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RIGIDITY AND UNLIKELY INTERSECTIONS FOR FORMAL GROUPS

by

Laurent Berger

Abstract. — Let K be a p -adic field and let F and G be two formal groups over \mathcal{O}_K . We prove that if F and G have infinitely many torsion points in common, then $F = G$. This follows from a rigidity result: any bounded power series that sends infinitely many torsion points of F to torsion points of F is an endomorphism of F .

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Introduction

Let K be a finite extension of \mathbf{Q}_p (or, more generally, a finite extension of $W(k)[1/p]$ where k is a perfect field of characteristic p). Let \overline{K} be an algebraic closure of K and let \mathbf{C}_p be the p -adic completion of \overline{K} . Let \mathcal{O}_K denote the ring of integers of K , and let $F(X, Y) = X \oplus Y \in \mathcal{O}_K[[X, Y]]$ be a formal group law over \mathcal{O}_K . Let $\text{Tors}(F)$ be the set of torsion points of F in $\mathfrak{m}_{\mathbf{C}_p} = \{z \in \mathbf{C}_p, |z|_p < 1\}$. The question that motivates this paper is: to what extent is a formal group F determined by $\text{Tors}(F)$? Our main result is an “unlikely intersections” result.

Theorem A. — *If F and G are two formal groups over \mathcal{O}_K and if $\text{Tors}(F) \cap \text{Tors}(G)$ is infinite, then $F = G$.*

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If $n \geq 2$ and $[n](X)$ denotes the multiplication by n map on F , then $\text{Tors}(F)$ is also the set of preperiodic points of $[n](X)$ in $\mathfrak{m}_{\mathbf{C}_p}$. We can therefore think of $\text{Tors}(F)$ as the set $\text{Preper}(F)$ of preperiodic points of a p -adic dynamical system attached to F . Theorem A then becomes a statement about preperiodic points of certain dynamical systems.

Some analogues of theorem A are known in other contexts. For example, if two elliptic curves over $\overline{\mathbf{Q}}$ have infinitely many torsion points in common (in a suitable sense), then they are isomorphic (Bogomolov and Tschinkel, see §4 of [BT07]). In another context, if f and g are two rational fractions of degree at least 2 with coefficients in the complex numbers, and if $\text{Preper}(f) \cap \text{Preper}(g)$ is infinite, then $\text{Preper}(f) = \text{Preper}(g)$ (Baker and DeMarco, theorem 1.2 of [BD11]). In this case, f and g have the same Julia set (corollary 1.3 of *ibid.*). One can then show that, if f and g are polynomials of the same degree, then in most cases they are equal up to a linear symmetry that preserves their common Julia set (see for instance [BE87] and [SS95]).

Our proof of theorem A relies on a rigidity result for formal groups. We say that a subset $Z \subset \mathfrak{m}_{\mathbf{C}_p}^d$ is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$ if every power series $h(X_1, \dots, X_d) \in \mathcal{O}_K[[X_1, \dots, X_d]]$ that vanishes on Z is necessarily equal to zero. For example, if $d = 1$ then $Z \subset \mathfrak{m}_{\mathbf{C}_p}$ is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}$ if and only if it is infinite.

Theorem B. — *If F is a formal group over \mathcal{O}_K and if $h(X) \in X \cdot \mathcal{O}_K[[X]]$ is such that $h(z) \in \text{Tors}(F)$ for infinitely many $z \in \text{Tors}(F)$, then $h \in \text{End}(F)$.*

More generally, if $h(X_1, \dots, X_d) \in \mathcal{O}_K[[X_1, \dots, X_d]]$ is such that $h(0) = 0$ and $h(z) \in \text{Tors}(F)$ for all z in a subset of $\text{Tors}(F)^d$ that is Zariski dense in $\mathfrak{m}_{\mathbf{C}_p}^d$, then there exists $h_1, \dots, h_d \in \text{End}(F)$ such that $h = h_1(X_1) \oplus \dots \oplus h_d(X_d)$.

This theorem generalizes corollary 4.2 of Hida’s [Hid14], which concerns the case $F = \mathbf{G}_m$. Our proof uses ideas coming from the theory of p -adic dynamical systems (developed in large part by Lubin, see [Lub94]) rather than the “special subvarieties” argument of Hida (which is in the spirit of Chai’s [Cha08]). Other kinds of “unlikely intersections” results for certain formal groups can be found in [Ser18].

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1. Formal groups

For the basic definitions and results about formal groups that we need, we refer for instance to Lubin's [Lub64, Lub67]. Let $F(X, Y) = X \oplus Y \in \mathcal{O}_K[[X, Y]]$ be a formal group law over \mathcal{O}_K . If $n \in \mathbf{Z}$, let $[n](X)$ denote the multiplication by n map on F . More generally, if $a \in \mathcal{O}_K$, let $[a](X)$ be the unique endomorphism of F such that $[a]'(0) = a$ if it exists. Let $\text{Tors}(F)$ be the set of torsion points of F . If F is of finite height, then $\text{Tors}(F)$ is infinite, while if F is of infinite height, then $\text{Tors}(F)$ is finite and our results are vacuous. We therefore assume from now on that F is of finite height h .

Let $T_p F = \varprojlim_n F[p^n]$ be the Tate module of F . If F is of height h , then $T_p F$ is a free \mathbf{Z}_p -module of rank h , equipped with an action of $\text{Gal}(\overline{K}/K)$. If we choose a basis of $T_p F$, this gives a Galois representation $\rho_F : \text{Gal}(\overline{K}/K) \rightarrow \text{GL}_h(\mathbf{Q}_p)$. Let E be the fraction field of $\text{End}(F)$. It is a finite extension of \mathbf{Q}_p whose degree e divides h (theorem 2.3.2 of [Lub64]), so that we can view $\text{GL}_{h/e}(E)$ as a subgroup of $\text{GL}_h(\mathbf{Q}_p)$.

Theorem 1.1. — *The image of ρ_F is an open subgroup of a conjugate of $\text{GL}_{h/e}(E)$.*

Proof. — This is an unpublished theorem of Serre (see however the remark after theorem 5 on page 130 of [Ser67]), which is proved in [Sen73] (see theorem 3 on page 168 and the remark that follows). \square

Corollary 1.2. — *The image of ρ_F contains an open subgroup of $\mathbf{Z}_p^\times \cdot \text{Id}$.*

If $\sigma \in \text{Gal}(\overline{K}/K)$ is such that $\rho_F(\sigma) = a \cdot \text{Id}$, then $\sigma(z) = [a](z)$ for all $z \in \text{Tors}(F)$.

2. p -adic dynamical systems

In this §, we prove a number of results about power series that commute under composition (sometimes also called permutable power series). These results are all inspired by Lubin's theory of p -adic dynamical systems (see [Lub94]).

A power series $h(X) \in X \cdot K[[X]]$ is said to be stable if $h'(0)$ is neither 0 nor a root of unity. If $h'(0) \neq 0$, then there exists a unique power series $h^{\circ-1}(X) \in X \cdot K[[X]]$ such that $h \circ h^{\circ-1} = h^{\circ-1} \circ h = X$. If in addition $h(X) \in X \cdot \mathcal{O}_K[[X]]$ and $h'(0) \in \mathcal{O}_K^\times$, then $h^{\circ-1}(X) \in X \cdot \mathcal{O}_K[[X]]$.

Theorem 2.1. — *Let $u(X) \in X \cdot K[[X]]$ be a stable power series.*

A power series $h(X_1, \dots, X_d) \in K[[X_1, \dots, X_d]]$ such that $h(0) = 0$ and such that $h \circ u = u \circ h$ is determined by $\{dh/dX_i(0)\}_{1 \leq i \leq d}$.

Proof. — Suppose that $h^{(1)}$ and $h^{(2)}$ are two such power series, and that they coincide in degrees $\leq m$. Let h_m be the sum of the terms of $h^{(i)}$ of total degree $\leq m$. We have $h^{(i)} = h_m + r^{(i)}$ with $r^{(i)}$ of degree $\geq m + 1$, and

$$\begin{cases} (h_m + r^{(i)}) \circ u = h_m \circ u + r^{(i)} \circ u \equiv h_m \circ u + u'(0)^{m+1} r^{(i)} \pmod{\deg(m+2)}, \\ u \circ (h_m + r^{(i)}) \equiv u \circ h_m + r^{(i)} u'(h_m) \equiv u \circ h_m + r^{(i)} u'(0) \pmod{\deg(m+2)}. \end{cases}$$

Since $u'(0)^m \neq 1$, the fact that $h^{(i)} \circ u = u \circ h^{(i)}$ implies that

$$r^{(i)} \equiv \frac{h_m \circ u - u \circ h_m}{u'(0) - u'(0)^{m+1}} \pmod{\deg(m+2)}.$$

If $h^{(1)}$ and $h^{(2)}$ coincide in degrees $\leq m$, they therefore have to coincide in degrees $\leq m+1$. This implies the theorem by induction on m . \square

Let us say that an endomorphism of a formal group is stable if the corresponding power series is stable.

Corollary 2.2. — *Let F be a formal group and let u be a stable endomorphism of F . If $h(X) \in X \cdot \mathcal{O}_K[[X]]$ is such that $h \circ u = u \circ h$, then h is an endomorphism of F .*

Proof. — The power series $F \circ h$ and $h \circ F$ both commute with u , and have the same derivatives at 0, so that $F \circ h = h \circ F$ by theorem 2.1. \square

Corollary 2.3. — *If u is a stable endomorphism of a formal group and if $h(X_1, \dots, X_d) \in \mathcal{O}_K[[X_1, \dots, X_d]]$ is such that $h(0) = 0$ and $h \circ u = u \circ h$, then there exists $a_1, \dots, a_d \in \mathcal{O}_K$ such that $h(X_1, \dots, X_d) = [a_1](X_1) \oplus \dots \oplus [a_d](X_d)$.*

Proof. — Let $h_i(X)$ be the power series h evaluated at $X_i = X$ and $X_k = 0$ for $k \neq i$. We have $h_i \circ u = u \circ h_i$ and hence by corollary 2.2, $h_i(X) = [a_i](X)$ where $a_i = h'_i(0) \in \mathcal{O}_K$. The two power series $h(X_1, \dots, X_d)$ and $[a_1](X_1) \oplus \dots \oplus [a_d](X_d)$ commute with u and have the same derivatives at 0, so that they are equal by theorem 2.1. \square

3. Rigidity and unlikely intersections

We first recall and prove theorem B.

Theorem 3.1. — *If F is a formal group over \mathcal{O}_K and if $h(X_1, \dots, X_d) \in \mathcal{O}_K[[X_1, \dots, X_d]]$ is such that $h(0) = 0$ and $h(z) \in \text{Tors}(F)$ for all z in a subset Z of $\text{Tors}(F)^d$ that is Zariski dense in $\mathfrak{m}_{\mathbb{C}_p}^d$, then there exists $h_1, \dots, h_d \in \text{End}(F)$ such that $h = h_1(X_1) \oplus \dots \oplus h_d(X_d)$.*

Proof. — Since $\text{Tors}(F)$ is infinite, F is of finite height. By corollary 1.2, there exists $\sigma \in \text{Gal}(\overline{K}/K)$ and a stable endomorphism u of F such that $\sigma(z) = u(z)$ for all $z \in \text{Tors}(F)$. If $z \in Z$, then we have $\sigma(h(z)) = u(h(z))$ as well as $\sigma(h(z)) = h(\sigma(z)) = h(u(z))$. The power series $u \circ h - h \circ u$ therefore vanishes on Z . Since Z is Zariski dense in $\mathfrak{m}_{\mathbb{C}_p}^d$, we have $u \circ h = h \circ u$. The theorem now follows from corollary 2.3. \square

Remark 3.2. — If Y_1, \dots, Y_d are infinite subsets of $\text{Tors}(F)$, then $Y_1 \times \dots \times Y_d$ is Zariski dense in $\mathfrak{m}_{\mathbb{C}_p}^d$.

We now recall and prove theorem A.

Theorem 3.3. — *If F and G are two formal groups over \mathcal{O}_K and if $\text{Tors}(F) \cap \text{Tors}(G)$ is infinite, then $F = G$.*

Proof. — By corollary 1.2, there exists an element $\sigma \in \text{Gal}(\overline{K}/K)$ and a stable endomorphism u of F such that $\sigma(z) = u(z)$ for all $z \in \text{Tors}(F)$. The set $\Lambda = \text{Tors}(F) \cap \text{Tors}(G)$ is stable under the action of $\text{Gal}(\overline{K}/K)$. If $z \in \Lambda$, we therefore have $\sigma(z) \in \Lambda$ and hence $u(z) \in \text{Tors}(G)$ for all $z \in \Lambda$, since $u(z) = \sigma(z)$. By theorem B applied to G , we get that $u \in \text{End}(G)$. The power series F and G commute with u and have the same linear terms, hence $F = G$ by theorem 2.1. \square

4. Generalizations and perspectives

4.1. Universal bounds. — In §4 of [BT07], Bogomolov and Tschinkel prove that two nonisomorphic elliptic curves over $\overline{\mathbb{Q}}$ have only finitely many torsion points in common. In [BFT18], the authors raise the question of the existence of a universal bound for the maximum number of torsion points that two nonisomorphic elliptic curves over $\overline{\mathbb{Q}}$ (or even over the complex numbers) can share. The same kind of question is raised, for preperiodic points of rational fractions, in the forthcoming paper [DKY].

The following proposition shows that in our situation, there is no straightforward refinement of theorem A.

Proposition 4.1. — *For all $m \geq 1$, there exists a formal group F over \mathbf{Z}_p , of height 1, such that F is not isomorphic to \mathbf{G}_m but such that $\text{Tors}(F) \cap \text{Tors}(\mathbf{G}_m)$ contains at least m points.*

Proof. — Take $n \geq 1$ and let $q(X) = (1 + X)^p - 1$ and $u(X) = 1 + ((1 + X)^{p^n} - 1)/X$ and $f(X) = u(X)q(X)$. We have $f(X) = p(1 + p^n)X + O(X^2)$ and $f(X) \equiv X^p \pmod{p}$.

By Lubin-Tate theory (see §1 of [LT65]) there exists a formal group F such that $[p(1 + p^n)](X) = f(X)$. This group is attached to the uniformizer $p(1 + p^n)$ of \mathbf{Q}_p . Likewise, \mathbf{G}_m is attached to p . The formal group F is not isomorphic to \mathbf{G}_m over \mathbf{Q}_p as $p \neq p(1 + p^n)$ and any Lubin-Tate group attached to a uniformizer π determines π .

However, we have $f(\zeta_p - 1) = 0$ and $f(\zeta_{p^k} - 1) = \zeta_{p^{k-1}} - 1$ for all $k \leq n$, so that $\zeta_{p^k} - 1 \in \text{Tors}(F)$ for all $k \leq n$. This proves the proposition. \square

If $\text{Tors}(F) \cap \text{Tors}(G)$ is large, then are F and G close to each other in some sense?

4.2. The logarithm of a formal group. — Using the logarithms of formal groups, we can give a very short proof of a weaker form of theorem A, namely: if $\text{Tors}(F) = \text{Tors}(G)$ (and this common set is infinite), then $F = G$. Indeed, Log_F is holomorphic on $\mathfrak{m}_{\mathbf{C}_p}$ and its zeroes are precisely the elements of $\text{Tors}(F)$, with multiplicity 1. In addition, Log'_F is a bounded power series since $d\text{Log}_F$ is the normalized invariant differential on F . If $\text{Tors}(F) = \text{Tors}(G)$, then Log_F and Log_G have the same zeroes, so that they differ by a unit u . A unit is necessarily bounded. We have $\text{Log}_G = u \cdot \text{Log}_F$ and hence $\text{Log}'_G = u \cdot \text{Log}'_F + u' \cdot \text{Log}_F$. Since Log'_G and Log'_F and u are bounded, but not Log_F , we must have $u' = 0$ (the sup norms $\|\cdot\|_r$ on circles are multiplicative). This implies that $u \in \mathcal{O}_K^\times$ and then that $u = 1$ since $\text{Log}'_F(0) = \text{Log}'_G(0) = 1$, so that $\text{Log}_F = \text{Log}_G$ and $F = G$. The same argument gives the following characterization of the logarithm of a formal group of finite height.

Proposition 4.2. — *If F is a formal group of finite height, then the power series Log_F is the unique element of $X + X^2 \cdot K[[X]]$ that is holomorphic on $\mathfrak{m}_{\mathbf{C}_p}$, whose zero set is precisely $\text{Tors}(F)$, with multiplicity 1, and whose derivative is bounded.*

4.3. More rigidity. — A common generalization of theorems A and B would be the assertion that if a power series h maps infinitely many torsion points of F to torsion points of G , then $h \in \text{Hom}(F, G)$. In order to prove this using the same method as in the proof of theorem B, we would need to show that there exists $\sigma \in \text{Gal}(\overline{K}/K)$ that acts on $\text{Tors}(F)$ and $\text{Tors}(G)$ by two power series u_F and u_G , satisfying some stability condition. If G is a Lubin-Tate formal group (for some finite extension of \mathbf{Q}_p contained in K), there is a character $\chi_G : \text{Gal}(\overline{K}/K) \rightarrow \mathcal{O}_K^\times$ such that $\sigma(z) = [\chi_G(\sigma)](z)$ for all $z \in \text{Tors}(G)$ (theorem 2 of [LT65]).

Theorem 4.3. — *If F is a formal group and G is a Lubin-Tate formal group, both defined over \mathcal{O}_K , and if $h(X) \in X \cdot \mathcal{O}_K[[X]]$ is such that $h'(0) \neq 0$ and $h(z) \in \text{Tors}(G)$ for infinitely many $z \in \text{Tors}(F)$, then $h \in \text{Hom}(F, G)$.*

Proof. — Since $\text{Tors}(F)$ is infinite, F is of finite height. By corollary 1.2, there exists an element $\sigma \in \text{Gal}(\overline{K}/K)$ and a stable endomorphism u_F of F such that $\sigma(z_F) = u_F(z_F)$ if $z_F \in \text{Tors}(F)$. Let $u_G(X) = [\chi_G(\sigma)](X)$, so that $\sigma(z_G) = u_G(z_G)$ if $z_G \in \text{Tors}(G)$.

If $z \in \text{Tors}(F)$ is such that $h(z) \in \text{Tors}(G)$, then $\sigma(h(z)) = u_G(h(z))$ and $\sigma(h(z)) = h(\sigma(z)) = h(u_F(z))$. The power series $u_G \circ h - h \circ u_F$ therefore vanishes at infinitely many points of $\mathfrak{m}_{\mathbb{C}_p}$, so that $u_G \circ h = h \circ u_F$. Since $h'(0) \neq 0$, we have $u'_F(0) = u'_G(0)$ and u_G is stable. The theorem now follows from lemma 4.4 below. \square

Lemma 4.4. — *Let F and G be two formal groups and let f and g be endomorphisms of F and G , with g stable. If $h(X) \in X \cdot \mathcal{O}_K[[X]]$ is such that $h'(0) \neq 0$ and $h \circ f = g \circ h$, then $h \in \text{Hom}(F, G)$.*

Proof. — Consider the power series $K(X, Y) = h \circ F(h^{\circ-1}(X), h^{\circ-1}(Y))$. We have

$$K \circ g = h \circ F \circ h^{\circ-1} \circ g = h \circ F \circ f \circ h^{\circ-1} = h \circ f \circ F \circ h^{\circ-1} = g \circ h \circ F \circ h^{\circ-1} = g \circ K$$

Since K and G commute with g and have the same derivatives at 0, we have $K = G$ by theorem 2.1 and hence $h \circ F = G \circ h$, so that $h \in \text{Hom}(F, G)$. \square

Note that the hypothesis of the lemma imply that $f'(0) = g'(0)$ so that if one series is stable, then both are.

4.4. Homotheties and stable p -adic dynamical systems. — If F is a formal group of finite height, then $\text{End}(F)$ is a set of power series that commute with each other under composition. One can forget about the formal group and study certain sets \mathcal{D} of elements of $X \cdot \mathcal{O}_K[[X]]$ that commute with each other under composition. This is the object of Lubin's theory of p -adic dynamical systems (see [Lub94]).

Let us say that $\mathcal{D} \subset X \cdot \mathcal{O}_K[[X]]$ is a stable p -adic dynamical system of finite height if the elements of \mathcal{D} commute with each other under composition, and if \mathcal{D} contains a stable series f such that $f'(0) \in \mathfrak{m}_K$ and $f(X) \not\equiv 0 \pmod{\mathfrak{m}_K}$ (i.e. f is of finite height) as well as a stable series u such that $u'(0) \in \mathcal{O}_K^\times$. We can then assume that \mathcal{D} is as large as possible, namely that any power series $g \in X \cdot \mathcal{O}_K[[X]]$ that commutes with the elements of \mathcal{D} belongs to \mathcal{D} . For example, if F is a formal group of finite height, then $\text{End}(F)$ is a stable p -adic dynamical system.

Given a stable p -adic dynamical system of finite height \mathcal{D} , the set $\text{Preper}(g)$ is independent of the choice of a stable $g \in \mathcal{D}$ (see §3 of [Lub94]). One can then define $\text{Preper}(\mathcal{D})$ as the preperiodic set of any stable element of \mathcal{D} . To what extent does $\text{Preper}(\mathcal{D})$ determine a stable p -adic dynamical system of finite height \mathcal{D} ?

In order to extend our results from formal groups to stable p -adic dynamical systems of finite height, we can ask whether the consequence of corollary 1.2 holds in more generality: for which stable p -adic dynamical systems of finite height \mathcal{D} is there a stable power series $w \in \mathcal{D}$ and an element $\sigma \in \text{Gal}(\overline{K}/K)$ such that $\sigma(z) = w(z)$ for all $z \in \text{Preper}(\mathcal{D})$?

References

- [BD11] M. BAKER & L. DEMARCO – “Preperiodic points and unlikely intersections”, *Duke Math. J.* **159** (2011), no. 1, p. 1–29.
- [BE87] I. N. BAKER & A. ERĚMENKO – “A problem on Julia sets”, *Ann. Acad. Sci. Fenn. Ser. A I Math.* **12** (1987), no. 2, p. 229–236.
- [BFT18] F. BOGOMOLOV, H. FU & Y. TSCHINKEL – “Torsion of elliptic curves and unlikely intersections”, in *Geometry and Physics: Volume I. A Festschrift in honour of Nigel Hitchin*, Oxford University Press, 2018, p. 19–38.
- [BT07] F. BOGOMOLOV & Y. TSCHINKEL – “Algebraic varieties over small fields”, in *Diophantine geometry*, CRM Series, vol. 4, Ed. Norm., Pisa, 2007, p. 73–91.
- [Cha08] C.-L. CHAI – “A rigidity result for p -divisible formal groups”, *Asian J. Math.* **12** (2008), no. 2, p. 193–202.
- [DKY] L. DEMARCO, H. KRIEGER & H. YE – “Lower bounds on dynamical height pairings”, in preparation.
- [Hid14] H. HIDA – “Hecke fields of Hilbert modular analytic families”, in *Automorphic forms and related geometry: assessing the legacy of I. I. Piatetski-Shapiro*, Contemp. Math., vol. 614, Amer. Math. Soc., Providence, RI, 2014, p. 97–137.
- [LT65] J. LUBIN & J. TATE – “Formal complex multiplication in local fields”, *Ann. of Math. (2)* **81** (1965), p. 380–387.
- [Lub64] J. LUBIN – “One-parameter formal Lie groups over p -adic integer rings”, *Ann. of Math. (2)* **80** (1964), p. 464–484.
- [Lub67] ———, “Finite subgroups and isogenies of one-parameter formal Lie groups”, *Ann. of Math. (2)* **85** (1967), p. 296–302.
- [Lub94] ———, “Nonarchimedean dynamical systems”, *Compositio Math.* **94** (1994), no. 3, p. 321–346.
- [Sen73] S. SEN – “Lie algebras of Galois groups arising from Hodge-Tate modules”, *Ann. of Math. (2)* **97** (1973), p. 160–170.
- [Ser67] J.-P. SERRE – “Sur les groupes de Galois attachés aux groupes p -divisibles”, in *Proc. Conf. Local Fields (Driebergen, 1966)*, Springer, Berlin, 1967, p. 118–131.
- [Ser18] V. SERBAN – “An infinitesimal p -adic multiplicative Manin-Mumford conjecture”, *J. Théor. Nombres Bordeaux* **30** (2018), no. 2, p. 393–408.
- [SS95] W. SCHMIDT & N. STEINMETZ – “The polynomials associated with a Julia set”, *Bull. London Math. Soc.* **27** (1995), no. 3, p. 239–241.

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