Demonstration of Riemann's Hypothesis

Yao Maxime Kayi

Demonstration of Riemann's Hypothesis

Yao Maxime KAYI

Abstract—.

Index Terms— complex, Riemann Hypothesis, zeta

1 Introduction

The Riemann hypothesis is one of the most important Hilbert mathematical problems. It concerns the non-trivial zeros of zeta function in a complex plane. He announced that the real part of non-trivial zeros of zeta function is exactly equal to *0.5*.

We will prove that his hypothesis is true using complex numbers properties.

2 Functional Equation

We won't demonstrate the functional equation but use his result. With Euler's function Γ we get:

$$\pi^{\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s)=\pi^{\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)$$

Through the functional, if s is a non-trivial zero of zeta then 1-s is also a non-trivial zero.

3 Demonstration

A simple demonstration of Riemann hypothesis. It is based on the fact that is a complex number $a+ib$ is equal to null then $a-ib$, is also equal to null.

3.1 Generalization

Let $s=x+iy$ be a zero of the zeta function then we have:

$$\zeta(s)=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))-i.\sin(y\log(n))\right)=0$$

We have:

$$\begin{cases}\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))\right)=0\\ \sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(-i.\sin(y\log(n))\right)=i.0\end{cases}\Rightarrow\begin{cases}\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))\right)=0\\ -\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(i.\sin(y\log(n))\right)=i.0\end{cases}$$

$$\begin{cases}\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))\right)=0\\ -\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(i.\sin(y\log(n))\right)=i.0\end{cases}\Rightarrow\begin{cases}\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))\right)=0\\ \sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(i.\sin(y\log(n))\right)=i.0\end{cases}$$

Now we sum lines obtained after simplification on both sides of equalities. Then we have:

$$\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))+i.\sin(y\log(n))\right)=0$$

And:

$$\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}e^{iy\log(n)}=0$$

$$\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}e^{-(-iy\log(n))}=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(e^{-iy\log(n)}\right)^{-1}=0$$

$$\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\left(\frac{1}{n}\right)^{iy}\right)^{-1}=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\frac{1}{n}\right)^{-iy}=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x-iy}=0$$

Then we get $\zeta(\bar{s})=0$. We derive the following propositions.

Proposition 1. If s is a non-trivial (resp. trivial) zeta function zero then $\bar{s}$ is also a zero (resp. trivial).

Proposition 2. If $\bar{s}$ is a non-trivial (resp. trivial) zeta function zero then s is also a zero (resp. trivial).

Proposition 3. Let s be a complex number not equal to 1. Then $\zeta(s)=0\Leftrightarrow\zeta(\bar{s})=0$

Proof. This a consequence of the first and the second proposition.

3.2 Non Trivial Zero

Let $s=x+iy$ be a non-trivial zero of the zeta function then we have:

$$\zeta(s)=\zeta(1-s)=\zeta(\bar{s})=0\ (\mathrm{R})$$

Let us find conditions that satisfies s from (R).

Then $\zeta(1-s)=\zeta(\bar{s})$ if only:

$$\begin{cases}\sum_{n\geq 1}\left(\frac{1}{n}\right)^{1-x}\left(\cos(y\log(n))\right)=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\cos(y\log(n))\right)\\ \sum_{n\geq 1}\left(\frac{1}{n}\right)^{1-x}\left(\sin(y\log(n))\right)=\sum_{n\geq 1}\left(\frac{1}{n}\right)^{x}\left(\sin(y\log(n))\right)\end{cases}$$

Which gives us:

$$\left(\frac{1}{n}\right)^{1-x}=\left(\frac{1}{n}\right)^{x},\forall n\geq 1$$

Then $x=1/2$. The relation (R) established is verified (i.e. we get the three equalization) if only the real part of s is equal to ½.

We get the following theorem.

Theorem 1. *Let* s *be a complex number not equal to 1. Then if s is a non-trivial zero of zeta function then his real part is exactly equal to ½.*

With Theorem 1 we show that, Riemann hypothesis is true.

4 CONCLUSION

.A great step has been done. The demonstration is simple but not evident also for me.