distance of a solution returned by each run. This plot shows that the returned approximation returned by the *all* and *leximax* configurations remains within close distance of the reference front. Comparing the various configurations, guiding the search with insertion criteria based on the longest route is less effective than with the proposed leximax based heuristics.

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