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HAL Id: hal-01922786
https://hal.science/hal-01922786
Submitted on 14 Nov 2018
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ABSTRACT

This work investigates the possibility of predicting the actual flow-induced vibration of a panel from a measurement of its sensitivity functions and a priori knowledge of the auto-spectral and cross-spectral density functions defining the excitation. The method takes advantage of an explicit separation in the wave number domain of the structure behavior (defined by the sensitivity functions) and of the random excitation characteristics (defined by the spectral density functions). The validity of the approach was previously confirmed for the case of an isotropic aluminium panel with controlled simply-supported boundary conditions, and excited by a diffuse acoustic field and a turbulent boundary layer.

To verify the robustness of the methodology, it is here applied on two plane panels with different aspect ratios, both made of a composite material representative of those used in interior trim panels. A blind approach is followed, in the sense that the mechanical properties of the panels are voluntarily kept unknown, their boundary conditions are not especially controlled and the test sequence is executed step by step without any intermediate verification. Tests are conducted in a wind tunnel facility, with two flow speeds considered. A direct vibration measurement under turbulent flow excitation is first conducted using an accelerometer fixed at a given point on each panel. The vibratory response of the panel when excited by the flow is then deduced at the same point, now using the measured wall-pressure fluctuations statistics that are combined with the sensitivity functions. The predicted vibration velocity autospectra using the proposed method are in very good agreement with those directly measured whatever the considered panel or flow velocity.
INTRODUCTION

As already stated by Lyon [1] in 1967, engineering methods have now been tested for decades 'to anticipate the dynamic environment of a flight vehicle, to predict the vehicle response to this environment, and to simulate the expected environment in the laboratory'. In-flight tests appear at first to be an ideal solution, but such tests are seldom conducted [2] since they are costly, time-consuming and usually hard to control. The simulation of the expected environment (i.e. the simulation of the external load) has been thus mainly conducted using wind tunnels, but they mostly share the same limitations that are encountered for flight tests, albeit to a lesser extent. Numerical simulations can help improving the prediction of structural response to this excitation [3], but the need of experimental validation tools still remains.

For these reasons, several studies have concerned the vibration or acoustic response testing of panels under a simulated turbulent boundary layer excitation in laboratory in order to replace in-flight or wind-tunnel tests. In a few rare cases, the possibility of mimicking flow-induced vibration by shaker-induced vibration has been studied [1, 4]. Most of the research conducted in order to reproduce a turbulent boundary layer excitation in laboratory conditions has considered either an array of loudspeakers or a source scanning approach, then combined with different sound field reproduction techniques [5–7]. In this case, the reproduction of flow-induced sound or vibration has shown to be mostly limited to the acoustic domain of the excitation, therefore reducing the representativeness of these approaches.

This work investigates the possibility of predicting the actual flow-induced vibration of a panel from a measurement of its sensitivity functions and a priori knowledge of the cross-spectral density function of the excitation. The method was previously validated for the case of a panel excited by a diffuse acoustic field [8]. The sensitivity functions are defined as the panel response to a set of wall-pressure acoustic plane waves. Since such a direct test is hard to realize in practice, a reciprocal measurement is used to characterize the structural response by exciting the panel with a normal force at the point of interest and measuring its spatial velocity response using a scanning laser vibrometer. The acoustic response (radiated pressure, acoustic intensity) can be characterized as well by exciting the panel with a monopole and a dipole source [8]. Indeed, the reciprocity principle states that the sensitivity functions at any point on the structure or in the acoustic medium are equivalent to the panel velocity response expressed in the wavenumber domain when the system is excited at the point of interest by specific elementary sources. The cross-spectral density function defining the excitation can be either be defined using a measurement or a model (like the sinc function for the case of a diffuse acoustic field [8], or the model of Mellen [9] for the case of a turbulent boundary layer). In the present case, a direct measurement of the wall-pressure fluctuations generated on a flat plate in a wind-tunnel is conducted using a dedicated microphone array. The autospectral density, the exponential decay rates in the flow and transverse direction and the convection velocity are all extracted from this measurement. Using this data set that characterize the wall-pressure fluctuations, the proposed methodology was validated on an academic test case (an isotropic aluminum plate with simply supported boundary conditions) in [10].

For complete validation purposes, this method is confronted to direct vibration measurements in an anechoic wind tunnel for two different panels with unknown properties and arbitrary boundary conditions. The two panels have different aspect ratios, and two flow speeds are considered (20 and 40 m/s). Using the measured wall-pressure fluctuations and the estimated sensitivity functions, the vibratory response of the panel when excited by a turbulent boundary layer is deduced at a point of interest for both panels. The predicted vibration velocity autospectra using the proposed method are in very good agreement with those directly measured whatever the considered panel or flow velocity. The method thus provides a convenient tool to characterize the vibroacoustic response of a panel under any random excitation, provided the sensitivity functions and the cross-spectral density function of the excitation are a priori known.
2 ESTIMATION OF PANELS VIBRATION RESPONSE UNDER A TURBULENT BOUNDARY LAYER EXCITATION BASED ON THE RECIPROCITY PRINCIPLE

As illustrated in Fig. 1, a baffled rectangular panel of surface Σ_p with arbitrary boundary conditions is considered in this study. It is excited on one of its sides by a fully developed TBL with a free flow velocity \( U_\infty \) outside the boundary layer. This excitation is considered stationary in time and spatially homogeneous. Let us define \( \mathbf{x} = (x, y) \) an observation point and \( \mathbf{\tilde{x}} = (\tilde{x}, \tilde{y}) \) an excitation point (where the surface pressure fluctuation induced by the TBL is prescribed). Both points are defined in a Cartesian coordinate system \((x, y, z)\) with its origin at the center of the panel, as shown in Fig. 1, and are located on the panel surface \((z = 0)\).

Figure 1. Illustration of a baffled panel (gray line) excited by a TBL and coordinate system [8].

The vibration response of the panel under this excitation is characterized by the one-sided vibration velocity spectrum \( v(x, f) \) at point \( x \), where \( f \) is the frequency (considered positive valued). As the excitation is random, this quantity is derived from the normal velocity auto-spectral density (ASD) function \( G_{vv}(x, f) \). An approach for evaluating this quantity based on the reciprocity principle as been thoroughly presented, and validated numerically and experimentally for the case of a diffuse acoustic field excitation in [8]. It has also been validated for the case of a TBL excitation on an academic test case in [10] (an isotropic aluminum panel with simply-supported boundary conditions [11]). The aim of this paper is to apply this methodology on two plane panels made of a composite material commonly used in interior trim panels. The panels have different aspect ratios and their mechanical properties are voluntarily kept unknown. The methodology is briefly summarized in the next section, along with a short description of the considered TBL excitation.

2.1 Vibration response of a panel under random pressure field

The vibration response of a panel excited by a random pressure field can be described by the auto spectral density (ASD) function of the velocity \( v \). The one-sided frequency ASD function of the velocity \( G_{vv}(x, f) \) at point \( x \) can be expressed as a discrete integral in the wavenumber domain [8]:

\[
G_{vv}(x, f) \approx \frac{1}{4\pi^2} \sum_{k \in \Omega_k} |H_v(x, k, f)|^2 G_{p_b p_b}(k, f) \delta k,
\]

where

\[
H_v(x, k, f) = \int_{\Sigma_p} H_v/F_n(x, \tilde{x}, f) e^{-j\mathbf{k} \cdot \mathbf{\tilde{x}}} d\mathbf{\tilde{x}},
\]

\( k = (k_x, k_y) \) is the wavevector defined in the plane \((x, y)\), \( \delta k \) represents the wavenumber resolution and \( \Omega_k \) is the finite wavenumber domain over which the discrete integration is performed. The function \( G_{p_b p_b}(k, f) \) corresponds to the cross spectral density (CSD) function of the wall-pressure field on the excitation side (for instance a TBL excitation). The term \( H_v/F_n(x, \tilde{x}, f) \) corresponds to the transfer function between the panel velocity \( v \) at point \( x \) and a normal point force \( F_n \) applied
at point \( \tilde{x} \). The \( H_v(x, k, f) \) function is called the sensitivity function [12] and characterize the vibration behavior of the panel. It can be determined using the reciprocity principle which, for the particular case of a normal force applied at point \( \tilde{x} \) and the normal vibration velocity observed at point \( x \), can be translated following the previous notations into [13]

\[
H_{v/F_n}(x, \tilde{x}, f) = H_{v/F_n}(\tilde{x}, x, f).
\]  (3)

Introducing Eq. (3) in Eq. (2) one obtains

\[
H_v(x, k, f) = \iint_{\Sigma_p} H_{v/F_n}(\tilde{x}, x, f)e^{-jk\tilde{x}} d\tilde{x}.
\]  (4)

The right hand side of Eq. (4) can be interpreted as the space-wavenumber transform of \( H_{v/F_n}(\tilde{x}, x, f) \) with respect to the space variable \( \tilde{x} \). The points \( \tilde{x} \) become observation points on the panel surface \( \Sigma_p \), which means that the space-wavenumber transform is performed over the vibration velocity field of the panel. To sum up, the sensitivity function \( H_v(x, k, f) \) may be obtained by exciting the panel with a normal effort \( F_n \) at point \( x \) and by calculating the space-wavenumber transform of the transfer function between the panel velocity at the observation points and the applied force (as illustrated in Fig. 2).

![Figure 2. Determination of the sensitivity functions \( H_v \) using the reciprocity principle.](image)

\[
H_v(x, k) = 2D - DFT[v(\tilde{x})]
\]

In practice, the vibration field has to be measured on a regular grid of points denoted \( \Gamma_{\tilde{x}} \), using a scanning laser vibrometer for example. The space-wavenumber transform is therefore approximated by a 2D discrete Fourier transform (2D-DFT). In order to avoid aliasing effects, the spatial resolution \( \delta \tilde{x} \) over \( \Gamma_{\tilde{x}} \) should be determined so that the spatial variations of the vibration field can be correctly represented by the grid of points. For a complex panel with unknown mechanical properties, the mesh should be refined as much as possible to avoid aliasing effects. The panel is known to have a filtering effect on the excitation [14] which can help defining the wavenumber domain \( \Omega_k \) providing the panel vibration behaviour is known [10]. As the properties of the panel are unknown, the wavenumber domain should be extended as much as possible to minimize errors linked to truncation effects in the wavenumber domain. The limits of the wavenumber domain \( \Omega_k \) are, therefore, defined by the maximum wavenumber that can be resolved in the determination of the sensitivity functions.

### 2.2 Description of the excitation

In addition to the knowledge of the vibration behavior of the panel through the sensitivity functions, solving Eq. (1) requires that CSD functions of the blocked wall-pressure of the excitation are known in the wavenumber domain. Over the past few years, numerous studies have shown that the coherent power of the wall-pressure fluctuations induced by a TBL decays exponentially with the increasing separation distances along flow and transverse directions [15]. It has also been shown that the phase of the cross-spectrum is directly related to the convection wavenumber \( k_c = \omega/U_c \),
where \( \omega \) is the angular frequency and \( U_c \) is the convection speed (usually defined as a constant fraction of the free flow velocity \( U_\infty \)). These dependencies are included in most of the semi-empirical models [16, 17] aiming at predicting the CSD functions of the wall-pressure fluctuations.

In order to validate the proposed methodology in comparison to actual measurements of the panel response in an anechoic wind tunnel, the wall-pressure fluctuations induced by a subsonic turbulent flow generated in a low-speed anechoic wind tunnel have been measured using a spiral-shaped microphone array [18]. These measurements are then used to fit the Mellen model [9] by extracting the exponential decay rates \((\alpha_x, \alpha_y)\) in \(x\) and \(y\) directions, respectively, and the convection speed \(U_c\). The parameters obtained for two free flow velocities [10] are presented in Fig. 3, and are used to feed the Mellen model [9]. With \( G_{p_b p_b} (f) \) the autospectral density, this model can be expressed in the wavenumber domain:

\[
G_{p_b p_b} (k, f) = G_{p_b p_b} (f) \frac{2\pi (\alpha_x \alpha_y k_c^2)^2}{\left[ (\alpha_x \alpha_y k_c^2)^2 + (\alpha_x k_c k_y)^2 + (\alpha_y k_c)^2 (k_c - k_x)^2 \right]^{3/2}}.
\]

(5)
2.3 Description of the proposed methodology

Following Eq. (1), the estimation of the sensitivity functions using a reciprocity principle and of the excitation both in the wavenumber domain, a simple methodology for evaluating the velocity ASD function $G_{vv}$ at a given point $x$ of the panel ($z = 0$) can be summarized as follows:

- Excite the panel with a normal mechanical force at point $x$ and measure the normal velocity response of the panel at points $\tilde{x} \in \Gamma_\tilde{x}$ to determine $H_{v/F_n}(\tilde{x}, x, f)$,
- Perform a 2D-DFT of the panel velocity response $H_{v/F_n}(\tilde{x}, x, f)$ (with respect to $\tilde{x}$) to obtain the sensitivity functions $H_v(x, k, f)$ at point $x$ for $k \in \Omega_k$,
- Use Eq. (1) and the fitted model of Mellen $G_{ppb}(k, f)$ to estimate the velocity ASD function $G_{vv}$ at point $x$ under the considered TBL excitation.

3 VALIDATION OF THE PROPOSED METHODOLOGY FOR AN UNKNOWN PANEL

For validation purposes, the methodology proposed in previous section has been applied on two composite panels following a blind study approach (panels properties remain voluntarily unknown, panels are glued on their perimeter without any special care and the methodology is applied step by step without any intermediate validation). The panels are made of the same material, have similar thickness ($2.2$ mm) and only differ on their aspect ratios (dimensions in $x$ and $y$ directions are $(L_x, L_y) = (0.23, 0.23)$ m for panel 'A' and $(L_x, L_y) = (0.45, 0.23)$ m for panel 'B'). The study was performed at two free flow velocities: $U_\infty = 20$ m.s$^{-1}$ and $U_\infty = 40$ m.s$^{-1}$.

3.1 Description of the measurements

The panels were mounted in the anechoic wind tunnel to measure their vibration response under the actual turbulent flow, which will serve as reference for the validation of the proposed methodology. Because the method based on the reciprocity principle does not theoretically depend on boundary conditions, the latter have voluntarily not been controlled but should be close to clamped boundary conditions since all edges were glued. These boundary conditions remained the same for the application of the proposed method (measurements of the sensitivity functions) and direct vibration measurements under turbulent flow, since the same setup was kept for both experiments. The experimental setup is presented in Fig. 4.

The vibration response of the panel under the actual turbulent flow was first measured in the anechoic wind tunnel. A $8 \times 4$ feet medium density fiberboard (MDF) panel of $3/4$ inch thickness was mounted at the end of the convergent from which air was flowing. A sandpaper strip was glued at the end of the convergent in order to help the TBL developing. The panel was flush-mounted in the fiberboard panel $1.8$ m away from the convergent, where the wall-pressure fluctuations were also measured. The vibration response was measured at point $x$ of coordinates $(x = -0.06, y = -0.065)$ m (arbitrarily chosen) using an accelerometer (PCB 353B18). Time signals of 30 seconds were acquired with a sampling frequency of $8192$ Hz, and the vibration velocity autospectral density functions were estimated using MATLAB ”cpsd” command (using a Hanning window, with 50% overlap).

The sensitivity functions were then experimentally determined based on the reciprocity principle (Sec. 2.3). Keeping the same experimental setup, a vibration shaker (TMS SmartShaker K2007E01) with an impedance head (PCB 288D01) was fixed at point $x$ (where the accelerometer was previously placed). The spatial vibration velocity of the panel was measured using a scanning laser vibrometer (Polytec PSV-300) on a grid of points uniformly distributed over the entire panel ($25 \times 25$ points for panel A and $49 \times 25$ points for panel B), leading to a spatial resolution of $\delta_x = \delta_y \approx 9.2$ mm in directions $x$ and $y$ for both panels. This measurement mesh has been defined.
to reach a compromise between a reasonable measurement time (between one hour and an half and
three hours, with ten averages at each points) and a sufficient point density (to avoid aliasing effect
when performing the 2D-DFT). The highest wavenumbers $k_{x}^{max}$ and $k_{y}^{max}$ that can be resolved in
directions $x$ and $y$, respectively, are given by:

$$k_{x}^{max} = k_{y}^{max} = \frac{\pi}{\delta_{x}} = \frac{\pi}{\delta_{y}} \simeq 341 \text{ m}^{-1}. \tag{6}$$

These wavenumbers define the wavenumber domain $\Omega_{k}$ over which Eq. (1) is solved. The
wavenumber resolutions are inversely proportional to the dimensions of the panel ($\delta_{k} = 2\pi/L$)
and differ for the two considered panels in the $x$ direction. The wavenumber resolution value
being too large to appropriately describe the sensitivity functions and the excitation, zero-padding
is used to finally obtain a wavenumber resolution of 1 m$^{-1}$ along $k_{x}$ and $k_{y}$.

Finally, the Mellen model requires an additional information on the wall-pressure ASD function.
It has been measured at point x (i.e., where $G_{vv}$ is estimated) using a MEMS microphone array:
AH+ Stick-on/Peel-off Conformal Array [19–21]. This measuring device has been chosen to lower
the spatial averaging effect due to the size of the transducer, which has already been observed in
the literature [21, 22]. The array was mounted in the spanwise direction, as shown in Fig. 5, and
8 seconds time signals have been acquired for each of the 32 microphones composing the array
with a sampling frequency of 21400 Hz (acquisition length and sampling frequency being defined
by the dedicated post-processing software). The wall-pressure ASD functions were then estimated
using "cpsd" MATLAB command while applying a Hanning window with 50% overlap to the time
signals.

The wall-pressure ASD function of the microphone closest to point x has been considereds.
The measured wall-pressure ASD functions are presented in Fig. 6 at the two considered flow velocities. An overall increase of the pressure level with the flow velocity can be observed over the all frequency range. The noisy aspect of the estimated wall-pressure ASD functions is mainly due to the limited acquisition time [23], which could not be increased. This effect will inevitably be repeated in the estimation of the vibration response of the panel based on the proposed methodology.

Figure 6: Measured ASD function of the blocked wall-pressure (dB, ref. \(4 \times 10^{-10} \text{ Pa}^2.\text{Hz}^{-1}\)) at a flow velocity \(U_\infty = 20 \text{ m.s}^{-1}\) (black line) and \(U_\infty = 40 \text{ m.s}^{-1}\) (red line).

The parameters presented in Figs. 3 and 6, and Eq. (5) entirely describe the wall-pressure fluctuations induced by the TBL reproduced in the anechoic wind tunnel.
3.2 Sensitivity functions

Examples of the sensitivity functions measured on panel 'A' are presented as a function of both wavenumber and frequency, along \( k_x \) direction (for \( k_y = 0 \)) in Fig. 7(a) and along \( k_y \) direction (for \( k_x = 0 \)) in Fig. 7(b). The convective wavenumbers for the two considered flow velocities are also indicated for the \( k_x \) direction case (i.e., flow direction).

Figure 7: Squared absolute value of the sensitivity functions \(|H_v(x, k, f)|^2\) (dB, ref. 1 m \(^2\).s\(^{-2}\).Hz\(^{-1}\)) of panel A: along \( k_x \geq 0 \) for \( k_y = 0 \). (b) along \( k_y \geq 0 \) for \( k_x = 0 \). The superimposed lines represent: \( k_0 \) (continuous line); \( k_c \) for \( U_\infty = 20 \) m.s\(^{-1}\) (dotted line); \( k_c \) for \( U_\infty = 40 \) m.s\(^{-1}\) (dashed line).

Figure 8: Squared absolute value of the sensitivity functions \(|H_v(x, k, f)|^2\) (dB, ref. 1 m \(^2\).s\(^{-2}\).Hz\(^{-1}\)) of panel B: along \( k_x \geq 0 \) for \( k_y = 0 \). (b) along \( k_y \geq 0 \) for \( k_x = 0 \). The superimposed lines represent: \( k_0 \) (continuous line); \( k_c \) for \( U_\infty = 20 \) m.s\(^{-1}\) (dotted line); \( k_c \) for \( U_\infty = 40 \) m.s\(^{-1}\) (dashed line).

The levels and shapes of the sensitivity functions along \( k_x \) and \( k_y \) are similar, with flexural
wavenumber dispersion curves that can be fairly well noticed. Since the dimensions of the panel are identical in both directions, this shows that the panel is nearly isotropic, if not perfectly.

Similarly, the sensitivity functions obtained for panel ‘B’ are presented in Fig. 8. The dimension of panel B in the \( x \) direction being larger than the one of panel A, a higher modal density is logically observed along with similar flexural wavenumber dispersion curves in both directions (that do not depend of the dimensions of the panel).

### 3.3 Confrontation of predicted response to direct measurements

In this section, the results obtained following the proposed method are confronted to direct measurements in the anechoic wind tunnel. The ASD functions of the vibration velocity estimated according to the methodology presented in Sec. 2.3 are thus compared to those obtained with direct measurements using an accelerometer in Fig. 9 for panel A and in Fig. 10 for panel B.

![Figure 9: Velocity ASD functions \( G_{vv} \) (dB, ref. 1 m².s⁻².Hz⁻¹) of panel A. For \( U_\infty = 20 \) m.s⁻¹: wind tunnel measurements (bold gray line) vs. proposed approach (light black line). For \( U_\infty = 40 \) m.s⁻¹: wind tunnel measurements (bold orange line) vs. proposed approach (light red line).](image)

Results for the two considered flow velocities are presented in each figure. Slight frequency shifts at the resonance peaks are observed and can be explained by the slightly larger weight of the impedance head (and also stinger and vibration shaker) compared with the one of the accelerometer alone. The higher modal density of panel B can also be noticed. Overall, the predicted responses are in good agreement with direct measurements in terms of frequency and amplitude over the whole considered frequency range, which shows that the proposed method is well suited for estimating the vibration response of a panel submitted to a TBL excitation.

### 4 CONCLUSION

Following the work presented in [8, 10], this study aims at applying a methodology to predict the vibration response of a panel submitted to a turbulent boundary layer excitation on a test case representative of aeronautical applications (two composite panels with unknown mechanical properties and arbitrary boundary conditions). This methodology is based on the mathematical formulation of the problem in the wavenumber domain, which allows separating the contributions of the excitation from those of the panel, represented by so-called "sensitivity functions". The sensitivity
functions at any point on the panel can be estimated based on the reciprocity principle by exciting it with a normal point force at a given point and by performing a discrete Fourier transform of the vibration field normalized by the injected force. By experimentally determining the sensitivity functions of the panel and by defining the excitation in terms of auto spectral and cross spectral density functions, the vibration response of the panel under the considered excitation can be estimated at a post-processing phase.

Results presented in this paper show that a good estimation of the vibration response of the panel is obtained following the proposed approach. The sensitivity functions have nevertheless to be precisely estimated by ensuring a proper definition of the grid of points over which the vibration response is measured. The wall-pressure cross spectral density functions have to accurately represent the considered excitation. In this study, the wall-pressure fluctuations have been measured in an anechoic wind tunnel in order to validate the proposed method by comparison with direct measurements of the vibration response under the actual turbulent flow.

It is, however, not mandatory and models presented in the literature [16] can be used unaltered to get a satisfactory estimation of the response of a panel under a turbulent boundary layer excitation. One of the main assets of the proposed methodology is that, once the excitation is known, ex situ vibratory characterizations can be conducted on any panel by simply measuring the sensitivity functions (provided they are representative of those achieved when the panel is mounted on a real structure). Since the excitation and the structure are explicitly separated in this approach, parametric studies can be also easily held by changing the properties of the excitation or those of the panel as examples.

ACKNOWLEDGMENTS

This work was supported by the Labex CeLyA of Université de Lyon, operated by the French National Research Agency (ANR-10-LABX-0060/ANR-11-IDEX-0007).
REFERENCES


