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Traffic State Estimation: Optimal Number and Location of Loop Detectors in the context of Data Assimilation

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ABSTRACT

Traffic managers need tools capable of providing online traffic flow monitoring and short-term predictions on large-scale networks. Data Assimilation (DA) techniques provide traffic state estimates based on a dynamic model and an observation model. Their performance are very sensitive to the observation model definition. The paper proposes a solution to optimize the number and the location of loop sensors in the context of DA. The optimality is defined based on travel time and travelled distance indicators, and Pareto efficiency is used to determine the optimal sensor allocation. The method is tested on two toy networks to assess the role and level of contribution of each sensor. The results provide a better understanding for deploying additional sensors or selecting sensors to be removed, for optimal traffic state estimation and traffic forecast.

INTRODUCTION

State of the art

Traffic managers need tools capable of providing online traffic flow monitoring and short-term forecasts on large-scale networks. These methods aim to obtain an estimate of current and future traffic conditions. Classically, they rely on traffic data collected by loop sensors, and the performance of the traffic state estimation depends on the number and location of sensors on the network. However, traffic managers are facing budgetary constraints : the price of each sensor does not allow for massive complementary deployment, and existing sensors need constant and expensive maintenance operations. In this context, the optimization of the number and the location of loop sensors is a critical issue for traffic managers.

Traffic monitoring and forecast can be achieved with Data Assimilation (DA) techniques. Since the 2000s, many DA solutions have been proposed. They combine observed data (also called observation model) with a dynamic traffic flow model to estimate traffic states at any point of the network. Observed and forecasted states are then combined through a data fusion method, which provide the most likely states on the network. Classically, observed traffic data are collected with a given frequency, which implies data assimilation methods to be sequential. Their are based on two key components: the dynamic traffic flow model, and the observation model.

Dynamic traffic flow models can be categorized according to various criteria (see (1) for an overview): level of detail, operationalization, representation of the processes. Three modeling scales can be identified. Macroscopic models provide the evolution of densities on the network with respect to time (*i.e.* Eulerian). Microscopic models provide the evolution of vehicles position with respect to time (*i.e.* Lagrangian Time). Finally mesoscopic models provide the passage time of vehicles with respect to position (*i.e.* Lagrangian Space). They have been introduced in (2, 3) to be the solution of the Lighthill-Whitham-Richards (LWR) model (4). The later scale seems to be the most promising solution for traffic monitoring and traffic forecast on a large scale network (5).

The observation model can be composed of two types of data (5, 6, 7): Eulerian and Lagrangian. Eulerian data, from spatially-fixed sensors, have been the most widely used for their high level of availability (8, 9, 10) since the fifties. Lagrangian data, from vehicles equipped with sensors, can provide positions at a fixed time intervals. The availability of Lagrangian data has increased during the recent years and solutions have been recently proposed to assimilate such data in the context of data assimilation (11, 12, 13, 5).

The observation model is critical in data assimilation applications, because it provides information on the current state on the network. Castillo and al. (14) describe the observation models as an observability problem. The optimisation problem contains three parts: (15)

- Network description

It can be a toy or a real network. A toy network is simple and helps to better understand the way the model works. For instance, freeway corridors with only merges and exits (5), or a mesh network (16), are often used for toy network. A real network is more complex and can show the operability of the model. The litterature provides many references that consider freeway corridors (17, 18) or larger networks on the scale of a city (19, 14, 20).

- Constraints to limit the solutions

(15, 21) have proposed a list of constraints for traffic observation models. They rely on the flow conservation principle and the positivity of speed. Topological constraint can be defined, for in-

stance Eisenman considers the impossibility for two successive links without entrance or exit to have an observation localisation (6).

- Objective function to select one optimal solution

Castillo and al. have proposed a global overview of objective functions, which must be adjusted according to the objective of the study. It can be : origin–destination matrix estimation (16, 22), flow reconstruction (23, 24, 18), travel time estimation (20, 25, 17), or freeway congestion monitoring (26). The present paper belongs to the last category.

Contributions and objectives

This paper presents a method for assessing the observation model in the context of Traffic State Estimation. The method to understand the role and the level on contribution of each sensor is proposed. This paper is organized as follows. Section 2 describes the methodology used to obtain the observation model, including the description of performance indicators. Section 3 presents the results for two toy networks. Section 4 proposes a discussion around the operational context for traffic managers. Section 5 closes the paper on the needs for further research.

METHODOLOGY

Methodological framework

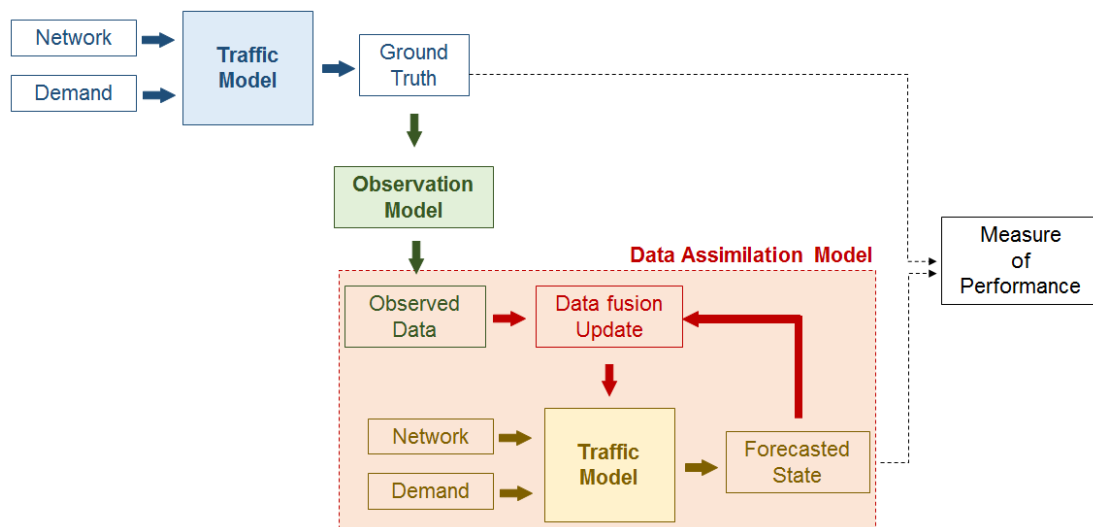


FIGURE 1 Flowchart of methodological framework

The paper proposes a methodological framework to test the performance of DA with respect to the observation model. Figure 1 provides a flowchart of this framework. It starts with the generation of a Ground Truth (GT) with a dynamic traffic flow model. Then an observation model is defined with the number and the location of sensors on the network. Observed data are then derived: flows and speeds collected every 60s at each loop sensor. Based on this observation model, the data assimilation model is applied (see (12) and (5) for more details). It should be noted that the demand-supply scenario of the data assimilation model contains errors, so that the TSE performance relies mainly on the observation model definition.

1 Ground truth generation

2 The ground is generated based on a mesoscopic LWR traffic flow model, which provide traffic
 3 states at any point of the network. Traffic states consist in passing times of every vehicle at ev-
 4 ery node of the network, which allows to derive easily traffic data classically measured by loop
 5 sensors (flow, speed). Based on the ground truth scenario, any observation model can be defined.
 6 This flexibility allows to test any combination of loop sensors, both in number and position. The
 7 resulting data is considered as an input of a DA model.

8 Data assimilation model

9 The DA sequential framework used in the paper is identical to the one recently proposed in (12, 5).
 10 Each sequence can be divided into 4 stages: (1) Collect the observed states from sensors and
 11 forecast states from the dynamic traffic flow model. (2) Estimate the most likely traffic states
 12 through a data fusion model. (3) Update the dynamic traffic flow model accordingly. (4) Run the
 13 model to obtain forecasts over the next sequence. The output of the DA framework also provides
 14 passing times of every vehicle at every node of the network. And the performance of the DA
 15 mainly relies on the observation model, which provides information on the current traffic state and
 16 update the DA model accordingly.

17 Performance indicators

18 To assess the performance of the observation model, traffic states estimated from the DA model
 19 are compared with traffic states from the GT. Two Performance Indicators (PI) are defined.

- The first PI, I_1 , is the root mean square error of travel time:

$$I_1(O) = \sqrt{\frac{1}{N} \sum_{i=t_1}^{t_N} (TT_i^{GT} - \widehat{TT}_i^{DA}(O))^2} \quad (1)$$

21 where N is the number time periods (60 s) during the scenario, $TT_i(O)$ the travel time for the
 22 period i from GT and \widehat{TT}_i^{DA} the travel time obtained from the DA model with the observation
 23 model O .

- The second PI is based on the total distance travelled, *i.e.* the distance travelled by all vehicles during the simulation (equation 2). I_2 measures the difference in total distance between the GT and DA model.

$$I_2(O) = \sum_{n=1}^N (D_n^{GT} - D_n^{DA}(O)) \quad (2)$$

24 where D_n the distance travelled by vehicle n , and N is the total number of vehicles during the
 25 simulation.

26 Pareto optimum

27 Two criteria have been defined to assess the performance of the observation model. For a given
 28 model with n loop sensors, both indicators will likely not provide the same optimal solution.
 29 Consequently, the Pareto optimality concept is used to find the optimal observation model $O^*(n)$.
 30 To do so, the Pareto front is estimated (27) to obtain a set of observation models that are Pareto
 31 efficient.

The optimal observation model $O^*(n)$ is selected by assigning a priority to the maximal marginal benefit of both criteria. The process is illustrated in Figure 2, for eight different observation models with n sensors. Three of them compose the Pareto front (green line) and two are in the area where a benefice for the both criteria (black dotted line) exists. Finally one is maximizing marginal benefit (red circle).

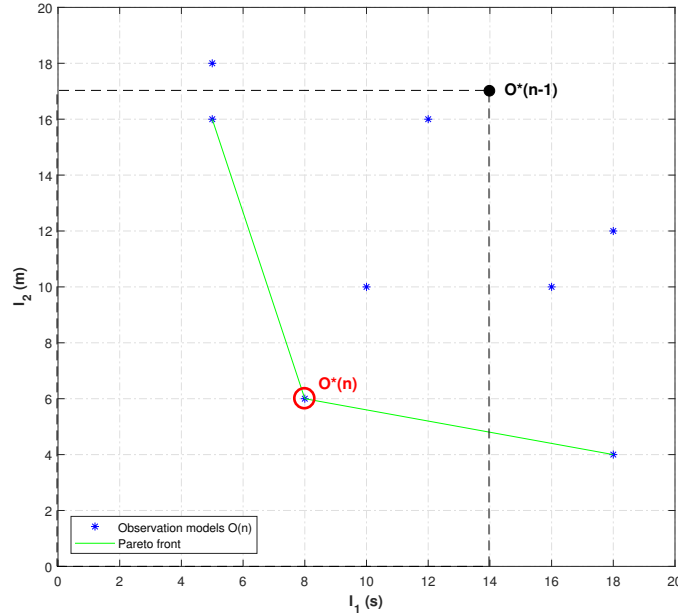


FIGURE 2 Pareto front

Algorithm

The methodological framework presented above has been implemented to assess the optimum number and the position of loop sensors on toy networks. To assess the marginal gain of an additional sensor to an existing observation model, the algorithm has been defined as follows. First, it tests a set of possible positions for $n = 1$ loop sensor on the network, and estimates the optimal observation model $O^*(1)$. Then, the position of the first loop sensor is set according to $O^*(1)$ and the process is repeated for an additional loop sensor on the network. The iterative process is repeated until $n = N$ loop sensors on the network. The different steps are described in Algorithm 1.

RESULTS

Two toy networks are used to assess the performance of the TSE with respect to the observation model.

- Network 1 represents a homogeneous road stretch
- Network 2 represents a freeway corridor with one merge and one diverge

Several demand-supply scenarios are defined for each networks.

Algorithm 1 Optimum number and locations of loop sensors

```

1: procedure INITIALIZATION
2:   set the maximum number of loop sensors  $N$ 
3:   set the initial number of loop sensor  $n = 0$ 
4: end procedure
5: procedure FIND OPTIMUM LOCATION FOR  $n$  SENSORS
6:   while  $n < N$  do
7:     Set  $n = n + 1$ 
8:     Set the position of  $n - 1$  loop sensors according to optimal observation model  $O^*(n - 1)$ 
9:     Define  $S$  the set of possible positions for the loop sensor  $n$ 
10:
11:    for all  $s$  in  $S$  do
12:      Set the observation model  $O(n, s)$ 
13:      Run the DA model with the observation model  $O(n, s)$ 
14:      Calculate indicators  $I_1(O(n, s))$  and  $I_2(O(n, s))$ 
15:    end for
16:    Run the Pareto algorithm and find optimal observation model  $O^*(n)$ 
17:  end while
18: end procedure

```

Network 1

Network 1 aims to identify the optimal number and the locations of loop sensors on a single link.

Scenario definition

Network 1 is a homogeneous road stretch (length = 1000m, 3 lanes). A ground truth has been generated with the following scenario: the upstream demand is piecewise constant: 10% until $t = 600s$, then 75% until $t = 1200s$, and 10% until $t = 3600s$; the downstream supply (exit) is constant for the entire duration of the simulation : 50%. Here, both the demand and the supply have been normalized with respect to the road capacity. For the ground truth scenario, the congestion is triggered starting from $t = 1200s$. Then the congestion propagates backward (until $x = 750m$) and gradually decreases. The free-flow situation is recovered at the time $t = 1700s$.

The demand scenario of the DA model has been intentionally changed (constant, 42.5%) for the DA model. No congestion occurs, which render the traffic states estimated by the DA model inaccurate. The impact of the observation model on the DA performance is illustrated in Figure 3.

Results

Figure 3 presents the performance of DA with respect to the observation model, both in number and in position.

One loop sensor On the left of Figure 3, the observation model is composed of a single loop sensor. The performance of the DA model is illustrated for a different loop sensor position between $x = 100m$ and $x = 900m$. Figure 3(a) shows that the travel time estimation is improved for all the tested positions. It also shows that the best performance is obtained when the loop sensor is

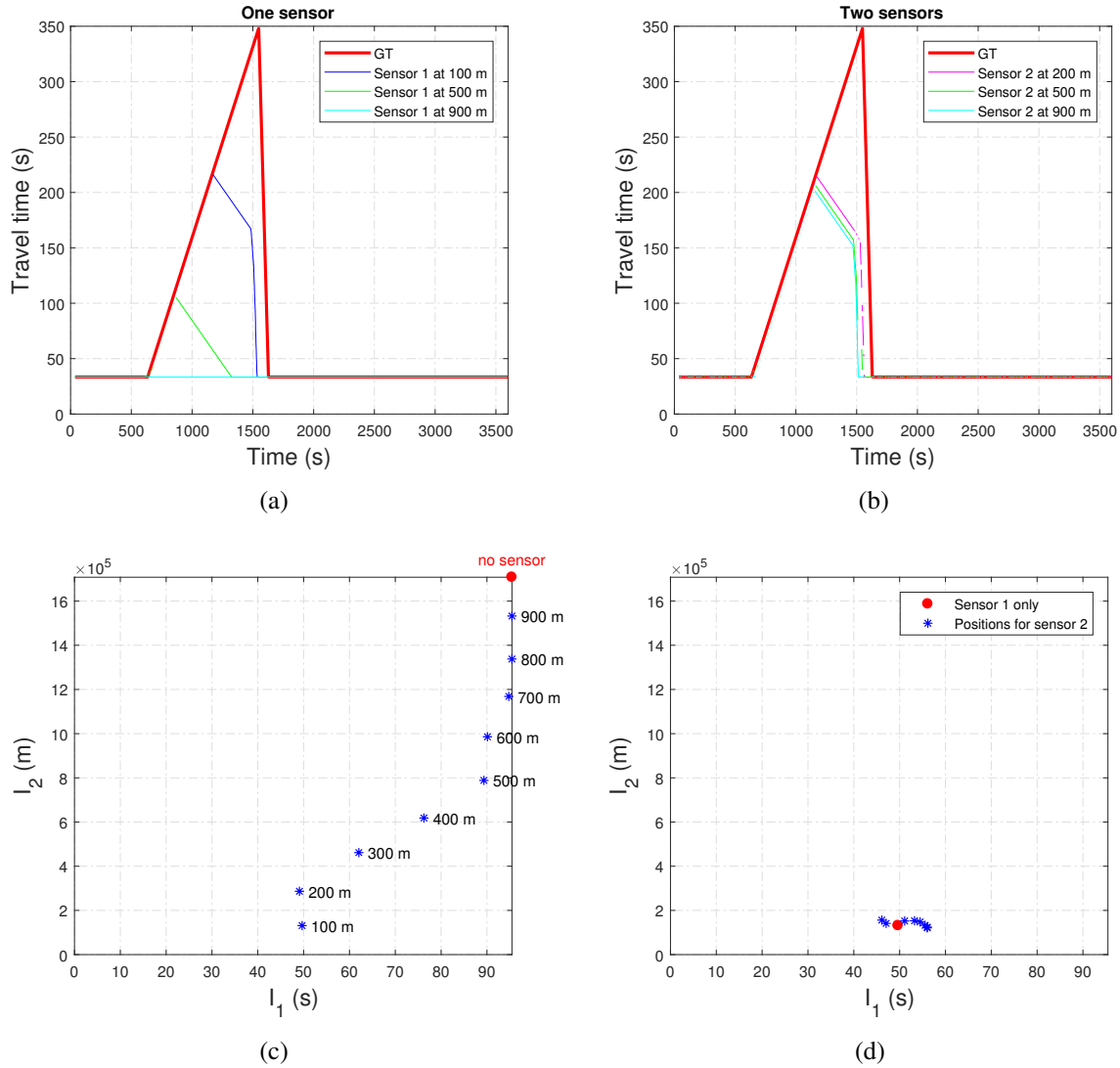


FIGURE 3 Performance of DA on Network 1: One sensor (left) and two sensors (right), Travel time estimation (top) and Performance Indicators (down)

positioned close to the entry.

The results are confirmed in Figure 3(c), where both indicators I_1 and I_2 are plotted with respect to the position of the loop sensor. I_1 , which is related to the travel time estimation, shows that the benefits in travel time reconstruction are optimal for $x = 100m$ and $x = 200m$. Both optimal positions are located upstream of the most upstream location of the congestion. When $x \geq 300m$, the travel time reconstruction becomes worse as the sensor moves away from the entry. I_2 , which is related to the total travelled distance, decreases linearly with respect to the distance to the entry. The Pareto efficiency constraint gives that the optimal position for sensor 1 is $x = 100m$. We conclude on an intuitive result: when the demand of the DA model is inaccurate, states observed close to the entry of the network improve TSE. The conclusion is similar for inaccurate supplies: the TSE is improved when the observation model has provided a sensor located close to

the head of the congestion.

Looking beyond this intuition, it also confirms that I_1 and I_2 are appropriate indicators to measure the performance of the DA model with respect to the observation model.

Two loop sensors Here, the observation model is composed of two sensors. Sensor 1 is set at its optimal position $x = 100m$ (see above), and all the possible positions are tested for sensor 2, between $200m$ and $900m$. The performance of the DA model is illustrated on the right of 3. The travel time estimation slightly improved for all the tested second observations (for a better readability, only 3 positions are plotted). The result is confirmed in Figure 3(d). Blue stars show the results for different positions for sensor 2, and the red circle shows the result with a single loop sensor. The cluster of blue stars is centered around the red circle, which means that the marginal benefit of the second sensor is poor. Again, it confirms an intuitive result: the inaccurate demand of the DA model is already adjusted from sensor 1 observation. Consequently, when the benefit of sensor 2 is marginal, we conclude that a single sensor should be sufficient to provide satisfying TSE. It should also be noted that when both the demand at entry and the supply at exit are inaccurate, the marginal benefit of sensor 2 is high.

From Network 1, we conclude that loop sensor deployment depends on the network topology because sensors must be located close to network discontinuities. The optimal number and location of sensors is also related to traffic conditions and traffic model performance with no observation model. Indeed, every additional sensor will help the model to rectify an inaccuracy related to the demand, the supply, or to a calibration defect.

Network 2

The objective is to understand the causal relationship between the observation model and the performance of the TSE for a more complex network, with on- and off- ramps. The network is illustrated in Figure 4(a). Links 1-2-3 share the same properties in length ($10km$), number of lanes (3). Links 4 and 5 are shorter ($5km$) with two lanes only.

Scenario definition

We use the same methodology as for network 1. First, a ground truth has been generated with a traffic flow model. The demand-supply scenario has been defined over one hour, see Figure 4(b). The upstream demand at entry 1 is piecewise constant: 10% -> 75% -> 10%. The upstream demand on entry 2 is also piecewise constant: 7% -> 67% -> 7%. The downstream supplies are constant: 30% for exit 1 and 7% for exit 2 (Figure 4(c)). Here again, demand and supply have been normalized with respect to the capacity of a 3-lane carriageway. At the diverge point (node 3), the demand is fairly distributed among the two directions. The simulation results show that three congestions are triggered: at node 2, propagating backwards along links 1 and 4; at node 4, propagating along link 3; at node 6, propagating along link 5.

The demand scenario for the DA model has been intentionally changed as follows. At entries 1 and 2, they have been set to be constant at 42.5%, and the splitting ratio at node 3 has been skewed: 90/10 instead of 50/50. Moreover, the capacities at exits have been increased. Considering this inaccurate demand-supply scenario, the DA model with no observation returns free-flowing conditions on the network.

In the following, ten possible locations for optimal loop detectors are used according the network 1 results (in red rectangle on Figure 4(a)).

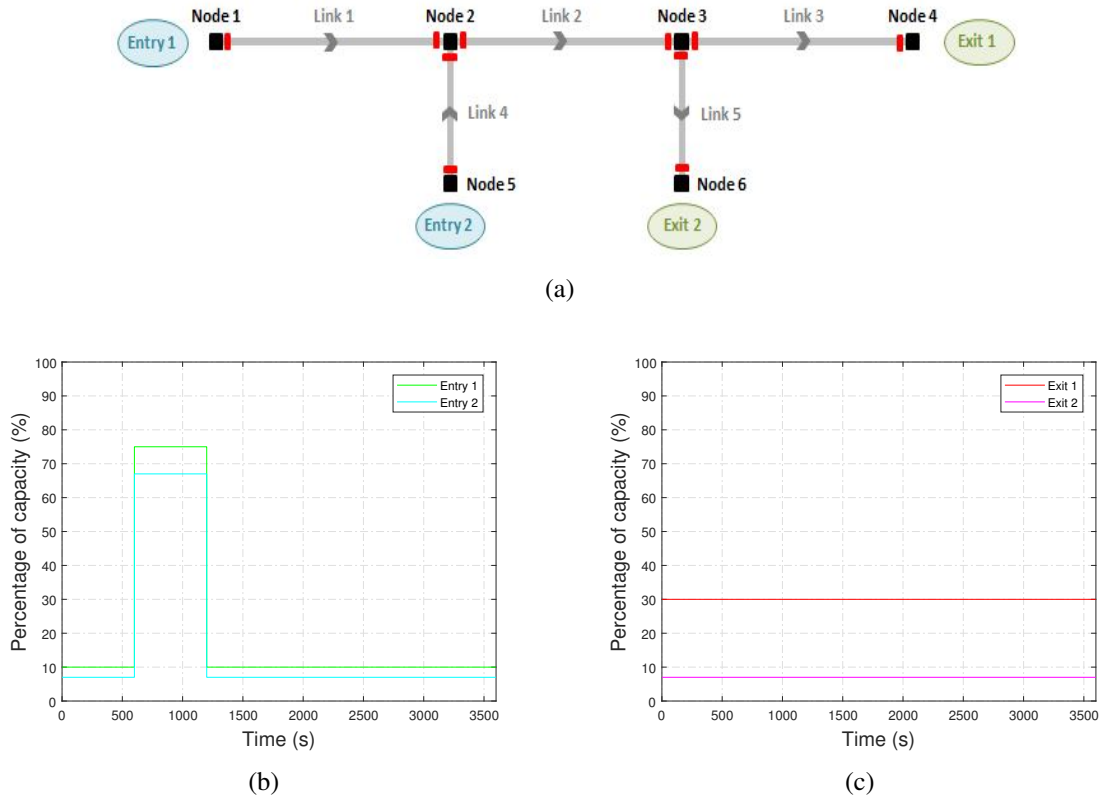


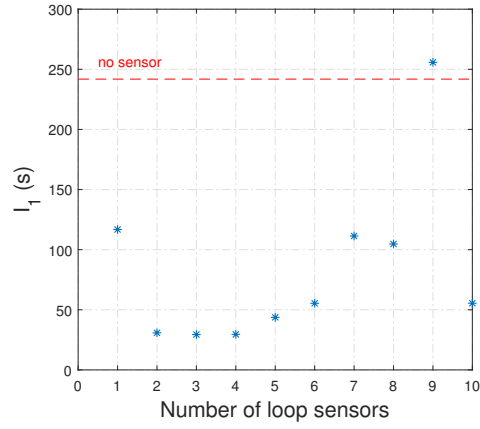
FIGURE 4 Network 2: (a) network, (b) demand at entries, (c) supply at exits

1 Results

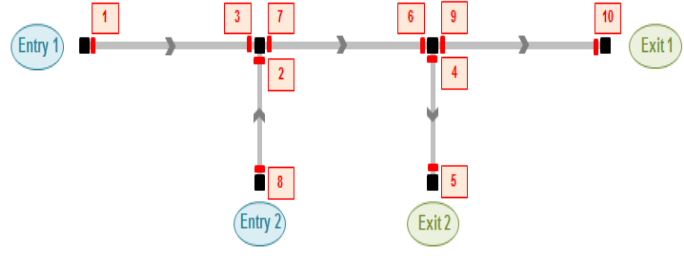
2 Three optimization processes has been tested : minimizing I_1 only, minimizing I_2 only, and mini-
 3 mizing both using the Pareto optimality concept.

4 Figure 5 presents the performance of DA with respect to the observation model, both in
 5 marginal benefit and in optimal position. On this figure, the left column represents the perfor-
 6 mances with respect to the number of loop sensors and the right column show the corresponding
 7 positions on the network.

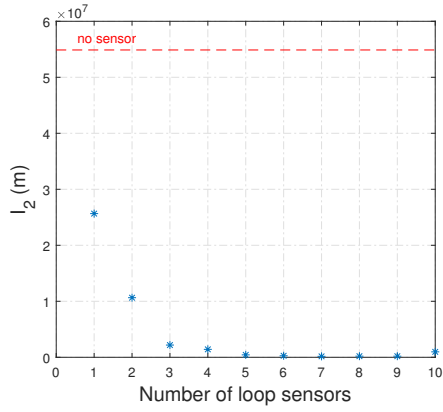
8 **Minimizing I_1 (only)** Figures 5(a) and 5(b) show the optimal results based on I_1 only. As a
 9 reminder, I_1 is based on travel time, leading to a sensor deployment that gives a priority to the
 10 quality of travel time estimation on the network. The top priority sensor (1) is located upstream
 11 node 2, where the major traffic jam is triggered. By doing so, the observation model provides
 12 decisive information to update the demand upstream the network. The second priority sensors (2
 13 and 3) are located immediatly upstream the node 2. The traffic data observed at these points play
 14 an important role to properly model the capacity at this node and update traffic volume from link 4
 15 accordingly. It should be noticed that the sensors 4 is located immediatly downstream the diverge.
 16 It helps to rectify the splitting ratio according to observed traffic volumes mesured immediatly
 17 downstream node 3. Additional sensors (5 or more) deteriorate the travel time estimation. It can
 18 be explained has follow : additional sensors are located close to already deployed sensors, which
 19 lead to unconstitencies in traffic data due to agregation procedure (classicaly between 1 and 5 min).



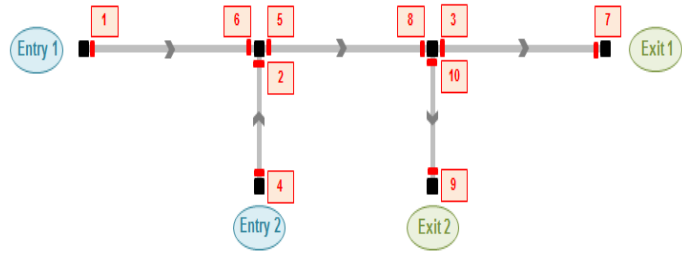
(a)



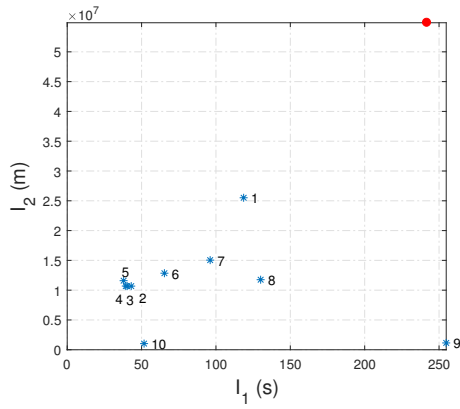
(b)



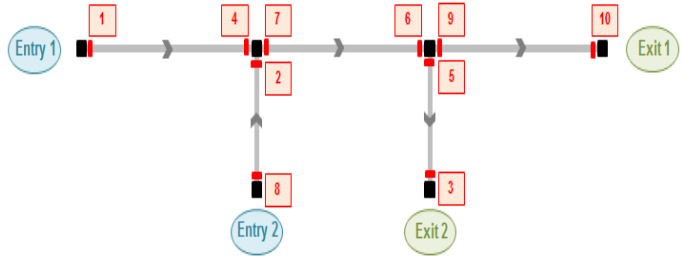
(c)



(d)



(e)



(f)

FIGURE 5 Performance of DA on Network 2: marginal benefit values (left) and optimal position (right), I_1 (top), I_2 (second line) and Pareto optimum (down)

1 Consequently, based on I_1 only, the optimal observation model offers a better representation of the

1 imbalance between demand and supply, which size the traffic jam on the network and provide a
 2 better travel time estimation. However, free-flowing situations are not well-adressed, which may
 3 lead to traffic volume errors when the network is free-flowing.

4 **Minimizing I_2 (only)** Figures 5(c) and 5(d) show the optimal results based on I_2 only. As a
 5 reminder, I_2 is based on total travelled distance, leading to a sensor deployment that gives a
 6 priority to the quality of traffic volumes on the network. Only 4 sensors present a major importance
 7 according to I_2 . Sensors 1 and 4 aim to offer a better representation of the traffic volume at
 8 entries of the network. Sensors 2 and 3 both improve the model performance near internal nodes.
 9 The results also show that the benefit of additional observation sensors (5 or more) is negligible.
 10 Consequently, based on I_2 only, the optimal observation model offers a better representation of the
 11 total traffic volume on the network. But the optimal solution does not guaranty that traffic jam and
 12 travel times are properly estimated.

13 **Pareto optimality concept** The optimality of both previous indicators is combined, using the
 14 Pareto optimality concept (Figures 5(c) and 5(d)) Sensors 1 and 2 are identical to the previous
 15 optimality conditions. Sensor 3 is located at immediatly upstream an exit, and sensors 4 and 5
 16 are positionned at internal nodes (upstream merge, downstream diverge). The results also confirm
 17 that additional sensors deteriorate the performance of the Traffic State Estimation, especially travel
 18 time.

19
 20 From Network 2, we conclude that loop sensor deployment must be based on:

- 21 - the topology of the network: near entries and exits for a better representation of total traffic
 22 volumes, upstream merge to properly update the model in case of congestion, and downstream
 23 diverge to rectify errors in splitting ratios.
- 24 - the performance of the traffic model: the optimal number of sensors is 5, which also correspond
 25 to the number of variables that have been intentionnaly changed (two demands at entries, two
 26 capacities at exits, one split ratio).

27 CONCLUSION

28 Main findings

29 The paper presents a simulation-based methodology to optimize the number and the positions of
 30 loop sensors, in order to improve Traffic State Estimated from loop sensor in the context of Data
 31 Assimilation The method has evaluated the performance of the DA model regarding its ability to
 32 estimate both the travel times and total travelled distance on the network. The results confirms
 33 the importance of the network discontinuities, where congestion are often trigered. It also confirm
 34 that the number of sensors to be deployed should be based on the network topology and the traffic
 35 model performance with no observation model. Based on these principles, both travel times and
 36 total travelled distance can be greatly improved, which demonstrates the ability of the method to
 37 be applied for free-flowing and congested networks. On this basis, sensor removal or addition
 38 strategies can be built for an optimal traffic state estimation.

39 Further research

40 The proposed methodology has been applied on toy networks, for inaccurate demand-supply sce-
 41 narios. Further research is still needed to confirm its practical use with real network.

1 • What is the importance of the calibration of the traffic model, and how to design the
2 observation model in order to rectify calibration errors?

3 • In the paper, the observation model is only composed of loop sensors. Floating Car Data
4 are more and more popular and the question of observability can be posed similarly : can the
5 propose methodology be replicated to size the FCD in the context of DA?

6 • In the paper, the authors have used the DA framework proposed in (12, 5). Further
7 research may be needed to generalise the conclusions for other traffic model (micro, macro) and
8 other DA frameworks.

9 And finally, the paper proposes to built the optimum observations model based on two indicators,
10 and the optimum is determined based on Pareto optimality concept. Other optimization techniques
11 could be explored, for instance bioinspired methods as genetic algorithms.

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