Generalized helical vortex pairs

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New solutions describing the interaction of helical pairs of counter-rotating vortices are obtained using a vortex filament approach. The vortices are assumed to have a small core size allowing the calculation of the self-induced velocities from Biot-Savart law using the cut-off theory.

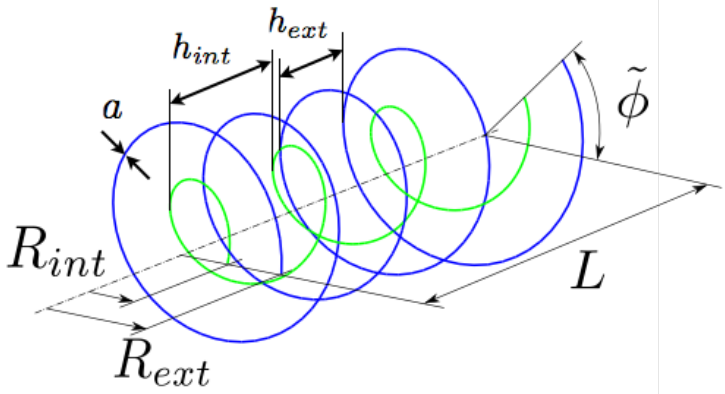

Figure 1: Parameters defining the generalized helical vortex pairs.

These new vortex structures do not possess any helical symmetry but they exhibit a spatial periodicity (figure 1)

$$F(r, \theta + \tilde{\phi}, z + L) = F(r, \theta, z)$$

and are stationary in a rotating and translating frame.

Their properties, such as radial deformation, frame velocity (figure 2) or induced flow, are provided as a function of the four geometric parameters characterizing each solution:

$$R^* = \frac{R_{int}}{R_{ext}}, \quad h^* = \frac{h_{ext}}{R_{ext}}, \quad \alpha = \frac{h_{int}}{h_{ext}}, \quad \varepsilon = \frac{a}{Rext}.$$

Approximate solutions are also obtained when the mutual interaction is weak. This allows us to provide explicit expressions for the rotation and translation velocities of the structure in this limit.

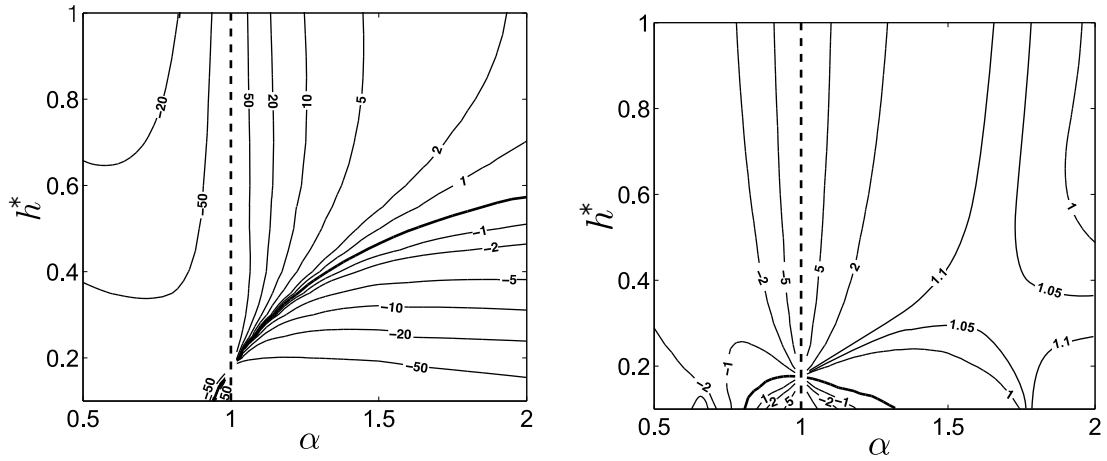

Figure 2: Angular velocity (right) and axial velocity (right) of the frame where the solution is stationary as a function of α and h^* for R^*=0.5 and ε=0.03. Both quantities have been normalized using the external radius R_{ext} , and the vortex circulation Γ.

The variation of the vortex core size induced by the helix deformation is also analysed. We show that these variations have a weak effect on the shape and characteristics of the solutions, for the range of parameters that we have considered.

The results are finally applied to rotor wakes. It is explained how these solutions could possibly describe the far wake of an helicopter rotor in vertical flight (figure 3).

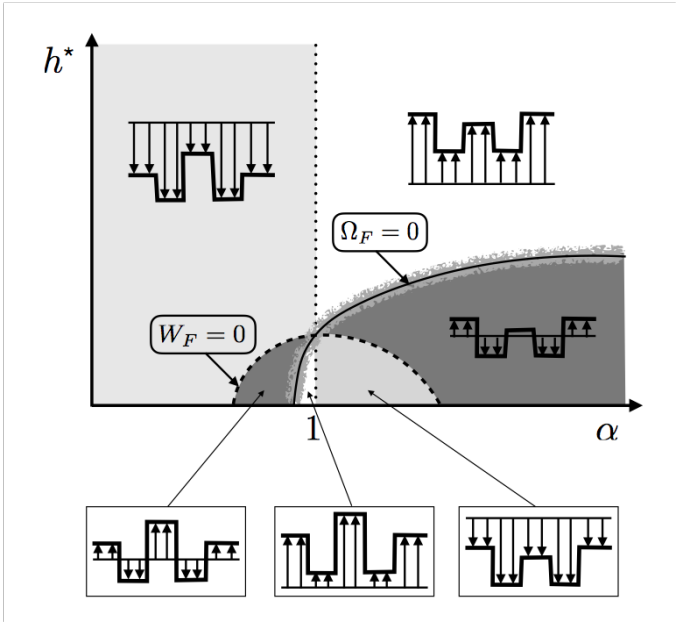

Figure 3: Diagram of the different rotor flow regimes. White regions: windmill brake regime or wind turbine regime (both vortices are going upwards as the external wind). Light gray regions: ascending regime (both vortices are going downwards as the external wind). Dark gray regions: slow descending regime and vortex ring state (both vortices are going downwards while the external wind is going upwards). Close to the line F = 0, there exists a small region where one vortex is going upwards while the other is going downwards: such a solution cannot describe the (far) wake of a rotor.