



HAL
open science

Studes of Hilbert's Epsilon Operator in the USSR

Sergei Soloviev

► **To cite this version:**

Sergei Soloviev. Studes of Hilbert's Epsilon Operator in the USSR. *IfColog Journal of Logics and their Applications (FLAP)*, 2017, 4 (2), pp.425-434. hal-01914086

HAL Id: hal-01914086

<https://hal.science/hal-01914086>

Submitted on 6 Nov 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Open Archive Toulouse Archive Ouverte

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in:

<http://oatao.univ-toulouse.fr/19099>

Official URL:

<http://www.collegepublications.co.uk/downloads/ifcolog00011.pdf>

To cite this version: Soloviev, Sergei *Studes of Hilbert's Epsilon Operator in the USSR*. (2017) The IfCoLog Journal of Logics and their Applications, 4 (2). 425-434. ISSN 2055-3706

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

STUDIES OF HILBERT'S ε -OPERATOR IN THE USSR

SERGEI SOLOVIEV

*IRIT, Université de Toulouse, Toulouse, France; associated researcher at ITMO
University, St.-Petersburg, Russia*

`soloviev@irit.fr`

Abstract

The aim of this paper is to give a short survey of the studies that concern the notion of Hilbert's ε -operator and its applications by the researchers in the USSR and continuation of their works abroad in post-soviet time.

Keywords: Hilbert's epsilon-operator, history of logic, research in the USSR.

The aim of this paper is to give a short survey of the studies that concern the notion of Hilbert's ε -operator and its applications by the researchers in the USSR. These works mostly belong to the domain of mathematical logics, but interactions with philosophical logics and mathematical linguistics cannot be ignored, all three domains being quite active.

The paper is based on a systematic historical investigation and my personal recollections, that helped to organize the search for information. I knew personally Grigori Mints, Albert Dragalin and Vladimir Smirnov mentioned below. Mints was the adviser of my graduate work, and I met Dragalin and Smirnov at various conferences. It is possible that my investigation is not completely exhaustive, but I recall whose names were mentioned then in connection with the ε -operator, and this knowledge was confirmed when I did look for references. I am reasonably sure that there were no other authors who did significant study of the ε -operator in the USSR.

My personal recollections were used also to reconstruct to some extent the atmosphere of Soviet times, at least as far as the relationship with the international science is concerned.

Three main aspects will be addressed.

The author would like to thank Christian Retoré for organizing the workshop on *Hilbert's Epsilon and Tau in Logic, Informatics and Linguistics* and Giselle Reis, Bruno Woltzenlogel Paleo, and Hans Leiss for fruitful discussions. Thanks also are due to the anonymous referees. This work was partially supported by the Government of the Russian Federation Grant 074-U01.

-
- How the interest for ε -operator had arisen, in particular, the external sources of logical research in the USSR.
 - Principal works by soviet researchers who studied ε -operator and continuation of their works in post-soviet time. There are three main names: A. G. Dragalin (1941-1998), G. E. Mints (1939-2014) and V. A. Smirnov (1931-1996).
 - Influence of these works on the research worldwide in their own time, and how they influence contemporary research.

Two volumes of German edition of “Grundlagen der Mathematik” by Hilbert and Bernays [5] were known to leading researchers in the USSR. Russian translation was done in 1968/70 and published in 1979/82 by “Nauka” (transl. from German by N. M. Nagorny). The full English translation still does not exist.

In general, the translation of scientific literature into Russian at this period was extremely active, and this fact mostly answers the question about sources. One may mention the publication of Kleene’s “Introduction to Metamathematics” in 1957 (translated by Essenin-Volpin), the translation of A. Robinson’s “Introduction to Model Theory and to the Metamathematics of Algebra” (1967), the translation of selected foundational papers in proof theory published as “Mathematical theory of logical deduction” (edited by A. V. Idelson and G.E. Mints) in 1967. In 1973 Kleene’s “Mathematical Logic” (translated by Yu. Gastev and edited by G. Mints) was published. In 1981 G. Kreisel’s selected papers in proof theory translated by Gastev and Mints [15] were published. For scientific literature at this time the interval between publication of an original and its Russian translation was often 5-7 years. The number of copies was usually at least several thousands, *e.g.*, 7800 for [15]. Partly (but only partly) this may be explained by the fact that the USSR did not sign most of the international copyright agreements ¹.

What else was accessible? Just as an illustration, one may mention that the library of Steklov Mathematical Institute included almost all “Mathematische Annalen” until June 1941 (with Gentzen’s foundational paper of 1936, and Ackermann’s paper of 1940), and again since 1949. It got also “Dissertationes Mathematicae” (published by Polish Academy), all issues 1953 - 1989 (including such papers as [13] by Kreisel and Takeuti)...

Personal contacts also should not be underestimated. For example, many prominent logicians attended the International Congress of Mathematicians in Moscow (1966), among them Tarski, Church, Kleene, Curry, Schütte, Feferman, Cohen.

¹A curious fact, mentioned in [8], is that some Finnish universities, for example, the University of Turku, purchased in 1960s and 1970s Russian translations of American or West-European research due to financial reasons, since they were much cheaper than the original editions.

An one-hour talk was given by Schütte, 30 minutes talks by Cohen, Ershov (from Novosibirsk), joint talk by Shanin (the head of the logic group at Leningrad Branch of Steklov Mathematical Institute), Tseitin, and Zaslavski. Junior logicians from Leningrad (Maslov, Matiyasevich, Mints, Orevkov, Slissenko) participated in ICM with 15 minutes talks and had many opportunities to discuss logical problems with western colleagues. Tarski mentions [4] his dinner at “Praha” restaurant with Mal’cev (Maltsev), Markov, Shanin, Ershov, Kleene, Curry, Chang, Feferman. After the ICM Tarski visited Leningrad and gave a talk. Earlier, in 1965, both Moscow and Leningrad were visited by John McCarthy. G. Mints, whose works on the ε -symbol we shall consider below, was greatly influenced by G. Kreisel. Mints considered Kreisel (who did several important works concerning the ε -symbol himself) as one of his teachers. Kreisel visited the USSR in 1976, but actively communicated with Mints before (cf. [14]).

I cannot go too much into detail in this short paper, but would like to recommend the book [19] for a more general picture of the East-West scientific and cultural exchanges during Cold War times.

Another illustration of active interaction of soviet researchers with worldwide research community is a series of biannual Finnish-Soviet Logic Conferences that started in 1976. “Somewhere around 1975 J. Hintikka and V.A. Smirnov have agreed to hold Finnish-Soviet Conference on logic” [9] ².

The proceedings of the first Soviet-Finnish Logic Conference included 6 papers by soviet participants. Among other contributions let us mention the papers by S. Feferman, J. Hintikka, J. Ketonen, G. Kreisel, D. Prawitz, R. Statman, D. Van Dalen. As Karpenko writes [9], the second Finnish-Soviet Logic Conference was held in Moscow, at the Institute of Philosophy, in 1979. The first Finnish-Soviet-Polish Logic Conference at Polanica-Zdrój was held in 1981. The series of Finnish-Soviet Logic Conferences continued (Helsinki, 1983; Telavi, Georgia, 1985; Helsinki, 1987; Moscow, 1989). It continued even after the fall of the USSR (until 1997, and restarted in 2012)³.

Among early works on the ε -operator one may cite V. A. Smirnov [38] (from Institute of Philosophy), and G. E. Mints [20] (from Leningrad Branch of Mathematical Institute). In Smirnov’s paper [38] (published in French) Hilbert, Bernays, Quine,

²Finland played a special role in this interaction. It was a liberal democracy, but it was not a NATO member and had a special relationship with the USSR because of the conditions of the Agreement of Friendship, Cooperation, and Mutual Assistance (1948). In times of the Cold War it was considered as a kind of “neutral ground”. For example, in 1975 it was the venue of the Conference on Security and Co-operation in Europe.

³I guess that the interruption was connected with the changes in the finance of scientific research.

Russell, Sloupiétzski⁴ were mentioned, but there is no bibliography. Mints [20] cited several publications, in particular [3], [16], [17], [18], [36]. From this brief outline it is clear that a large variety of sources was accessible, at least to researchers working in academic institutions. It is interesting to notice the role of Japanese and Polish sources, in addition to German, English and American.

We may agree that the most important contributions of the USSR researchers concerning the ε -symbol belong to mathematical logics. This is true also for continuation of this line of research after the fall of the USSR. As B. H. Slater writes: “A good deal of technical work has been done, as a result of such proofs, to create epsilon extensions of Intuitionistic Logic which are conservative.” [37] It is more difficult to agree with Slater that this work is now mostly of academic interest. The research on ε in the post-soviet time included the studies of ε -substitutions for analysis [25] and other theories [1], [29] (see the discussion in Stanford Encyclopedia of Philosophy [2]).

As said above, the first paper on the ε -operator by a soviet researcher was the paper [38] published in “Revue Internationale de Philosophie” in 1971. In 1974 two more mathematical papers were published: the paper [3] by Albert Dragalin and [20] by Grigori Mints.

Here is a brief outline of the main points of Mints’ paper (which takes into account the two others):

- It is known, that adding the ε -axiom $A[t] \rightarrow A[\varepsilon xA]$ to Heyting’s (intuitionistic) predicate calculus HA gives a non-conservative extension - for example, the formula $\exists x(\neg Px \rightarrow \neg Pb \wedge \neg Pa)$ becomes derivable⁵.
- We know (says Mints) two conservative ε -extensions of HA . In one of them, due to V. A. Smirnov, a rather limiting constraint is imposed on the notion of proof. We shall use another formulation, due to A. G. Dragalin; in this formulation functions defined by ε -expressions, are seen as partially defined.
- A. G. Dragalin...uses model-theoretical methods; we shall use proof-theoretical methods, that may be extended to stronger systems, and obtain a supplementary theorem about cut-elimination for proofs of arbitrary formulas, not only ε -free.
- These results admit natural extension to Intuitionistic Predicate Calculus with decidable equality (*i.e.*, with supplementary axiom $\forall x\forall y(x = y \vee \neg x = y)$ and

⁴In fact it has to be Jerzy Słupecki, who published a book with Ludwik Borkowski on logic and set theory; its russian edition appeared in 1965.

⁵Here HA may be a misprint, since below Mints speaks about Heyting’s arithmetic, and calls Heyting’s predicate calculus HPC .

to Heyting's arithmetic with *free* functional variables and a choice principle in the form

$$\frac{\Gamma \rightarrow \forall x \exists y A \quad \forall x A_y[f(x)], \Gamma \rightarrow C}{\Gamma \rightarrow C}$$

where f does not occur in Γ, C, A . An extension to Heyting's arithmetic with *bound* variables of higher types and with a respective choice principle requires new ideas.

- The system studied in the paper is denoted HPC^ε . It is obtained from Heyting Predicate Calculus HPC with two sorts of variables (free and bound), with functional symbols but without equality by addition of the following term formation rule: for a formula A , a free and x bound variable $\varepsilon x A_a[x]$ is a term. The system HPC^ε has the same postulates as Hentzen's LJ (except modified $\exists \rightarrow$), but the definition of proof is different.
- An occurrence V of some sequent in a tree-form figure (of deduction) is called meaningful⁶ if for every quasiterm $\varepsilon x A$ in V , there is in the antecedent of V or in some sequent lying below V a member

$$(1) \quad \forall \alpha_1 \dots \forall \alpha_n \exists x A$$

where $\alpha_1, \dots, \alpha_n$ is the full list of free variables of quasiformula $\exists x A$. The (1) is denoted $!\varepsilon x A$.

- The figure (of deduction) that is built from axioms $A \rightarrow A$ using deduction rules is called meaningful if all sequents in it are meaningful.
- There are three theorems: the cut elimination theorem for HPC^ε , the conservativity theorem for HPC^ε w.r.t. HPC and the theorem that the following rule

$$\frac{\forall x A_y[f(x)], \Gamma \rightarrow C}{\forall x \exists y A, \Gamma \rightarrow C}$$

is admissible in HPC^ε and HPC for function symbols f that do not occur in A, Γ, C .

This work by Mints keeps its relevance for modern research. A surprising fact is that later he might underestimate its relevance himself. Bruno Woltzenlogel Paleo met Mints in 2012 and discussed his current work about epsilonization (in collaboration with Giselle Reis). As he wrote to the author, concerning [20]: “the work you pointed to us is much more relevant than the work Grigori Mints recommended. He must have forgotten.”⁷ Mints recommended [34].

⁶“Osmyslennyi” in Russian.

⁷E-mail to the author, 6 Oct. 2015.

Next paper by V. A. Smirnov [39], presented initially as a talk at the First Soviet-Finnish Conference, was published in 1979. In this paper Smirnov recalls the history of his intuitionistic system of natural deduction with the ε -symbol (a conservative extension of the system without ε). Dragalin's technique is used to obtain a similar result for the system of natural deduction with identity. At the same conference was presented also the talk by Yu. Gladkich "Singular Terms, Existence and Truth: Some Remarks on a First Order Logic of Existence" published later in the same proceedings as [39].

In 1982 and 1989 Mints published two papers [21], [22].

- **Zbl 0523.03043** [21]: The paper contains a simplified proof of Ackermann's theorem about the consistency of Peano Arithmetic. The proof is based on Hilbert's idea to apply so called ε -substitutions to systems of arithmetical formulas (...). A careful analysis of the behaviour of the above substitutions and (...) give a proof of the convergence of a suitable system of ε -substitutions. Hence Ackermann's theorem follows. (E. Adamowicz)
- **Zbl 0677.03040** [22]: The Hilbert epsilon-substitution method is extended to some formalizations of the theory of hereditarily finite sets. Applying the methodology developed in an earlier paper (...) the convergence (...) for the theory of hereditarily finite sets is established which generalizes the Ackermann theorem of the convergence of the epsilon-substitution method for first-order arithmetic. (B.R. Boricić)

After the end of the USSR, G. Mints (with some co-authors of the next generation) was, no doubt the most active (among the researchers formed in the USSR) to pursue the studies of ε . V. A. Smirnov also continued to work in this direction [40], [41], [42]⁸.

The papers of this period are easily accessible, so there is less need for a detailed presentation.

G. Mints did work on versions of ε -substitution method in various systems, problems of completeness, termination and cut elimination [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35]. Among his collaborators were W. Buchholz, D. Sarenac, S. Tupailo.

Some of these papers concerned other aspects of ε -methods, such as their relationship to Kripke semantics. Here is the abstract of [34]:

⁸It must be noticed that in the USSR and post-soviet Russia mathematical logic interacted very little with philosophical logic and philosophy of language. The V. A. Smirnov's works had considerable influence on philosophical community.

-
- A natural deduction system for intuitionistic predicate logic with the existential instantiation rule presented here uses Hilbert’s ε -symbol. It is conservative over intuitionistic predicate logic. We provide a completeness proof for a suitable Kripke semantics, sketch an approach to a normalization proof, survey related work and state some open problems. Our system extends intuitionistic systems with the ε -symbol due to A. Dragalin and Sh. Maehara.

In the paper with Sarenac [31] Mints studied context-dependent descriptions and so called “salience hierarchy”. Here is the abstract:

- Epsilon terms indexed by contexts were used by K. von Heusinger⁹ to represent definite and indefinite noun phrases as well as some other constructs of natural language. We provide a language and a complete first order system allowing to formalize basic aspects of this representation. The main axiom says that for any finite collection S_1, \dots, S_k of distinct definable sets and elements a_1, \dots, a_k of these sets there exists a choice function assigning a_i to S_i for all $i \leq k$. We prove soundness and completeness theorems for this system S_{ε_i} .

The method of the proof proposed in [31] is based on a modification of the completeness proof for first-order predicate calculus given by Henkin [7]. Recently a critical analysis of [31] was presented in a talk by Hans Leiss¹⁰. His criticism concerns two important points: one technical (there is a gap in the proof), and one conceptual (whether the first-order theory proposed by Mints and Sarenac is an adequate representation of the second-order theory of von Heusinger).

It is clear, however, that independently of the results of this controversy [31] turns out to be a very stimulating work.

In his talk at Montpellier conference mentioned above Hans Leiss proposed what he himself described as a non-conclusive counterexample, and tried to “fill the gap” in the Mints-Sarenac proof (to modify the construction of term model).

An attempt to reflect a second-order theory in a first-order system also looks promising even if the details need to be fixed. One may hope that this theme will see new interesting developments.

It reminds the story of another proof proposed by G. Mints that was included in [43]. One may read there (p. 281): “Our treatment follows Mints [1992d], with a correction in proposition 8.4.12. (The correction was formulated after exchanges between Mints, Solovjov and the authors.)” It is meant here a “short proof” of so

⁹in his paper [6]

¹⁰At the same conference where the initial version of this work was presented, *Epsilon 2015*. Hilbert’s epsilon and tau in logic, informatics and linguistics, Montpellier, LIRMM, 10-12 June 2015.

called Coherence Theorem for Cartesian Closed Categories. First proof that was considerably longer was published by the author of this article and A. Babaev in 1979. G. Mints suggested a much shorter proof in 1992.

Conclusion. If I wanted to make a “lesson” from the history of studies of the ε -symbol outlined above, I would stress the role of continuity and interaction. Indeed, the papers by Smirnov, Mints and Dragalin could hardly appear in 1970s without continuity of the development of logic inside and outside the USSR, and strong collaboration between researchers on an international level.

The role of these factors remains vital even after the end of the USSR, as it is clearly seen from the examples above.

References

- [1] Arai, T. Epsilon substitution method for theories of jump hierarchies. *Archive for Mathematical Logic*, 2, 2002, 123-153.
- [2] Avigad, J., Zach, R. The Epsilon Calculus. (First published May 3, 2002; substantive revision Nov 27, 2013.) *Stanford Encyclopedia of Philosophy*. <http://plato.stanford.edu/entries/epsilon-calculus/>
- [3] Dragalin, A. Intuitionistic Logic and Hilbert’s ε -symbol, (Russian) *Istoriia i Metodologiiia Estestvennykh Nauk*, Moscow, 1974, 78-84. Republished in: Albert Grigorevich Dragalin, *Konstruktivnaia Teoriia Dokazatelstv i Nestandardnyi Analiz*, s. 255-263, Moscow, Editorial Publ.,2003.
- [4] Feferman, A., Feferman, S. *Alfred Tarski. Life and Logic*. Cambridge University Press, 2004.
- [5] Hilbert, D., Bernays, P. *Grundlagen der Mathematik*. I, Die Grundlehren der mathematischen Wissenschaften 40, 1934. II, Die Grundlehren der mathematischen Wissenschaften 50, 1939. Berlin, New York: Springer-Verlag.
- [6] Von Heusinger, K. Reference and Saliience. In F. Hamm, J. Kolb, A. von Stechow (eds.). *Proc. Workshop on Recent Developments in the Theory of Natural Language Semantics*, 149-172. Sfs, Tübingen 1995.
- [7] Henkin, L. The completeness of the first-order functional calculus. *J. Symbolic Logic*, 14 (1949), 159-166.
- [8] Kaataja, S. Expert Groups Closing the Divide: Estonian-Finnish Computing Cooperation since the 1960s. In [19], pp.101-121.
- [9] Karpenko, A.S. Preface. (The History of Finnish-Soviet Logic Colloquium.) *Logical Investigations (Logicheskie Issledovaniya)*. 20. Moscow, July 2013.
- [10] Kreisel, G. On the interpretation of non-finitist proofs, part I. *J. of Symbolic Logic*, 16, 1951, 241-267.
- [11] Kreisel, G. On the interpretation of non-finitist proofs, part II. *J. of Symbolic Logic*, 17, 1952, 43-58.

-
- [12] Kreisel, G. Mathematical logic. *Lectures in Modern Mathematics III*(T.L. Saaty ed.), Wiley and Sons, New York, 1965, 95-195.
- [13] Kreisel, G., Takeuti, G. Formally self-referential propositions for cut-free analysis and related systems. *Dissertationes mathematicae*, 118, Warsaw, 1974, 50 pp.
- [14] Kreisel, G., Mints, G. E., and Simpson, S. G. The use of abstract language in elementary metamathematics: some pedagogic examples. Springer Lecture Notes, 453 (1975), 38-131.
- [15] . Kreisel, G. Issledovaniya po teoriii dokazatelstv. Moscow, "Mir", 1981.
- [16] Leisenring, A. C. Mathematical logic and Hilbert's ε -symbol. London, MacDonald, 1969.
- [17] Leivant, D. Existential instantiation in a system of natural deduction for intuitionistic arithmetics. Technical Report ZW 13/73, Stichting Mathematische Centrum, Amsterdam, 1973
- [18] Maehara, Sh. A General Theory of Completeness Proofs. Ann. Jap. Assoc. Phil. Sci., no. 3, 1970, p. 242-256.
- [19] Mikkonen, S. and Koivunen, P. (eds.). Beyond the Divide. Entangled Histories of Cold War Europe. Berghahn, N.-Y., Oxford, 2015, viii+324 pp.
- [20] Mints, G. Heyting Predicate Calculus with Epsilon Symbol (Russian). Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR, Vol. 40, 1974, pp. 110-118. English Translation in [23] p. 97-104
- [21] Mints, G. A simplified consistency proof for arithmetic. (Russian, English summary), Izv. Akad. Nauk. Ehst. SSR, Fiz., Mat. 31, 376-381, 1982.
- [22] Mints, G. Epsilon substitution method for the theory of hereditarily finite sets (Russian, English Summary). Izv. Akad. Nauk Ehst. SSR, Fiz., Math. 38, N 2, 154-164, 1989.
- [23] Mints, G. Selected Papers in Proof Theory, Bibliopolis/North-Holland, 1992.
- [24] Mints, G. Hilbert's Substitution Method and Gentzen-type Systems. In Proc. of 9th International Congress of Logic, Methodology and Philosophy of Science, Uppsala, Sweden, 1994.
- [25] Mints, G., Tupailo, S., Buchholz, W. Epsilon substitution method for elementary analysis. Archive for Mathematical Logic, 35, 1996, p. 103-130.
- [26] Mints, G. Strong termination for the epsilon substitution method. J. of Symb. Logic, 61:1193-1205, 1996.
- [27] Mints, G. Thoralf Skolem and the epsilon substitution method for predicate logic. Nordic J. of Philos. Logic, 1:133-146, 1996.
- [28] Mints, G. Strong termination for the epsilon substitution method for predicate logic. J. of Symb. Logic, 1998.
- [29] Mints, G., Tupailo, S. Epsilon-substitution method for the ramified language and Δ_1^1 -comprehension rule. In A. Cantini et al. (eds.), Logic and Foundations of Mathematics (Florence, 1995), Dordrecht: Kluwer, 1999, p. 107-130.
- [30] Mints, G. A termination proof for epsilon substitution using partial derivations. Theor. Comput. Sci. 1(303): 187-213 (2003)

-
- [31] Mints, G., Sarenac, D. Completeness of indexed epsilon-calculus. *Arch. Math. Log.* 42(7): 617-625 (2003)
- [32] Mints, G. Cut Elimination for a Simple Formulation of PAepsilon. *Electr. Notes Theor. Comput. Sci.* 143: 159-169 (2006)
- [33] Mints, G. Cut elimination for a simple formulation of epsilon calculus. *Ann. Pure Appl. Logic* 152(1-3): 148-160 (2008)
- [34] Mints, G. Intuitionistic Existential Instantiation and Epsilon Symbol. arXiv:1208.0861v1 [math.LO] 3 Aug 2012.
- [35] Mints, G. Epsilon substitution for first- and second-order predicate logic. *Ann. Pure Appl. Logic* 164(6): 733-739 (2013)
- [36] Shirai, K. Intuitionistic Predicate Calculus with ε -symbol. *Annals of the Japan Association for Philosophy*, no.4, 1971, p.49-67.
- [37] Slater, B. H. Hilbert's Epsilon Calculus and its Successors. In: Dov Gabbay (ed.), *The Handbook of the History of Logic*. Elsevier. 385-448 (2009)
- [38] Smirnov, V. A. Elimination des termes ε dans la logique intuitioniste. "Revue internationale de Philosophie", 98, 1971, 512-519.
- [39] Smirnov, V. A. Theory of Quantification and ε -calculi. Proceedings of the Fourth Scandinavian Logic Symposium and the First Soviet-Finnish Conference, Jyväskylä, Finland, June 29 - July 6, 1976. *Essays on mathematical and philosophical logic*. Synthese Library, vol.122, D. Reidel P.C.1979.
- [40] Smirnov, V. A. (with Anisov, A. M., Bystrov, P. I.). *Logika i Komputer*. Moscow, 1990 (Russian). In this book Smirnov contributed a paper on proof search.
- [41] Smirnov, V. A. Cut elimination in ε -calculus // Book of abstracts, N. 1. XIX World Congress of Philosophy. Section 4. Moscow, 1993.
- [42] Smirnov, V. A.. Proof search in intuitionistic natural deduction with ε -symbol and existence predicate. (Russian) // *Logical investigations*, 3, Moscow, 1995, 163-173.
- [43] Troelstra, A. S., Schwichtenberg, H. *Basic Proof Theory* (second edition). *Cambridge Tracts in Theoretical Computer Science*, 2000.