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Demonstration of Riemann's Hypothesis

Yao Maxime KAYI

Abstract—.

Index Terms— complex, Riemann Hypothesis, zeta



1 INTRODUCTION

The Riemann hypothesis is one of the most important Hilbert mathematical problems. It concerns the non-trivial zeros of zeta function in a complex plane. He announced that the real part of non-trivial zeros of zeta function is exactly equal to 0.5.

We will prove that his hypothesis is true using complex numbers properties.

2 FUNCTIONAL EQUATION

We won't demonstrate the functional equation but use his result. With Euler's function Γ we get:

$$\pi^{\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s) = \pi^{\frac{1-s}{2}} \Gamma\left(\frac{1-s}{2}\right) \zeta(1-s)$$

Through the functional, if s is a non-trivial zero of zeta then $1-s$ is also a non-trivial zero.

3 DEMONSTRATION

A simple demonstration of Riemann hypothesis. It is based on the fact that is a complex number $a + ib$ is equal to null then $a - ib$, is also equal to null.

3.1 Generalization

Let $s = x + iy$ be a zero of the zeta function then we have:

$$\zeta(s) = \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n)) - i \sin(y \log(n))) = 0$$

We have:

$$\begin{cases} \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n))) = 0 \\ \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (-i \sin(y \log(n))) = i \cdot 0 \end{cases} \Rightarrow \begin{cases} \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n))) = 0 \\ -\sum_{n \geq 1} \left(\frac{1}{n}\right)^x (i \sin(y \log(n))) = i \cdot 0 \end{cases}$$

$$\begin{cases} \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n))) = 0 \\ -\sum_{n \geq 1} \left(\frac{1}{n}\right)^x (i \sin(y \log(n))) = i \cdot 0 \end{cases} \Rightarrow \begin{cases} \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n))) = 0 \\ \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (i \sin(y \log(n))) = i \cdot 0 \end{cases}$$

Now we sum lines obtained after simplification on both sides of equalities. Then we have:

$$\sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n)) + i \sin(y \log(n))) = 0$$

And:

$$\sum_{n \geq 1} \left(\frac{1}{n}\right)^x e^{iy \log(n)} = 0$$

$$\sum_{n \geq 1} \left(\frac{1}{n}\right)^x e^{-iy \log(n)} = \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (e^{-iy \log(n)})^{-1} = 0$$

$$\sum_{n \geq 1} \left(\frac{1}{n}\right)^x \left(\left(\frac{1}{n}\right)^{iy}\right)^{-1} = \sum_{n \geq 1} \left(\frac{1}{n}\right)^x \left(\frac{1}{n}\right)^{-iy} = \sum_{n \geq 1} \left(\frac{1}{n}\right)^{x-iy} = 0$$

Then we get $\zeta(\bar{s}) = 0$. We derive the following propositions.

Proposition 1. If s is a non-trivial (resp. trivial) zeta function zero then \bar{s} is also a zero (resp. trivial).

Proposition 2. If \bar{s} is a non-trivial (resp. trivial) zeta function zero then s is also a zero (resp. trivial).

Proposition 3. Let s be a complex number not equal to 1. Then $\zeta(s) = 0 \Leftrightarrow \zeta(\bar{s}) = 0$

Proof. This a consequence of the first and the second proposition.

3.2 Non Trivial Zero

Let $s = x + iy$ be a non-trivial zero of the zeta function then we have:

$$\zeta(s) = \zeta(1-s) = \zeta(\bar{s}) = 0 \text{ (R)}$$

Let us find conditions that satisfies s from (R).

Then $\zeta(1-s) = \zeta(s)$ if only:

$$\begin{cases} \sum_{n \geq 1} \left(\frac{1}{n}\right)^{1-x} (\cos(y \log(n))) = \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\cos(y \log(n))) \\ \sum_{n \geq 1} \left(\frac{1}{n}\right)^{1-x} (\sin(y \log(n))) = \sum_{n \geq 1} \left(\frac{1}{n}\right)^x (\sin(y \log(n))) \end{cases}$$

Which gives us:

$$\left(\frac{1}{n}\right)^{1-x} = \left(\frac{1}{n}\right)^x, \forall n \geq 1$$

Then $x = 1/2$. The relation (R) established is verified (i.e. we get the three equalization) if only the real part of s is equal to $1/2$.

We get the following theorem.

Theorem 1. *Let s be a complex number not equal to 1. Then if s is a non-trivial zero of zeta function then his real part is exactly equal to $\frac{1}{2}$.*

With Theorem 1 we show that, Riemann hypothesis is true.

4 CONCLUSION

.A great step has been done. The demonstration is simple but not evident also for me.