Logistic imperatives and mode choice

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LOGISTIC IMPERATIVES AND MODE CHOICE

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1. INTRODUCTION.

Many transport externalities depend strongly on the transport mode used, so that modal shift is perceived as a priority to diminish the social, economic and environmental impacts of freight transport. The relative lack of success of modal shift transport policies meet in practice indicates that mode choice may be partially misunderstood. Why do shippers use a given transport mode? Why is it difficult to induce them to modify their choices? These are central questions of freight transport economics and modeling.

The classic modeling approach to the question of modal choice is based on a representation of the transport modes by their characteristics, in the tradition of the consumer theory of Lancaster (1966). These characteristics are typically the transport price and the travel duration. This approach yields the well-known value of time, or value of travel time savings, the marginal substitution rate between time and price (Quinet and Vickermann, 2004), which is a key parameter of spatialised transport models in general (Small and Verhoef, 2006). This approach generally proves econometrically insufficient, so that other variables are introduced: mode specific constants in many cases, attempts to introduce some other characteristics, such as reliability (Fowkes et al., 2004). However, it has two main shortcomings: first, there is no notion of shipment size, yet a crucial decision variable in freight transport, closely related to the choice of vehicle, and to the logistic imperatives of the shipper. Second, it is difficult to give a precise theoretical explanation to the value of time in freight transport - in other words, there are no microeconomic model deriving the value of time of a shipper endogenously.

The objective of this paper is to address - at least partially - these two shortcomings. To do so, we will follow the direction taken by Baumol and Vinod, i.e. importing in the field of microeconomics a model initially belonging to the field of inventory theory. We will do so with a model representing a very simple supply chain: the single commodity periodic review model with backlogging. This model is analysed in detail in Arrow et al. (1958). Its basic idea is that the shipper has to decide how many commodities to send from a plant to a retail center, in order to fulfill the requests of its customers. These requests are not known in advance, so that the shipper has to predict the demand in order to take decisions.

Despite the simplicity of the model, it proves possible to investigate the relationship between the choice of shipment size and that of transport mode, and to derive endogenously the value of time of the shipper, on the basis of exogenous cost parameters. In particular, the transport time plays an important role: a longer transport time implies the need for the shipper to take decisions further in advance. In other words, the shipper loses reactivity, and

this loss comes with all kinds of increased logistic costs. As a matter of fact, the model illustrates how the preferences of shippers with respect to transport modes stem from their own logistic imperatives. Furthermore, this model takes into account the influence of the preferences of the shippers' customers on the own preferences of the shippers[1].

The paper proceeds as follows: the framework of the model is presented in Section 2. Then, Section 3 derives the optimal logistic policy of the shipper. Both Section 2 and Section 3 present, in a simplified manner, results from Arrow et al. (2004). Section 4 analyses this optimal policy from a transport economics perspective, then some numerical applications are provided Section 5. The paper is concluded in Section 6.

2. MODEL FRAMEWORK

Consider a firm, thereafter called the shipper, owning a production unit in A, and a retail center in B. This shipper produces commodities of many distinct types in A, and sells them to its customers in B. There is only one transport mode available to carry all these commodities. This mode is used with a given frequency, say, without loss of generality, once per day, to carry all these commodities. The transport lead-time[2] is denoted l.

As a profit maximizer, whatever the market structure, the shipper minimizes its costs (including, if any, the user's costs[3]; Mohring, 1985). The analysis is focused here on the supply chain associated to a unique commodity type. By supply chain, we mean the commodity flow and stocks, from production to delivery.

Customers order each day a given amount of this commodity type. If they are not served straightaway, they wait. However, they regard waiting as a discomfort; in other words, they are willing to pay to reduce or avoid waiting time. As a consequence, pending orders incur a cost the shipper takes into account. The shipper can decrease the number of customers waiting by increasing the inventory level.

However, a high inventory implies two types of costs. First, warehousing requires space, buildings, monitoring, and handling, which are costly. Second, commodities owned by the shipper imply a capital opportunity cost as well as, depending on the commodity type, a depreciation cost.

The problem for the shipper is to decide the amount of commodities to ship each day, so as to minimize the related transport and inventory costs.

2.1. Demand distribution

During each time period $t \geq 0$, the customers order a amount D_t of the considered commodity at the retail center. The demands $\{D_t\}_{t\geq 0}$ are assumed stochastic, independent and identically distributed. The expected value of D_t is d, and its variance is σ. Its cumulative distribution function is denoted F_D.

2.2. System dynamics

The framework is modeled as a single-product single-location inventory system under periodic review, where excess demand is kept and the replenishment lead time is positive.

The destination inventory at time t is denoted I_t. Note that this variable can be negative. In that case, it stands for the amount of orders pending or, equivalently, the number of customers waiting. The shipment sent at time t is denoted s_t. The pipeline inventory (i.e. the amount of commodities being carried at time t) is denoted I_t^p.

The destination inventory dynamics are simple: at the beginning of period $t+1$, the destination inventory is the destination at time t plus the shipment sent at time $t-l$ minus the demand ordered during period t:

$$I_{t+1} = I_t + s_{t-l} - D_t \tag{1}$$

The pipeline inventory dynamics are even simpler:

$$I_{t+1}^p = I_t^p + s_t - s_{t-l} \tag{2}$$

For analytical convenience, the following conventions are taken: $\forall t < 0$, $D_t = d$, and $s_t = d$.

2.3. Costs

In this framework, four types of costs are distinguished. Each of them is thereafter defined and calculated over period t.

The first cost is the direct transport cost C_t, i.e. the amount paid by the shipper to the carrier for the commodities to be actually moved. In the framework considered, the transport system is designed for a large number of commodity types. With respect to the structure of costs, this means that a higher shipment size s_t means more vehicle capacity, but no change in the shipment frequency, for example, and a negligible change in the loading and unloading times. As a consequence, the daily transport cost is assumed proportional to the shipment size:

$$C_\tau = c_\tau s_t \tag{3}$$

Second, the pipeline inventory cost C_p, the inventory cost due to the time the commodities wait while being transported. This cost is assumed proportional to the time spent by the commodities during transport, up to a coefficient a_c standing for the amount the shipper would be ready to pay to decrease by 1 day the time waited by one ton of commodity in the shipper's inventory (a_c encompasses the capital opportunity cost and the depreciation cost, but not the warehousing cost).

During period t, this cost is equal to the amount of commodities currently being carried (it thus depends on the transport lead-time l) times the unit commodity value of time.

$$C_p = a_c \sum_{i=t-l}^{t-1} s_i \tag{4}$$

Third, the destination inventory cost: proportional to the time the commodities wait in the retail center, before they are sold, this cost consists of the capital and depreciation cost a_c, and of the warehousing cost, assumed proportional to the amount of commodities being stocked, up to a coefficient denoted a_w:

$$C_d = (a_c + a_w)(I_t)^+ \tag{5}$$

where $(.)^+$ denotes the positive value.

Fourth, the customer cost: it is assumed proportional to the number of customers waiting for their orders at time t, equal to $|I_t|$ if $I_t < 0$, times the unit customer value of time (the amount the customers would be ready to pay to reduce their waiting time of one day), denoted by a.

$$C_d = a(I_t)^- \tag{6}$$

The shipper's objective is to minimise the sum of these four costs[4]. The sum of these costs is thereafter called the full logistic cost C:

$$C = C_\tau + C_p + C_d + C_c \tag{7}$$

From Equations (3) to (6), Equation (7) becomes:

$$C = c_\tau s_t + a_c \sum_{i=t-l}^{t-1} s_i + (a_c + a_w)(I_t)^+ + a(I_t)^- \tag{8}$$

The objective of the shipper is to determine at each period t the shipment size s_t which minimizes the logistic cost C under the system dynamics given by Equations (1) and (2).

The detailed approach will not be reported here. We will take for granted that in this framework, the optimal logistic policy is an order-up-to policy. In other words, the amount sent in each period from the production unit to the retail center exactly compensates the quantity ordered at the retail center. As a consequence, the destination inventory plus the pipeline inventory, i.e. the overall inventory, remains constant. The level of the overall inventory is chosen so as to minimize the expected cost for any time t. See Arrow et al. (1958) for a formal analysis.

3. THE OPTIMAL LOGISTIC POLICY

In this framework, the difficulty for the shipper stems from the demand uncertainty and the transport lead-time: the system's state at time t derives

from decisions the shipper has taken at least l days earlier. The objective of the shipper is to address this uncertainty optimally with respect to the costs introduced earlier.

The best way to do so is to follow an optimal order-up-to policy. In this section, we show how this result stems intuitively from the problem's structure. Then, the optimal inventory level is derived.

3.1. Description of the optimal logistic policy

Given the regularity of the problem, one can expect the optimal logistic policy to be such that the destination inventory's expected value is equal to a given target. Let I_s denote this target, called the safety stock. In other words, the objective is:

$$E(I_t) = I_s$$

Now consider the information available to the shipper at time t: the shipper knows the inventory I_t and the shipments sent over the l previous periods. The shipper decides at time t the size of the shipment s_t which will arrive at $t+l$.

By combining Equation (1) $l+1$ times we obtain:

$$I_{t+l+1} = I_t - \sum_{i=t}^{t+l} D_i + \sum_{i=t-l}^{t-1} s_i + s_t \qquad (9)$$

where I_t and $\{s_i\}_{i=t-l,\dots,t-1}$ are given, s_t to be decided, $\{D_i\}_{i=t,\dots,t+l}$ yet unobserved.

By taking the expected value of Equation (9):

$$E\left(I_{t+l+1}\right) = I_t - (l+1)d + \sum_{i=t-l}^{t-1} s_i + s_t$$

so that to obtain $E\left(I_{t+l+1}\right) = I_s$, s_t must be equal to:

$$s_t = I_s + (l+1)d - I_t - \sum_{i=t-l}^{t-1} s_i \qquad (10)$$

If this policy is applied, then, by replacing s_t from the equation above in (9) and translating $l+1$ periods backwards, the destination inventory at time t is:

$$I_t = I_s + \sum_{i=t-l-1}^{t-1} \left(d - D_i\right) \qquad (11)$$

(Note that this is true only for $t \geq l$. We will assume that in the following.)

This equation is of central importance. First, it says that under the optimal logistic policy, the destination inventory is a random variable centered on I_s,

and of which the variance is directly related to the variance of D_t and to the transport lead time. Second, the distribution of I_t is identical for all $t \geq l$. Third, as it will be explained in more detail later, the expected full logistic cost depends almost entirely on this equation.

Let us introduce right away the following notations:

$$\begin{cases} \mu_I = E(I_t) \\ \sigma_I = V(I_t) \end{cases} \tag{12}$$

From Equation (11):

$$\mu_I = I_s \tag{13}$$

and:

$$\sigma_I = \sqrt{l+1}\sigma \tag{14}$$

The policy described by Equation (10) is clearly an order-up-to policy: if the predicted demand minus the already present inventory plus the already sent shipments is below the safety stock level, the shipment sent at period t is sized to exactly compensate this gap. In fact, by calculating $I_{t+1} - I_t$ from Equation (1), then from Equation (11), and comparing the results, one easily deduces that:

$$s_t = D_{t-1} \tag{15}$$

The shipment sent at time t exactly compensates the demand during period $t-1$. Incidentally, this also implies:

$$E(s_t) = d \tag{16}$$

3.2. The optimal safety stock

The choice of a target value for the safety stock remains to be done. It relies solely on Equations (8), (11) and (15).

Let us first consider the transport and pipeline inventory costs. Both of them only depend on the shipment size s_t. From Equation (15), the expected value is d, so that these two costs do not depend on I_s.

As a consequence, denote:

$$E(C)(I_s) = E(C_d)(I_s) + E(C_c)(I_s) + K$$

where K is a constant. The I_s variable is omitted from now on, unless necessary.

From Equations (5) and (11), the expected destination inventory cost is:

$$E(C_d) = (a_c + a_w)E\left[\left(I_s + \sum_{i=t-l-1}^{t-1}(d - D_i)\right)^+\right]$$

and from Equation (6), the expected customer cost is:

$$E(C_d) = aE\left[\left(I_s + \sum_{i=t-l-1}^{t-1}(d - D_i)\right)^-\right]$$

These two equations reveal the role of I_s: a large safety stock means fewer customers waiting, but a higher inventory cost, and conversely. Note that I_s can be negative: the shipper decides that the consumers usually wait, its only objective is that they do not wait too long.

Let D_t^r denote the sum of the orders placed over $\{t-l-1,...,t-1\}$, so that Equation (11) becomes:

$$I_t = I_s + (l+1)d - D_t^r$$

The D_t^r are identically distributed, but not independent. Due to the central limit theorem, whatever the distribution of D_t, the distribution of D_t^r is almost Gaussian, provided the travel time l is not too short. As a consequence, in order to avoid tedious considerations, which would bring little to the analysis, D_t^r is considered normally distributed.

In that case, then I_t is also normally distributed, of mean μ_I and variance ${\sigma_I}^2$. The c.d.f. of I_t, denoted F_I, is thus approximately:

$$F_I = \Phi\left(\frac{x - \mu_I}{\sigma_I}\right)$$

and its distribution density, denoted f_I, is:

$$f_I = \frac{1}{\sigma_I}\phi\left(\frac{x - \mu_I}{\sigma_I}\right)$$

where Φ and ϕ are the c.d.f. and density of the centered unit normal distribution, respectively. It is now possible to determine how $E(C_d)$ varies with I_s. Indeed:

$$E\left[(I_t)^+\right] = \int_0^{+\infty} x f_I(x)dx$$

so that, by variable substitution:

$$E\left[(I_t)^+\right] = \int_{-\frac{\mu_I}{\sigma_I}}^{+\infty} (\sigma_I v + \mu_I)\phi(v)dv \qquad (17)$$

From a few simple transformations (the detail is available in Combes, 2009) we obtain that:

$$\frac{\partial E\left[(I_t)^+\right]}{\partial I_s} = \Phi\left(\frac{\mu_I}{\sigma_I}\right)$$

and similarly that:

$$\frac{\partial E\left[(I_t)^-\right]}{\partial I_s} = \Phi\left(\frac{\mu_I}{\sigma_I}\right) - 1$$

so that the behaviour of $E(C)$ with respect to I_s is given by:

$$\frac{\partial E(C)}{\partial I_s} = (a_c + a_w + a)\Phi\left(\frac{\mu_I}{\sigma_I}\right) - a \tag{18}$$

which, given Equations (13) and (14), is equivalent to:

$$\frac{\partial E(C)}{\partial I_s} = (a_c + a_w + a)\Phi\left(\frac{\mu_I}{\sqrt{l+1}\sigma}\right) - a \tag{19}$$

Therefore, the expected full logistic cost is a convex function of I_s, minimized when the derivatives of the expected destination inventory cost and the expected customer cost are equal except for the sign. The optimal safety stock is derived straightforwardly from Equation (19):

$$I_s^* = \sigma\sqrt{l+1}\Phi^{-1}\left(\frac{a}{a_c + a_w + a}\right) \tag{20}$$

The role of the safety stock is clearly illustrated by this result: I_s^* does not depend on d, and is proportional to σ. Apart from the pure transport cost, all the logistic costs considered in this framework stem from the demand's uncertainty, which plays a crucial role. Indeed, if the demand were certain, there would be no need for a safety stock; in fact, there would be neither inventory nor customer costs: the shipper would send each day the exact amount which would fulfill the needs of his customers.

However, the demand uncertainty is not the only source of the need for a safety stock: the safety stock is made necessary by the positive transport lead time. A greater transport lead time means a less reactive system, and the need for a greater safety stock. As a consequence, the transport lead time is a central parameter for the shipper.

Up to this point, we have just provided very classical elements of inventory theory. The remaining paragraphs of this section aim at analyzing these results from a microeconomic, freight transport modeling perspective.

4. TRANSPORT DEMAND ANALYSIS: THE VALUE OF TIME

Now that the shipper's logistic policy has been introduced, it is analyzed from a microeconomic standpoint. This section is focused on how the safety stock and the full logistic cost depend on the model parameters. In particular, we examine how the preferences of the shippers with respect to freight transport can be derived from its full logistic cost function.

The transport mode used by the shipper is described by two of its characteristics: its unit cost c_τ and its lead time l. The roles these two parameters play are not symmetric.

Indeed, from Equation (20), the cost parameter c_τ has no influence on the safety stock level. Provided the shipper does not change from transport mode, an increase in c_τ does not impact the shipper's transport demand. The only consequence is a change in the full logistic cost. Under the optimal policy (as it is assumed from now on):

$$\frac{dE(C)}{dc_t} = d \tag{21}$$

On the contrary, the transport lead-time has a non-trivial impact both on the shipper's logistic policy and full cost. First, it appears straightforwardly from Equation (20) that the safety stock increases with l. Consistently with what was explained at the end of Section 3, the safety stock addresses the demand uncertainty and the system's lack of reactivity. An increase in the transport lead time means a loss of reactivity, which comes at a cost which is not limited to an increase in the pipeline inventory cost.

To calculate the derivative of the full logistic cost with respect to l, first note that I_s^* is a function of l.

However, from the envelop theorem:

$$\frac{dE(C)}{dl} = \left.\frac{\partial E(C)}{\partial l}\right|_{I_s = I_s^*}$$

As a consequence, μ_I can be considered fixed in Equation (17), and we have:

$$\frac{\partial E\left[(I_t)^+\right]}{\partial l} = \frac{\partial \sigma_I}{\partial l}\int_{-\frac{\mu_I}{\sigma_I}}^{+\infty} v\phi(v)dv$$

Given that $\phi'(v) = -v\phi(v)$, and from the symmetry of $\phi(v)$:

$$\frac{\partial E\left[(I_t)^+\right]}{\partial l} = \frac{\partial \sigma_I}{\partial l}\phi\left(\frac{\mu_I}{\sigma_I}\right)$$

The same result is obtained for $(I_t)^-$. As a consequence, and from Equation (14) and (20):

$$\frac{dE(C)}{dl} = a_c d + \frac{\sigma}{2\sqrt{l+1}}(a + a_c + a_w)\phi \circ \Phi^{-1}\left(\frac{a}{a_c + a_w + a}\right)$$

In order to clarify notations, the following function is introduced:

$$\zeta : (x, y) \mapsto (x + y)\phi \circ \Phi^{-1}\left(\frac{x}{x + y}\right) \tag{22}$$

It can be easily demonstrated (Combes, 2010) that ζ is positive, increasing in x and y, and that $\zeta(x, y) = \zeta(y, x)$. In the following, $\zeta(a, a_c + a_w)$ is simply written ζ.

Now, using these notations, the derivative of the full logistic cost with respect to the transport lead time can be written:

$$\frac{dE(C)}{dl} = a_c d + \frac{\zeta\sigma}{2\sqrt{l+1}} \tag{23}$$

When the transport lead time increases, the full logistic cost increases because the time the commodities spend in the transport operation increases, but not only. The second reason why the full logistic cost increases if the transport lead time is one day longer is that the sizes of shipments must be decided one day earlier, which means the variability of the destination inventory is greater. This greater variability implies a change in the safety stock, which is not a problem in itself since, due to the envelop theorem, such a shift leaves the full logistic cost unchanged, but it also means that $E\left[(I_t)^+\right]$ and $E\left[(I_t)^-\right]$ and, as a direct consequence, the inventory and customer costs, increase. Note that ζ can be interpreted the marginal cost of variability. From the properties of ζ, and as expected, it is an increasing function of each of its parameters.

The shipper's value of time can then be calculated, as the cost increase the shipper would be ready to accept for the travel time to be decreased by one unit. Denote it α.

$$\alpha = a_c + \frac{\zeta}{2\sqrt{l+1}}\frac{\sigma}{d} \tag{24}$$

As expected, α encompasses the commodity value of time, but it also encompasses another term, which can be interpreted as the value of speed in the frame of a supply chain.

It is of great importance to note that this value of time actually depends on the speed of the considered transport mode. As a consequence, a shipper cannot be characterized by a fixed, unique value of time helping discrimination between transport modes. On the contrary, a shipper which chooses a slow transport mode will present a lower value of time than if he had chosen a fast one, all other things equal. This suggests there might be some endogeneity in the relationship between observed values of time and mode choices.

Note that this value of time is not necessarily closely related to the carrier's value of time as it is observed in classic roadside surveys for example. Indeed, the transport lead time offered by the carrier can rely on a complex organization, so that the linkage between the transport lead time and the actual transport operation durations is not trivial.

5. NUMERICAL APPLICATIONS

The model presented here is not intended to be used in an operational context, and has not been the object of econometric investigations. However, some numerical applications are presented here, to illustrate the model's behavior and to provide orders of magnitude. The values given to the various parameters are purely indicative. Two parts of supply chains are considered: laptop computers, and cars.

5.1. Laptop computers

Consider a shipper producing laptop computers in China and selling them in Europe[5].

Since a laptop weights by and large 2 kg and is sold by and large 1000€, its value density is about 500 k€ per ton of commodity. Assume an opportunity cost of capital of about 20% and a depreciation cost of 20%; a_c is then about 200 k€ per ton per year, or 550 € per ton and per day.

The warehousing cost can be roughly estimated on the basis of average warehouse rents, about 50€ per year and m² in France. On the hypothesis that it is possible to stock about 3 tons of commodities per m², a_w is estimated at 0.05€ per ton and per day.

The customer cost is much trickier to estimate. According to many industry studies, 8% of retail items are out of stock at any one time (Su and Zhang, 2009). In our model, if the probability that I_t is negative is 8%, then $a/(a_c + a_w + a)$ is 0.92, so that a is approximately $11(a_c + a_w)$. In the absence of more accurate data, the following value is retained: a = 6000€ per day and per ton, which stands, in this case, for 12€ per laptop per day.

Estimates of the daily demand expected value and variance are unavailable. The following assumption is made: $d = 5$ units, or 0.01 ton, and $\sigma = 3$ units, or 0.006 ton. Given that the example is focused on a single computer brand, in a given retail center, small numbers are reasonable.

Now assume the computers are transported by plane and by truck for the pickup and delivery movements, in 5 days, at a cost of 2000€ per ton.

Given these parameters, the safety stock is (in units) $I_s = 10.1$ units. As a is much larger than $a_c + a_w$, we find as expected that the average amount of computers actually stocked at a given time, $E\left[(I_t)^+\right] = 10.4$, is much larger than the average number of customers waiting at a given time $E\left[(I_t)-\right] = 0.3$. The per day cost components are:

$$C_\tau = 20.0€/day$$
$$C_p = 27.5€/day$$
$$C_d = 11.4€/day$$
$$C_c = 3.4€/day$$

The direct transport costs constitute less than half of the full logistic cost, but the generalised transport cost (including the pipeline inventory cost) more than 75% of the full logistic cost. Finally, the shipper's value of time is $\alpha = 673.7€/day$ or $\alpha = 28.1€/h$.

Now assume the computers are transported by sea. The transport rate is assumed to be approximately 100€ per ton, and the transport lead time approximately 45 days. The safety stock is then of 28.1 units. There are on average 28.8 computers stocked, and 0.8 customers waiting. The cost elements are:

$$C_\tau = 1.0€/day$$
$$C_p = 247.5€/day$$
$$C_d = 31.7€/day$$
$$C_c = 9.4€/day$$

so that the full logistic cost is 289.6€ per day, for 62.3€ per day in the previous case, so that air transport is more competitive in this example, due to the pipeline inventory cost. Note that in this case, the value of time is 594.7€ per day, not very different from the previous one. The elements are summarized in Table 1.

Table 2: Laptop supply chain example

Variable	Air	Sea
I_s^*	10.1	28.1
$E\left[(I_t)^+\right]$	10.4	28.8
$E\left[(I_t)^-\right]$	0.3	0.8
α	673.7	594.7
C_τ	20.0	1.0
C_p	27.5	247.5
C_d	11.4	31.7
C_c	3.4	9.4
C	62.3	289.6

5.2. Cars

Consider now a shipper sending cars from a producing unit located in Europe to a central distribution platform also located in Europe, at about, from which the cars are then sent to the car dealers. Car dealers will be considered as the customers of the model, which is justified by the fact that they have no stock: if they wait for their deliveries, so do the final customers. We focus on a particular car model.

The following assumptions are taken. The cars are worth 25 000€. Their weight is one ton (for simplicity). The capital cost is 15%, and the depreciation cost 15%, so that $a_c = 20€/day$. The warehousing cost is $a_c = 0.05€/day$. We assume: $a = 8(a_c + a_w) = 160$ (the decision to buy a car is taken in a wider time scale, and other parameters such as the attachment to a particular brand, or the need of a particular car type, can mitigate the importance of the waiting time to the customer; anyway, all these assumptions are of course very coarse). The daily demand of the considered car model is $d = 10$, and the variance $\sigma = 6$.

Two transport modes will be compared. First, motor carriers: the lead time is 2 days, and the rate about 200€ per ton. Second, railroad transport: the lead time is 5 days, and the rate about 50€ per ton.

The model variables in the two cases are compared in Table 2. It appears again that the higher lead time of rail transport implies higher pipeline inventory costs, and higher costs due to the destination inventory variability. These effects do not compensate the price decrease, so that rail transport is competitive.

Table 2: Car supply chain example

Variable	Road	Rail
I_s^*	12.7	20.7
$E\left[(I_t)^+\right]$	13.2	21.6
$E\left[(I_t)^-\right]$	0.6	0.91
α	25.9	23.6
C_τ	2000.0	500.0
C_p	400.0	1400.0
C_d	265.3	433.2
C_c	89.6	146.4
C	2755.0	2479.6

6. CONCLUSION

This paper is based on a very classic inventory theory model. No innovative results are presented from the perspective of inventory theory. However,

examining such a model from the microeconomic perspective of freight transport demand analysis yields new highlights on the behavior of shippers.

The model indeed allows to make explicit the transport and other logistic costs of a shipper for a given commodity type. The way the full logistic cost depends on the model parameters illustrates the linkage between the logistic constraints of the shipper and its preferences with respect to freight transport. In particular, it illustrates the particular role of the preferences of the shipper's customers (both the willingness to pay to avoid stock shortages, and the general variability of the demand.)

The model shows how the shipper's value of time derives from its logistic imperatives. The shipper is ready to pay to reduce the transport lead time not only because it reduces the time commodities spend in the transport operation, but also, and in certain cases mainly, because it improves the system's reactivity: a lower transport lead time allows the inventory to stick more closely to the demand, so that both the inventory and the customer costs are decreased. The amount of costs which are thus avoided can be very large, as suggests the general orientation of many industries toward any direction that might improve their reactivity (also referred to as 'time-to-market') and reducing their inventories). Besides, this trend is all the more necessary as the number of brands, and commodity variants which are available to customers increases very fast.

However, the limitation of the model used here limits the generality of the results. In particular, the model assumes there are no fixed transport costs (the transport cost is indeed assumed strictly proportional to shipment size; this hypothesis is not very realistic). Introducing fixed transport costs in the model would improve its generality, as well as that of the results presented in this paper. Econometric investigations of the various parameters the model involves would also be fruitful advances of this work.

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NOTES

[1] This particular feature generally absent from other models. For example, in Baumol and Vinod, the decisions of shippers derive from their decision that the probability of stock shortage (i.e. the inability of the shipper to fulfill the requests of its customers at a given time) must be equal to a certain, exogenous value. They do not explain how this value is chosen, on the basis of other parameters such as inventory costs and stock shortage costs (which consist mainly of the disappointment of the customers facing the shortages).

[2] The model presented in this paper is built from the standpoint of the shipper. As a consequence, apart from some aspects such as the vehicle type (the shipper needs berths to load and unload trucks quickly, railway sidings to have access to rail transport) the way the transport operation is practically realized is not relevant. In particular, if the carrier combines several vehicle movements in addition to break-bulk operations (e.g. for a motor carrier carrying LTL shipments) or moves railcars through marshalling yards (for a rail carrier), the actual time the commodities spend moving can be much smaller than the whole transport operation duration. This is why we prefer to speak about transport lead time rather than about travel time.

[3] Mohring's statement is based on the following intuition: if it costs the shipper less than 1 to reduce by 1 the total cost for customers, the shipper may proceed to this reduction, increase its price by 1, have the same number of customers and increase its own profit, even though the shipper's cost function does not encompass all the elements of the customers' cost function, such as waiting time.

[4] Strictly speaking, there is a cost component for each period of time. Typically, the objective for the shipper would then be to minimize an actualized sum of these costs. However, this simple analysis will be limited to the minimization of the expected cost per time period.

[5] In the framework presented above, the shipper produces the commodities and sells them in the retail center. However, the producer and the retailer of a given commodity are often distinct firms. In a perfectly competitive environment, and provided the firms communicate each other all the information they have, the results previously presented still hold. Of course, this is generally not the case, thus the strategic dimension of supply chain management.