The scale of cosmic homogeneity as a standard ruler

Pierros Ntelis, Anne Ealet, Stephanie Escoffier, Jean-Christophe Hamilton, Adam James Hawken, Jean-Marc Le Goff, James Rich, Andre Tilquin

The scale of cosmic homogeneity as a standard ruler

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

The scale of cosmic homogeneity as a standard ruler

Pierros Ntelis,[a,c] **Anne Ealet,**[c] **Stephanie Escoffier,**[c] **Jean-Christophe Hamilton,**[a] **Adam James Hawken,**[c] **Jean-Marc Le Goff,**[b] **James Rich**[b] **and Andre Tilquin**[c]

[a]APC, Université Paris Diderot-Paris 7, CNRS/IN2P3, CEA, Observatoire de Paris, 10, rue A. Domon & L. Duquet, Paris, France
[b]IRFU/CEA, Université Paris-Saclay, D36, 91190 Saclay, Gif-sur-Yvette, France
[c]CPPM, Aix Marseille University, 163, avenue de Luminy, Marseille, France

E-mail: pntelis@cppm.in2p3.fr, ealet@cppm.in2p3.fr, escoffier@cppm.in2p3.fr, hamilton@apc.in2p3.fr, hawken@cppm.in2p3.fr, jean-marc.le-goff@cea.fr, james.rich@cea.fr, atilquin@cppm.in2p3.fr

Received October 23, 2018
Accepted November 26, 2018
Published December 11, 2018

Abstract. In this paper, we study the characteristic scale of transition to cosmic homogeneity of the universe, $\mathcal{R}_H$, as a standard ruler, to constrain cosmological parameters on mock galaxy catalogues. We use mock galaxy catalogues that simulate the CMASS galaxy sample of the BOSS survey in the redshift range $0.43 \leq z \leq 0.7$. In each redshift bin we obtain the homogeneity scale, defined as the scale at which the universe becomes homogeneous to 1%, i.e. $D_2(\mathcal{R}_H) = 2.97$. With a simple Fisher analysis, we find that the performance of measuring the cosmological parameters with either the position of the BAO peak or the homogeneity scale is comparable. We show that $\mathcal{R}_H$ has a dependence on the galaxy bias. If the accuracy and precision of this bias is achieved to 1%, as expected for future surveys, then $\mathcal{R}_H$ is a competitive standard ruler.

Keywords: cosmological parameters from LSS, galaxy clustering, gravity, baryon acoustic oscillations

ArXiv ePrint: 1810.09362

https://doi.org/10.1088/1475-7516/2018/12/014

JCAP12(2018)014

Contents

1 Introduction

The standard model of cosmology, known as flat ΛCDM, describes a Universe mainly composed of Cold Dark Matter (CDM) and a cosmological constant Λ. The two main assumptions of this model are the validity of General Relativity as an accurate description of gravity and the *Cosmological Principle* [1] that states that the Universe is isotropic and homogeneous on large enough scales. This model shows excellent agreement with current data, be it from type Ia supernovae [2–5], temperature and polarisation anisotropies in the Cosmic Microwave Background [6] or Large Scale Structure Clustering [7–11].

In [12] (henceforth N17), we studied the characteristic scale of transition to cosmic homogeneity of the universe, the homogeneity scale, assuming the standard model of cosmology. Historically, the concept of homogeneity in the large scale structure of the Universe can be traced back to [13]. In the modern era, [14] were the first to measure the homogeneity scale in the sky, suggesting a homogeneity scale larger than 100 h^{-1}Mpc. Since then, several methods have been developed to study the homogeneity scale [14–25]. In this work, we follow the method first proposed by [23] and further developed by N17. However, what we really measure in these studies is the combination between the volume distance in the fiducial cosmology and a characteristic scale, similar to BAO studies [26]. Therefore, we will present the results from N17 divided by the volume distance in the fiducial cosmology.

Objects with a characteristic luminosity, such as type Ia supernovae, can be used as standard candles to probe cosmology [2–4]. Likewise, characteristic scales in the statistics of the clustering of galaxies, such as the position of the BAO peak [27] can be used as standard rulers. Standard rulers are important in cosmology since they allow us to measure cosmological distances as a function of redshift. The relationship between distance and redshift is dictated by the rate of cosmic expansion and curvature. Thus, by studying standard rulers, we can improve upon our understanding of cosmology.

JCAP12(2018)014

In this paper, we perform a proof of concept study. Using mock galaxy catalogues, we demonstrate that the homogeneity scale can be used as a standard ruler to constrain cosmology. We define the homogeneity scale as the scale at which the galaxy distribution is homogeneous to 1%, according to its fractal properties. A full definition of what we mean by "homogeneous" is given in section 3.

This document is structured as follows: in section 2, we describe the mock galaxy catalogues. In section 3, we describe the use of the homogeneity scale as a standard ruler and we compare it with the BAO standard ruler, i.e. the position of the BAO peak. In section 4, we explain how we can extract cosmological information from this new standard ruler. Finally, in section 5, we discuss our conclusions.

2 Mock galaxy catalogues

In this study, we use 1000 Quick Particle Mesh (QPM) mock galaxy catalogues [28] designed to simulate the CMASS galaxy sample of the Baryon Oscillation Spectroscopy Survey (BOSS) [29–31]. Each mock catalogue has a sky coverage of $10,400$ deg^2. Objects were selected following the CMASS colour cuts described in [32]. We selected objects in the redshift range of $0.43 < z < 0.7$. For this study, we used only the North Galactic Cap (NGC). The NGC has a larger area compared with the South Galactic Cap which allows us to measure our observable more precisely, as we have shown in N17. NGC has an area ~ 7000 deg^2. The flat ΛCDM cosmology used to obtain these catalogues is given by:

$$\mathbf{p}_{\rm F} = (h, \omega_b, \omega_{\rm cdm}, n_s, \ln\left[10^{10} A_s\right], \Omega_k) = (0.7, 0.0225, 0.11172, 0.95, 3.077, 0.0)\ . \tag{2.1}$$

where $h = H_0/[100\ {\rm km\ s^{-1}\ Mpc^{-1}}]$ is the dimensionless Hubble constant with H_0 the Hubble constant, $\omega_b = \Omega_b h^2$, is the reduced baryon density ratio, $\omega_{\rm cdm} = \Omega_{\rm cdm} h^2$ is the reduced cold dark matter density ratio, n_s the spectral index, A_s the amplitude of the primordial scalar power spectrum and Ω_k is the curvature density ratio. In this framework, the Dark Energy density ratio is defined via $\Omega_\Lambda = 1 - \Omega_m - \Omega_k$, where Ω_m is the total matter density ratio.

This defines the fiducial cosmology,[1] that we are using, to convert the $z, R.A., Dec$[2] information in the catalogue, into comoving coordinates, using the comoving distance relation, see appendix A. This gives an effective volume of $V_{\rm eff} \simeq 3\ h^{-3}{\rm Gpc}^3$, for the entire survey. However, this fiducial cosmology biases the $\mathcal{R}_H^{\rm fid}$ towards itself. Therefore, we need to correct for this effect as we explain in section 4.2. Note that a simple extension of this cosmology is a time varying Dark Energy density ratio, parametrized by w. This extension allows us to investigate models of modified gravity [33, 34] and we explore this extension in section 4.1.

3 Methodology

In this section, we describe how we can use the homogeneity scale, $\mathcal{R}_H$, as a standard ruler. Following N17 (and [23]), we use the fractal dimension, $\mathcal{D}_2(r)$, as a metric of homogeneity and estimate this in the mock galaxy catalogues. Our observable is the fractal dimension. The fractal dimension is related to the counts-in-spheres $N(<r)$ according to $N(<r) \propto r^{D_2}$. For a completely homogeneous distribution $\mathcal{D}_2 = 3$. While for a fractal distribution, it deviates

[1] This fiducial cosmology is in agreement with [6].

[2] The R.A. and Dec are the abbreviations of Right Ascension and Declination measured in degrees.

JCAP12(2018)014

	$\mathcal{R}_H^{\rm fid}/d_V^{\rm fid}$	$d_V^{\rm fid}[h^{-1}{\rm Mpc}]$
$0.430-0.484$	0.108 ± 0.007	1165
$0.484-0.538$	0.094 ± 0.005	1275
$0.538-0.592$	0.089 ± 0.005	1381
$0.592-0.646$	0.083 ± 0.005	1482
$0.646-0.700$	0.082 ± 0.005	1577

Table 1. Mean and standard deviation of the normalised homogeneity scale, $\mathcal{R}_H^{\rm fid}/d_V^{\rm fid}$, as a function of redshift, z, for the galaxy distributions in the north galactic cap (NGC), for the 1000 QPM mock catalogues, as explained in section 3.1. The last column is the fiducial volume distance, $d_V^{\rm fid}$ appendix A.

from this value. Our observable is related to the two-point correlation function according to:

$$\mathcal{D}_2(r) = 3 + \frac{d\ln}{d\ln r}\left[1 + \frac{3}{r^3}\int_0^r \xi(s)s^2 ds\right] , \tag{3.1}$$

where $\xi(s)$ is the usual two-point correlation function that it is used in large scale structure studies.[3] Now we are able to construct a standard ruler according to a characteristic scale of homogeneity, as:

$$\mathcal{D}_2(\mathcal{R}_H) = 2.97 . \tag{3.2}$$

This defines the scale at which the Universe becomes homogeneous to within 1%.

We use the perturbed einstein-boltzman equations (implemented in CLASS [35], which includes implicit assumption on the primordial power spectrum and the matter transfer function) to compute the theoretical matter power spectrum, $P(k)$, for our fiducial cosmology. Applying a fourier transform, we get $\xi(r)$. Then using eq. (3.1) and eq. (3.2), we compute the fractal dimension and the homogeneity scale, respectively.

We extracted the homogeneity scale, $\mathcal{R}_H$ and the position of the BAO peak, $\mathcal{R}_{\rm BAO}$, from the same mock galaxy catalogues for comparison purposes. We studied the homogeneity scale using the galaxy distribution rather than the total matter distribution. Therefore, we need to take into account the bias in the final analysis as we explain in section 4.2.

3.1 Estimation of $\mathcal{R}_H$ and $\mathcal{R}_{\rm BAO}$

We first used the Landy & Szalay estimator [36] to measure the two-point correlation function in the mock galaxy catalogues. Then in the range of $r = [40, 180]h^{-1}$Mpc, we fitted the two point correlation function around the position of the BAO peak, following [37], see appendix B.

Using eq. (3.1), we integrated the two point correlation function and we obtained the fractal dimension as a function of scale. To estimate the homogeneity scale, we fit a spline function to $\mathcal{D}_2$ (eq. (3.1)) over the range $r = [90, 200]h^{-1}$Mpc. Using the definition in eq. (3.2), we then obtained the homogeneity scale, $\mathcal{R}_H^{\rm fid}$ in the fiducial cosmology.[4] We took the values of $\mathcal{R}_H^{\rm fid}$, in the redshift range $0.430 < z < 0.700$ from N17. However, these values are measured using the fiducial cosmology. Therefore we note the values with the normalisation according to the volume distance, d_V (see appendix A). We show the results in table 1.

[3] This observable is calculated using a publicly available code https://github.com/lontelis/cosmopit.

[4] We also used a polynomial fit to estimate this scale over the ranges, $r = [10, 1300]h^{-1}$Mpc, and we found no significant disagreement.

JCAP12(2018)014

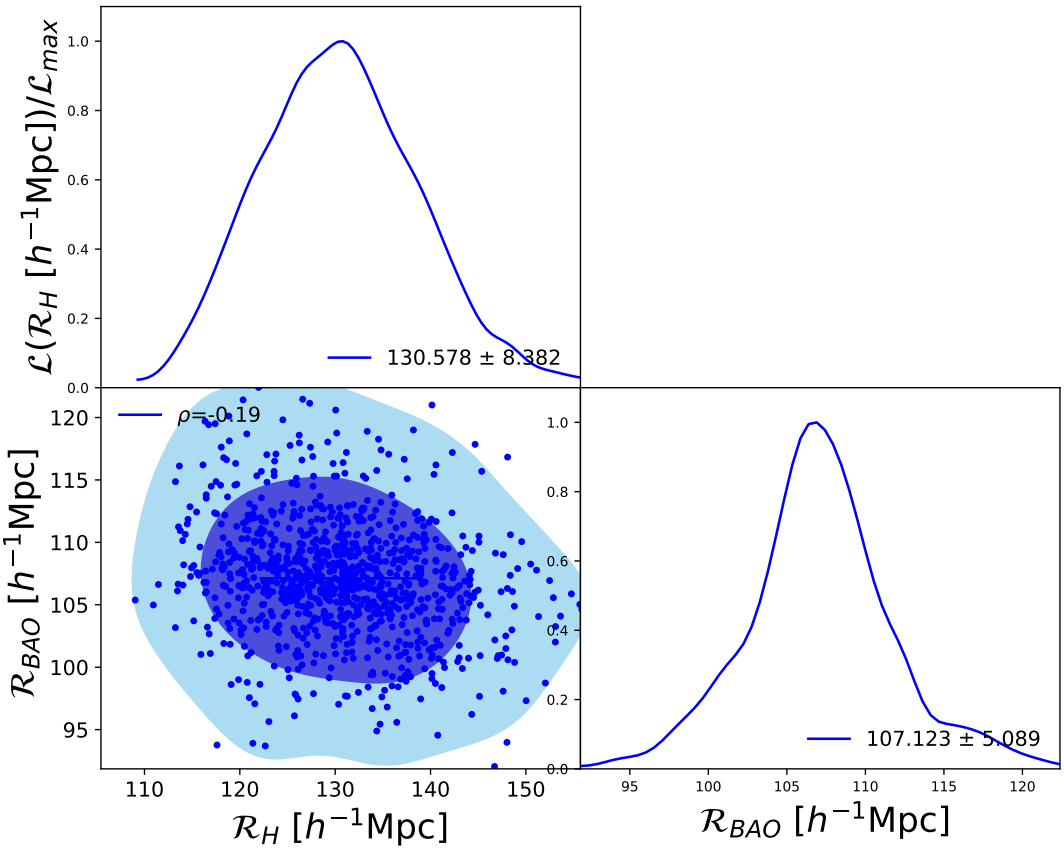

Figure 1. Contour plot with 1σ (2σ) in dark blue (light blue) for the Homogeneity scale versus the position of the BAO peak for the $0.538 \leq z < 0.592$, using 1000 mock catalogues. There is negligible correlation between the two scales, $|\rho| < 0.20$. The number on the top right up corner of the panels correspond to mean and the standard deviation value of the observables.

In [33], we have shown that the homogeneity scale has no dependence on the observational systematic effects of our survey.

3.2 Correlation of $\mathcal{R}_H - \mathcal{R}_{\mathrm{BAO}}$ estimates

In this section, we study the correlation coefficient between the estimate of the Homogeneity scale and the estimate of the position of the BAO peak, $\rho = C_{\mathcal{R}_H \mathcal{R}_{\mathrm{BAO}}}/\sqrt{\sigma_{\mathcal{R}_H}\sigma_{\mathcal{R}_{\mathrm{BAO}}}}$, where C denotes the covariance of the two scales and σ denotes the standard deviation of each scale.

To study the correlation, we estimated the two standard rulers in 1000 QPM mock catalogues. We measured the correlation between the Homogeneity scale and the position of the BAO peak both determined via the methods as described above. We found only a small correlation between the two scales, for example, in the redshift bin $0.538 \leq z < 0.592$, $\rho \simeq -0.19$, as shown in figure 1. We find similar results in the rest of the redshift bins. Explicitly, we find that $|\rho| < 0.3$, for all redshift bins.

The Homogeneity scale and the position of the BAO peak both have units of h^{-1}Mpc and are of a similar magnitude but the fact that the correlation coefficient between the two is $|\rho| < 0.3$ means that they are fairly independent of one another. Therefore, we can investigate the use of the homogeneity scale as an independent standard ruler.

JCAP12(2018)014

68%	CMB	$+\mathcal{R}_H$	$+\mathcal{R}_{\rm BAO}$
σ_{Ω_Λ}	0.06	0.04	0.03
$\sigma_{\rm w}$	0.30	0.20	0.20

Table 2. Precision of 1σ (68% C.L.) of the $\Omega_\Lambda - {\rm w}$ plane from the Fisher analysis, for the different probes, the CMB, and the combination of CMB with the homogeneity scale, $\mathcal{R}_H$, or the position of the BAO peak, $\mathcal{R}_{\rm BAO}$. [see text for details].

4 Cosmology with $\mathcal{R}_H$

We implement two techniques to assess the performance of $\mathcal{R}_H$ and $\mathcal{R}_{\rm BAO}$ as standard rulers to constrain cosmological parameters. We proceed with the two following steps. First, we perform a Fisher analysis to investigate the sensitivity of $\mathcal{R}_H$ and $\mathcal{R}_{\rm BAO}$ to cosmological parameters [38]. However, this is a Gaussian approximation of estimating the errors. Therefore, in the second step, we perform a Monte Carlo Markov Chain (MCMC) analysis to investigate the performance of the two probes as standard rulers in mock galaxy catalogues.

4.1 Fisher analysis

The Fisher matrix is defined by the derivatives of an observable at different redshifts, $O(z)$, as a function of the parameters, p_i, that this observable depends upon [38]. This is defined as:

$$F_{ij} = \sum_z \frac{1}{\sigma_O^2(z)} \frac{\partial O(z)}{\partial p_i} \frac{\partial O(z)}{\partial p_j} \tag{4.1}$$

where $\sigma_O(z)$ is the error on the observable at each redshift. For simplicity, we have assumed the same precision of each observables in all redshift bins. A simple extension to the ΛCDM-model, as discussed in section 1, is one with a varying Dark Energy density, i.e. the wCDM-model. Therefore, we look at the following parameters:

$$\mathbf{p}_{\rm wCDM} = \left\{h, n_s, \omega_b, \omega_{\rm cdm}, n_s, \ln 10^{10} A_s, \Omega_\Lambda, {\rm w}\right\} \tag{4.2}$$

and compare the amount of information on them gained by using the standard rulers, $\mathcal{R}_H$ and $\mathcal{R}_{\rm BAO}$. From observations we have a linear dependence on the cosmic linear bias, b. This parameter is degenerate with the A_s parameter via $b^2 A_s$, therefore we do not consider it here. However, in section 4.2, we take bias into account.

We find that the Homogeneity scale improves cosmological constraints, when used in combination with the CMB [39]. These constraints are comparable to those of the position of the BAO peak combined with the CMB. The constraints on A_s, Ω_Λ, w, h are significantly improved, while the contribution to $\omega_{\rm cdm}, \omega_b$ and n_s is negligible for both probes.[5] As we present in table 2, for Ω_Λ, $\mathcal{R}_{\rm BAO}$ provides better constraints than $\mathcal{R}_H$. When we look at w, $\mathcal{R}_H$ provides comparable constraints to $\mathcal{R}_{\rm BAO}$.

The homogeneity scale is sensitive to Ω_Λ and h since an accelerating expansion damps the growth of structures, rapidly decreasing the homogeneity scale. However, the expansion rate and the acceleration are two correlated phenomena. Therefore, we can constrain only one of them. In our case we choose Ω_Λ. These results show that given the improvement on Ω_Λ, the homogeneity scale can be used to explore external models such as non-flat universes

[5] See figure 1 of [40] proceedings.

JCAP12(2018)014

or universes with variable Dark Energy equation of state. The Fisher analysis also suggests that the constraints from the homogeneity scale and the position of the BAO peak are not orthogonal and so the combination of $\mathcal{R}_H$ and $\mathcal{R}_{\rm BAO}$ does not provide significantly improved constraints relative to $\mathcal{R}_{\rm BAO}$ alone.

4.2 MCMC analysis

From the Fisher analysis we learnt that the homogeneity scale is sensitive to Ω_Λ providing negligible constraints on the other parameters of the model. Therefore, in this section, we implement an MCMC analysis to determine the constraints provided by the homogeneity scale in the cosmological parameter space $p_C = (\Omega_\Lambda, \Omega_m)$. This is the open-ΛCDM model.

The χ^2 that we explore with an MCMC algorithm[6] is given by:

$$\chi^2(A, B, p_C) = \sum_{z=1}^{5} \left(\frac{\mathcal{R}_H^G(z; p_F) - \mathcal{R}_H^{M,th}(z; p_F) \times b_{\mathcal{R}_H}(z; A, B) \times \alpha(z; p_C, p_F)}{\sigma_{\mathcal{R}_H^G}(z)} \right)^2 \tag{4.3}$$

where p_F are the cosmological parameters fixed to their fiducial values; $\mathcal{R}_H^G(z; p_F)$ is the homogeneity scale of the galaxy distribution as a function of redshift as measured in the fiducial cosmology; $\sigma_{\mathcal{R}_H^G}(z)$ is the error on $\mathcal{R}_H^G(z; p_F)$; $\mathcal{R}_H^{M,th}(z; p_M)$ is the prediction of the homogeneity scale of the matter distribution as a function of redshift; $\alpha(z; p_C, p_F) = d_V(z; p_C)/d_V(z; p_F)$ is the ratio of the volume distance in a given cosmology to the fiducial value. The homogeneity scale of the galaxy distribution is biased with respect to the homogeneity scale of the total matter distribution. To account for this bias, we use a linear bias model, originally designed for the two point correlation function [41], adapted to fit the homogeneity scale:

$$b_{\mathcal{R}_H}(z; A, B) = A \left(\frac{1+z}{1+z_{\rm eff}} \right)^B , \tag{4.4}$$

where $z_{\rm eff}$ is the value of the intermediate redshift bin (where our estimate of A is the most accurate). The factor $(1 + z_{\rm eff})$ is included to reduce the degeneracy between A and B parameters.[7]

We estimated the homogeneity scale on the mock galaxy catalogues, as described in section 2. We measured the mean and standard deviation of these values. We then performed an MCMC using the mean of the mocks as our data and the standard deviation as our error. First we fixed the cosmological parameters to their fiducial values to find the best fitting values of the bias parameters in our cosmology, i.e. that of the mock galaxy catalogue. We find $(A, B) = (1.975 \pm 0.052, 0.999 \pm 0.597)$. The A parameter is close to the normal linear galaxy bias.

We then freed our cosmological parameters and ran the MCMC with the addition of Gaussian priors on A and B centred on our previous estimates, and priors from the CMB+Lensing [39].[8] We assessed the ability of priors of different widths (on A and B) to recover our fiducial cosmology in order to infer the precision to which our bias parameters must be known. We found that in order to improve upon the constraints from the CMB+Lensing alone the required precision is less than 2%. We then performed an MCMC analysis by applying a 1% prior to our bias parameters. The precision and accuracy considered for this bias parametrisation is not obtainable from current observations. However, it will be obtainable

[6] We use the publicly available code, pymc https://pymc-devs.github.io/pymc/.

[7] Notice that this is a new way of parametrising the linear galaxy bias.

[8] We extract graphically the $(\Omega_m, \Omega_\Lambda)$-information from figure 26 page 38 of [39].

JCAP12(2018)014

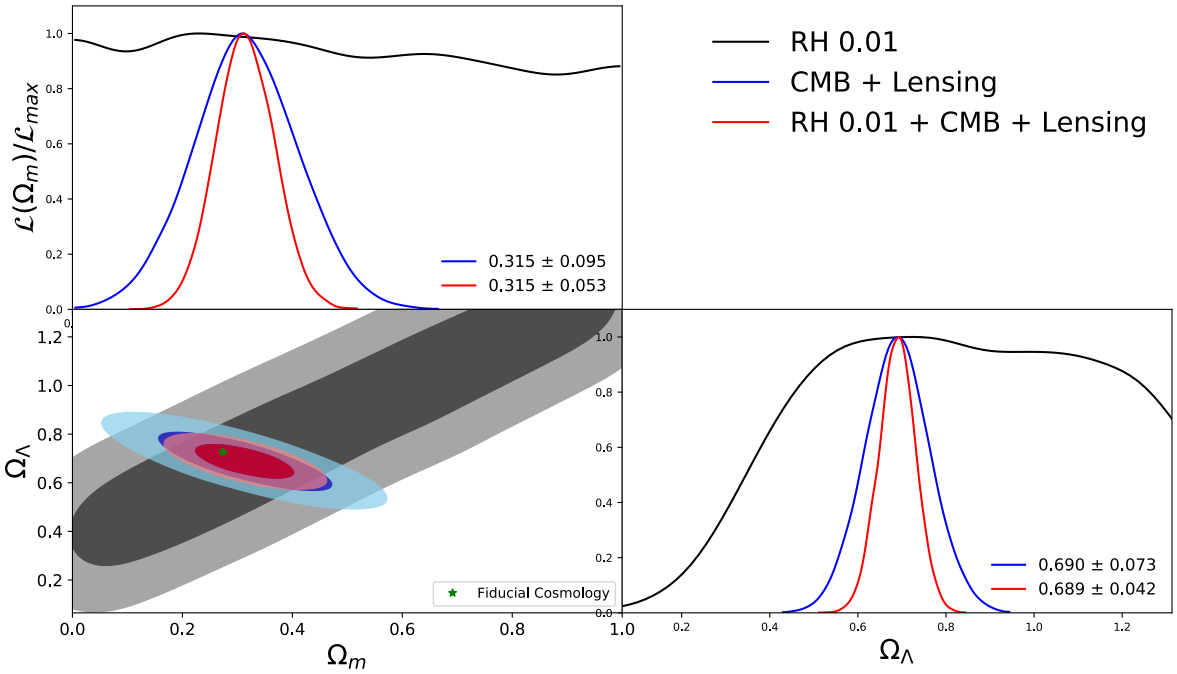

Figure 2. Contours of 68% (dark) and 95% (light) of the $(\Omega_m, \Omega_\Lambda)$ plane using $\mathcal{R}_H$ (black), CMB+Lensing (blue), $\mathcal{R}_H$+CMB+Lensing. The green star denotes the values of the fiducial cosmology of the mock galaxy catalogues. The diagonal panels show the normalised likelihood and the mean and the standard deviation of each parameter colour-coded for each probe. The precision on bias parameters is 1%. [see text for details].

from future surveys such as Euclid [42]. In figure 2, we show the marginalised contours of the $(\Omega_m, \Omega_\Lambda)$ plane for $\mathcal{R}_H$ alone (with 1% precision on the bias parameters A and B) (black), CMB+Lensing (blue), the combination of $\mathcal{R}_H$+CMB+Lensing (red). The green star denotes the values of the fiducial cosmology used to generate the mock galaxy catalogues. The addition of information from $\mathcal{R}_H$ improves constraints relative to the CMB+Lensing alone by a factor of 0.56 for Ω_m and 0.56 for Ω_Λ. We find, the mean of the marginalised likelihood of $\Omega_m^{\mathcal{R}_H+\mathrm{CMB+Lensing}} = 0.318 \pm 0.054$ and $\Omega_\Lambda^{\mathcal{R}_H+\mathrm{CMB+Lensing}} = 0.688 \pm 0.042$. When we compare figure 2 with, for example, [43], we see that the orientation of the constraints provided by $\mathcal{R}_H$ is comparable to the orientation of those obtained using BAO measurements, using galaxies. However, considering the fisher analysis, described in section 4.1, constraints obtained from the two standard rulers are not orthogonal to one another, limiting their use in combination.

This demonstrates that $\mathcal{R}_H$ can be used as a standard ruler to recover the input cosmology. Thus, $\mathcal{R}_H$ can be used as a standard ruler to constrain cosmological parameters. In particular, it can be used to improve the measurement of the $(\Omega_m, \Omega_\Lambda)$ plane.[9]

5 Conclusion and discussion

We have demonstrated that the characteristic scale of transition to cosmic homogeneity, $\mathcal{R}_H$ can potentially be used as a standard ruler to probe cosmology with large scale structure surveys.

[9]Our analysis is available under GNU licence https://github.com/lontelis/CoHo.

JCAP12(2018)014

We have compared the precision of cosmological parameters obtained using $\mathcal{R}_H$ with those obtained with the position of the BAO peak, $\mathcal{R}_{\rm BAO}$, using mock galaxy catalogues. We have found that there is only a small correlation between the two probes, $|\rho| < 0.30$, making the homogeneity scale a complementary cosmological probe.

In order to quantify the additional information contained in the homogeneity scale, we have performed a simple Fisher analysis with a set of cosmological parameters of the flat wCDM model. We have found that the homogeneity scale gives comparable information to that of the position of the BAO peak for this set of cosmological parameters. In particular, the homogeneity scale is sensitive to the dimensionless Hubble constant, h, the amplitude of the primordial fluctuations, A_s, the Dark energy density ratio, Ω_Λ, and the equation of state, w. This shows the implicit dependence of the homogeneity scale on the shape of the transfer function. However, constraints obtained from the two standard rulers are not orthogonal to one another, limiting their use in combination.

Using an MCMC algorithm, and applying a 1% prior to our bias parameters, we explored the open ΛCDM model on the mean of the mock galaxy catalogues. We found $\Omega_m^{\mathcal{R}_{\rm H}+{\rm CMB}+{\rm Lensing}} = 0.318 \pm 0.054$ and $\Omega_\Lambda^{\mathcal{R}_{\rm H}+{\rm CMB}+{\rm Lensing}} = 0.688 \pm 0.042$, consistent with the input flat ΛCDM-model cosmology of the mock galaxy catalogues. The inclusion of $\mathcal{R}_H$ improves CMB+Lensing constraints by a factor of 0.56 for the total matter density ratio and by a factor of 0.56 for Dark Energy density ratio constraints. These results show the sensitivity of our probe to cosmic bias.

In summary, we have shown the dependence of the homogeneity scale to the matter power spectrum by showing the parameter dependence. In a future study, we are going to investigate the observational selection effects dependence of our probe [44]. Therefore, we have revealed the complementarity of the homogeneity scale with respect to other cosmological probes.

Finally, we stress that this analysis can be performed and improved upon in the light of more observational data from current and future experiments such as SDSS-IV (eBOSS) [45], Euclid [46], LSST [47] and DESI [48]. Furthermore, analogous methods could be applied to data from SKA [49]. We relegate this analysis to future work.

Note added. A recent paper [24] appeared simultaneously with this work. They measured the homogeneity scale in the eBOSS DR14 QSO sample with a similar methodology to the one presented here, but at a higher redshift, $0.80 < z < 2.24$. They acquired a similar precision to our measurement on mocks, but on real data. Therefore, we can apply our analysis, the homogeneity scale as a standard ruler, to their measurement. On the other hand, another recent paper [50], appeared simultaneously with this work. They measured several quantities related to the fractal dimension in a galaxy catalogue of SDSS-DR7 at lower redshifts $z < 0.5$. They find values that do not agree with the homogeneity scale found and used in this work. Thus, we do not expect that our method will give reliable results to this chosen galaxy catalogue. However, several updates on the construction of the galaxy catalogue in their redshift bin have been made since then [32].

Acknowledgments

We would like to thank the anonymous referee, for his constructive comments that improved our manuscript.

JCAP12(2018)014

PN would like to thank Jean-Paul Kneib, Marc Lachieze-Ray, Stavros Katsanevas, Stéphane Plaszczynski, Nicolas G. Busca, Cyrille Doux, Mikhail Stolpovskiy, Hector Gil Marin and Charling Tao for useful suggestions and discussions.

PN was funded by Ecole Doctoral 560, university Paris Diderot for the early stages of this analysis and currently he is funded by 'Centre National d'étude spatiale' (CNES), for the Euclid project.

AJH acknowledges the financial support of the OCEVU LABEX (Grant No. ANR-11-LABX-0060) and the A*MIDEX project (Grant No. ANR-11-IDEX- 0001- 02) funded by the Investissements d'Avenir French government program managed by the ANR. This work also acknowledges support from the ANR eBOSS project (ANR-16-CE31-0021) of the French National Research Agency.

This research used also resources of the IN2P3/CNRS and the Dark Energy computing Center funded by the OCEVU Labex (ANR-11-LABX-0060).

The French Participation Group of SDSS-III was supported by the Agence Nationale de la Recherche under contracts ANR-08-BLAN-0222 and ANR-12-BS05-0015-01.

Funding for SDSS-III has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, and the U.S. Department of Energy Office of Science. The SDSS-III web site is http://www.sdss3.org/.

SDSS-III is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS-III Collaboration including the University of Arizona, the Brazilian Participation Group, Brookhaven National Laboratory, Carnegie Mellon University, University of Florida, the French Participation Group, the German Participation Group, Harvard University, the Instituto de Astrofisica de Canarias, the Michigan State/Notre Dame/JINA Participation Group, Johns Hopkins University, Lawrence Berkeley National Laboratory, Max Planck Institute for Astrophysics, Max Planck Institute for Extraterrestrial Physics, New Mexico State University, New York University, Ohio State University, Pennsylvania State University, University of Portsmouth, Princeton University, the Spanish Participation Group, University of Tokyo, University of Utah, Vanderbilt University, University of Virginia, University of Washington, and Yale University.

JCAP12(2018)014

A Cosmography

From $z, R.A., Dec$ we need to infer distances. Therefore we reconstruct the z information according to standard cosmology and then we make our measurements in comoving space. In standard cosmology, we define the following distances.

The *comoving distance*:

$$d_{\rm C}(z) = c\int_0^z \frac{dz'}{H(z')}\ , \tag{A.1}$$

where c is the speed of light and

$$H(z) = H_0\sqrt{(\Omega_{\rm cdm}+\Omega_{\rm b})(1+z)^3+\Omega_\Lambda+\Omega_k(1+z)^2}\ ,$$

is the usual Hubble expansion rate.

We define the *volume distance*:

$$d_V(z) = \left[czH^{-1}(z)d_M^2(z)\right]^{1/3}\ , \tag{A.2}$$

where d_M is the *motion distance*:

$$d_M(z) = \left\{ \begin{array}{c} \frac{d_H}{\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k}\frac{d_C(z)}{d_H}\right) \ , \ \Omega_k > 0 \\ d_C(z) \, , \ \Omega_k = 0 \\ \frac{d_H}{\sqrt{\Omega_k}} \sin\left(\sqrt{\Omega_k}\frac{d_C(z)}{d_H}\right) \ , \ \Omega_k < 0 \end{array} \right\} \ , \tag{A.3}$$

where d_C is given by eq. (A.1) and $d_H = c/H_0$.

B Determination of the position of BAO peak

We used two methods to determine the position of the BAO peak. For the first method, we model the 2-point correlation function in such a way that the result is only driven by the position of the BAO peak [37]. We then apply a broadband model. This model can be described by the following formula:

$$bb(r) = p_1 + \frac{p_2}{r} + \frac{p_3}{r^3} \tag{B.1}$$

where (p_1, p_2, p_3) are the broadband parameters.

We model the measurement of the position of the BAO peak using the usual Gaussian model [51], described by:

$$\xi^{(1)}(r) = A \exp\left[-\frac{1}{2}\left(\frac{r - \mathcal{R}_{\rm BAO}}{\sigma_{\rm peak}}\right)^2\right] + bb(r) \tag{B.2}$$

where $\mathcal{R}_{\rm BAO}$ is the position of the BAO peak parameter, A and $\sigma_{\rm peak}$ are the amplitude and the smoothing scale of a Gaussian function, respectively.

In order to determine the position of the BAO peak, we measure the parameter $\mathcal{R}_{\rm BAO}$ by marginalising over the rest of the parameters. We marginalise the nuisance parameters, $(A, \sigma_{\rm peak}, p_{bb})$.

For the second method, we follow the same steps but now we use the usual correlation function with the fiducial cosmology as a template, $\xi(r; p_F)$, instead the Gaussian model. We model the correlation function as:

$$\xi^{(2)}(r) = b^2 \xi(\alpha_{\rm iso} * r; p_F) + bb(r; p_{bb}) \tag{B.3}$$

where we model the *isotropic dilatation* parameter as:

$$\alpha_{\rm iso} = r_s / r_s^{\rm fid} \ . \tag{B.4}$$

Note that now $\mathcal{R}_{\rm BAO} = r_s$. In this case the nuisance parameters are (b, p_{bb}). The two methods, explained above, provide similar conclusions.

References

[1] G.F. Ellis, *Cosmology and verifiability*, *Quart. J. Roy. Astron. Soc.* **16** (1975) 245.

[2] Supernova Cosmology Project collaboration, S. Perlmutter et al., *Measurements of* Ω *and* Λ *from* 42 *high redshift supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133] [INSPIRE].

JCAP12(2018)014

[3] Supernova Search Team collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201] [INSPIRE].

[4] SDSS collaboration, M. Betoule et al., *Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples*, *Astron. Astrophys.* **568** (2014) A22 [arXiv:1401.4064] [INSPIRE].

[5] DES collaboration, T.M.C. Abbott et al., *First cosmology results using type Ia supernovae from the dark energy survey: constraints on cosmological parameters*, arXiv:1811.02374 [INSPIRE].

[6] Planck collaboration, N. Aghanim et al., *Planck* 2018 *results. VI. Cosmological parameters*, arXiv:1807.06209 [INSPIRE].

[7] 2dFGRS Team collaboration, W.J. Percival et al., *Parameter constraints for flat cosmologies from CMB and 2dFGRS power spectra*, *Mon. Not. Roy. Astron. Soc.* **337** (2002) 1068 [astro-ph/0206256] [INSPIRE].

[8] D. Parkinson et al., *The WiggleZ dark energy survey: final data release and cosmological results*, *Phys. Rev.* **D 86** (2012) 103518 [arXiv:1210.2130] [INSPIRE].

[9] C. Heymans et al., *CFHTLenS tomographic weak lensing cosmological parameter constraints: mitigating the impact of intrinsic galaxy alignments*, *Mon. Not. Roy. Astron. Soc.* **432** (2013) 2433 [arXiv:1303.1808] [INSPIRE].

[10] É. Aubourg et al., *Cosmological implications of Baryon Acoustic Oscillation measurements*, *Phys. Rev.* **D 92** (2015) 123516 [arXiv:1411.1074] [INSPIRE].

[11] BOSS collaboration, S. Alam et al., *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR*12 *galaxy sample*, *Mon. Not. Roy. Astron. Soc.* **470** (2017) 2617 [arXiv:1607.03155] [INSPIRE].

[12] P. Ntelis et al., *Exploring cosmic homogeneity with the BOSS DR*12 *galaxy sample*, *JCAP* **06** (2017) 019 [arXiv:1702.02159] [INSPIRE].

[13] I. Newton, *Philosophiae naturalis principia mathematica*, volumes 1–4, (1760).

[14] V.J. Martinez, M.-J. Pons-Borderia, R.A. Moyeed and M.J. Graham, *Searching for the scale of homogeneity*, *Mon. Not. Roy. Astron. Soc.* **298** (1998) 1212 [astro-ph/9804073] [INSPIRE].

[15] D.W. Hogg et al., *Cosmic homogeneity demonstrated with luminous red galaxies*, *Astrophys. J.* **624** (2005) 54 [astro-ph/0411197] [INSPIRE].

[16] J. Yadav, S. Bharadwaj, B. Pandey and T.R. Seshadri, *Testing homogeneity on large scales in the Sloan Digital Sky Survey data release one*, *Mon. Not. Roy. Astron. Soc.* **364** (2005) 601 [astro-ph/0504315] [INSPIRE].

[17] P. Sarkar, J. Yadav, B. Pandey and S. Bharadwaj, *The scale of homogeneity of the galaxy distribution in SDSS DR*6, *Mon. Not. Roy. Astron. Soc.* **399** (2009) L128 [arXiv:0906.3431] [INSPIRE].

[18] L. Guzzo, *Is the universe homogeneous? (On large scales)*, *New Astron.* **2** (1997) 517 [astro-ph/9711206] [INSPIRE].

[19] R. Scaramella et al., *The ESO slice project [ESP] galaxy redshift survey: V. Evidence for a* $d = 3$ *sample dimensionality*, *Astron. Astrophys.* **334** (1998) 404 [astro-ph/9803022] [INSPIRE].

[20] L. Amendola and E. Palladino, *The scale of homogeneity in the Las Campanas Redshift Survey*, *Astrophys. J.* **514** (1999) L1 [astro-ph/9901420] [INSPIRE].

[21] J. Pan and P. Coles, *Large scale cosmic homogeneity from a multifractal analysis of the PSCz catalogue*, *Mon. Not. Roy. Astron. Soc.* **318** (2000) L51 [astro-ph/0008240] [INSPIRE].

JCAP12(2018)014

[22] T. Kurokawa, M. Morikawa and H. Mouri, *Scaling analysis of galaxy distribution in the Las Campanas Redshift Survey data*, *Astron. Astrophys.* **370** (2001) 358.

[23] M. Scrimgeour et al., *The WiggleZ dark energy survey: the transition to large-scale cosmic homogeneity*, *Mon. Not. Roy. Astron. Soc.* **425** (2012) 116 [arXiv:1205.6812] [INSPIRE].

[24] R.S. Gonçalves, G.C. Carvalho, C.A.P. Bengaly, J.C. Carvalho and J.S. Alcaniz, *Measuring the scale of cosmic homogeneity with SDSS-IV DR*14 *quasars*, arXiv:1809.11125 [INSPIRE].

[25] F. Avila, C.P. Novaes, A. Bernui and E. de Carvalho, *The scale of homogeneity in the local universe with the ALFALFA catalogue*, arXiv:1806.04541 [INSPIRE].

[26] J. Rich, *Which fundamental constants for Cosmic Microwave Background and Baryon Acoustic Oscillation?*, *Astron. Astrophys.* **584** (2015) A69 [arXiv:1503.06012] [INSPIRE].

[27] SDSS collaboration, D.J. Eisenstein et al., *Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies*, *Astrophys. J.* **633** (2005) 560 [astro-ph/0501171] [INSPIRE].

[28] M. White, J.L. Tinker and C.K. McBride, *Mock galaxy catalogues using the quick particle mesh method*, *Mon. Not. Roy. Astron. Soc.* **437** (2014) 2594 [arXiv:1309.5532] [INSPIRE].

[29] BOSS collaboration, K.S. Dawson et al., *The Baryon Oscillation Spectroscopic Survey of SDSS-III*, *Astron. J.* **145** (2013) 10 [arXiv:1208.0022] [INSPIRE].

[30] SDSS-III collaboration, S. Alam et al., *The eleventh and twelfth data releases of the Sloan Digital Sky Survey: final data from SDSS-III*, *Astrophys. J. Suppl.* **219** (2015) 12 [arXiv:1501.00963] [INSPIRE].

[31] SDSS collaboration, D.J. Eisenstein et al., *SDSS-III: massive spectroscopic surveys of the distant universe, the Milky Way galaxy and extra-solar planetary systems*, *Astron. J.* **142** (2011) 72 [arXiv:1101.1529] [INSPIRE].

[32] B. Reid et al., *SDSS-III Baryon Oscillation Spectroscopic Survey data release* 12*: galaxy target selection and large scale structure catalogues*, *Mon. Not. Roy. Astron. Soc.* **455** (2016) 1553 [arXiv:1509.06529] [INSPIRE].

[33] P. Ntelis, *Probing cosmology with the homogeneity scale of the universe through large scale structure surveys*, theses, Astroparticule and Cosmology Group, Physics Department, Paris Diderot University, Paris, France, September 2017.

[34] F. Montanari and R. Durrer, *Measuring the lensing potential with tomographic galaxy number counts*, *JCAP* **10** (2015) 070 [arXiv:1506.01369] [INSPIRE].

[35] D. Blas, J. Lesgourgues and T. Tram, *The Cosmic Linear Anisotropy Solving System (CLASS) II: approximation schemes*, *JCAP* **07** (2011) 034 [arXiv:1104.2933] [INSPIRE].

[36] S.D. Landy and A.S. Szalay, *Bias and variance of angular correlation functions*, *Astrophys. J.* **412** (1993) 64 [INSPIRE].

[37] L. Anderson et al., *The clustering of galaxies in the SDSS-III Baryon Oscillation Spectroscopic Survey: Baryon Acoustic Oscillations in the data release* 9 *spectroscopic galaxy sample*, *Mon. Not. Roy. Astron. Soc.* **427** (2013) 3435 [arXiv:1203.6594] [INSPIRE].

[38] A. Albrecht et al., *Report of the dark energy task force*, astro-ph/0609591 [INSPIRE].

[39] PLANCK collaboration, P.A.R. Ade et al., *Planck* 2015 *results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13 [arXiv:1502.01589] [INSPIRE].

[40] EUCLID collaboration, P. Ntelis and A. Ealet, *Euclid: homogeneity in the search of the dark sector*, arXiv:1805.10441 [INSPIRE].

[41] S. Basilakos, M. Plionis and C. Ragone-Figueroa, *The halo mass-bias redshift evolution in the* Λ*CDM cosmology*, *Astrophys. J.* **678** (2008) 627 [arXiv:0801.3889] [INSPIRE].

JCAP12(2018)014

[42] L. Amendola, E. Menegoni, C. Di Porto, M. Corsi and E. Branchini, *Constraints on a scale-dependent bias from galaxy clustering*, *Phys. Rev.* **D 95** (2017) 023505 [arXiv:1502.03994] [INSPIRE].

[43] J.E. Bautista et al., *Measurement of Baryon Acoustic Oscillation correlations at $z = 2.3$ with SDSS DR12 Lyα-forests*, *Astron. Astrophys.* **603** (2017) A12 [arXiv:1702.00176] [INSPIRE].

[44] P. Ntelis, *Observational systematics on cosmic homogeneity*, in preparation.

[45] K.S. Dawson et al., *The SDSS-IV extended Baryon Oscillation Spectroscopic Survey: overview and early data*, *Astron. J.* **151** (2016) 44 [arXiv:1508.04473] [INSPIRE].

[46] L. Amendola et al., *Cosmology and fundamental physics with the Euclid satellite*, *Living Rev. Rel.* **21** (2018) 2 [arXiv:1606.00180] [INSPIRE].

[47] LSST Science and LSST Project collaborations, P.A. Abell et al., *LSST science book, version* 2.0, arXiv:0912.0201 [INSPIRE].

[48] DESI collaboration, A. Aghamousa et al., *The DESI experiment part I: science, targeting and survey design*, arXiv:1611.00036 [INSPIRE].

[49] P.E. Dewdney, P.J. Hall, R.T. Schilizzi and T.J.L. Lazio, *The Square Kilometre Array*, *Proc. IEEE* **97** (2009) 1482.

[50] J. Gaite, *The projected mass distribution and the transition to homogeneity*, arXiv:1810.03539 [INSPIRE].

[51] E. Sánchez et al., *Tracing the sound horizon scale with photometric redshift surveys*, *Mon. Not. Roy. Astron. Soc.* **411** (2011) 277 [arXiv:1006.3226] [INSPIRE].

JCAP12(2018)014