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# Traffic State Estimation: Optimal Number and Location of Loop Detectors in the context of Data Assimilation

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1 Traffic State Estimation: Optimal Number and Location  
2 of Loop Detectors in the context of Data Assimilation

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**1 ABSTRACT**

2 Traffic managers need tools capable of providing online traffic flow monitoring and short-term  
3 predictions on large-scale networks. Data Assimilation (DA) techniques provide traffic state esti-  
4 mates based on a dynamic model and an observation model. Their performance are very sensitive  
5 to the observation model definition. The paper proposes a solution to optimize the number and the  
6 location of loop sensors in the context of DA. The optimality is defined based on travel time and  
7 travelled distance indicators, and Pareto efficiency is used to determine the optimal sensor alloca-  
8 tion. The method is tested on two toy networks to assess the role and level of contribution of each  
9 sensor. The results provide a better understanding for deploying additional sensors or selecting  
10 sensors to be removed, for optimal traffic state estimation and traffic forecast.

## 1 INTRODUCTION

### 2 State of the art

3 Traffic managers need tools capable of providing online traffic flow monitoring and short-term  
4 forecasts on large-scale networks. These methods aim to obtain an estimate of current and future  
5 traffic conditions. Classically, they rely on traffic data collected by loop sensors, and the perfor-  
6 mance of the traffic state estimation depends on the number and location of sensors on the network.  
7 However, traffic managers are facing budgetary constraints : the price of each sensor does not allow  
8 for massive complementary deployment, and existing sensors need constant and expensive main-  
9 tenance operations. In this context, the optimization of the number and the location of loop sensors  
10 is a critical issue for traffic managers.

11 Traffic monitoring and forecast can be achieved with Data Assimilation (DA) techniques. Since  
12 the 2000s, many DA solutions have been proposed. They combine observed data (also called ob-  
13 servation model) with a dynamic traffic flow model to estimate traffic states at any point of the  
14 network. Observed and forecasted states are then combined through a data fusion method, which  
15 provide the most likely states on the network. Classically, observed traffic data are collected with  
16 a given frequency, which implies data assimilation methods to be sequential. They are based on  
17 two key components: the dynamic traffic flow model, and the observation model.

18  
19 Dynamic traffic flow models can be categorized according to various criteria (see (1) for  
20 an overview): level of detail, operationalization, representation of the processes. Three modeling  
21 scales can be identified. Macroscopic models provide the evolution of densities on the network  
22 with respect to time (*i.e.* Eulerian). Microscopic models provide the evolution of vehicles position  
23 with respect to time (*i.e.* Lagrangian Time). Finally mesoscopic models provide the passage time  
24 of vehicles with respect to position (*i.e.* Lagrangian Space). They have been introduced in (2, 3) to  
25 be the solution of the Lighthill-Whitham-Richards (LWR) model (4). The later scale seems to be  
26 the most promising solution for traffic monitoring and traffic forecast on a large scale network (5).

27 The observation model can be composed of two types of data (5, 6, 7): Eulerian and La-  
28 grangian. Eulerian data, from spatially-fixed sensors, have been the most widely used for their  
29 high level of availability (8, 9, 10) since the fifties. Lagrangian data, from vehicles equipped with  
30 sensors, can provide positions at a fixed time intervals. The availability of Lagrangian data has in-  
31 creased during the recent years and solutions have been recently proposed to assimilate such data  
32 in the context of data assimilation (11, 12, 13, 5).

33  
34 The observation model is critical in data assimilation applications, because it provides in-  
35 formation on the current state on the network. Castillo and al. (14) describe the observation models  
36 as an observability problem. The optimisation problem contains three parts: (15)

- 37 • Network description

38 It can be a toy or a real network. A toy network is simple and helps to better understand the way the  
39 model works. For instance, freeway corridors with only merges and exits (5), or a mesh network  
40 (16), are often used for toy network. A real network is more complex and can show the operability  
41 of the model. The literature provides many references that consider freeway corridors (17, 18) or  
42 larger networks on the scale of a city (19, 14, 20).

- 43 • Constraints to limit the solutions

44 (15, 21) have proposed a list of constraints for traffic observation models. They rely on the flow  
45 conservation principle and the positivity of speed. Topological constraint can be defined, for in-

1 stance Eisenman considers the impossibility for two successive links without entrance or exit to  
 2 have an observation localisation (6).

3 • Objective function to select one optimal solution

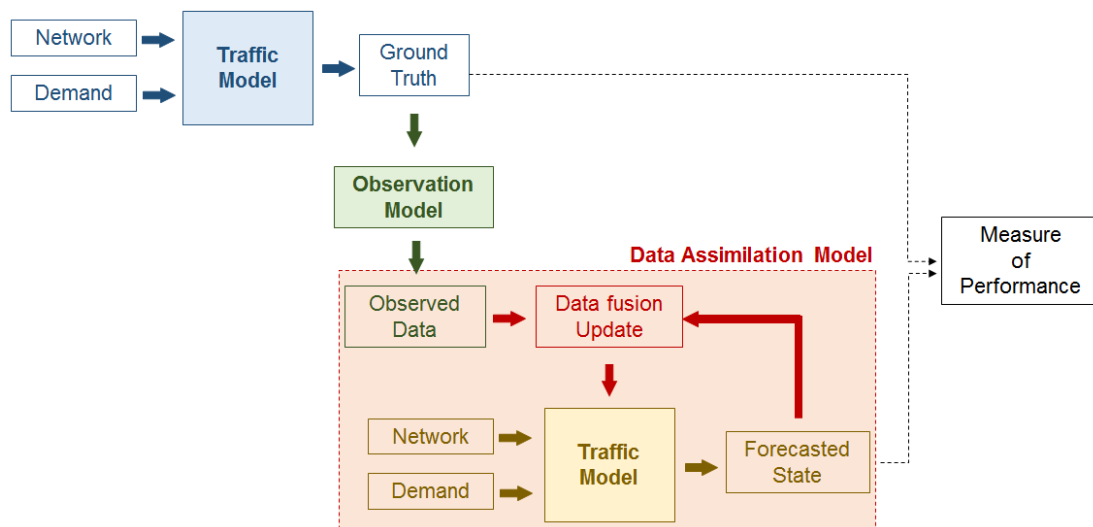
4 Castillo and al. have proposed a global overview of objective functions, which must be adjusted ac-  
 5 cording to the objective of the study. It can be : origin–destination matrix estimation (16, 22), flow  
 6 reconstruction (23, 24, 18), travel time estimation (20, 25, 17), or freeway congestion monitoring  
 7 (26). The present paper belongs to the last category.

## 8 Contributions and objectives

9 This paper presents a method for assessing the observation model in the context of Traffic State  
 10 Estimation. The method to understand the role and the level on contribution of each sensor is  
 11 proposed. This paper is organized as follows. Section 2 describes the methodology used to obtain  
 12 the observation model, including the description of performance indicators. Section 3 presents the  
 13 results for two toy networks. Section 4 proposes a discussion around the operational context for  
 14 traffic managers. Section 5 closes the paper on the needs for further research.

## 15 METHODOLOGY

### 16 Methodological framework



**FIGURE 1** Flowchart of methodological framework

17 The paper proposes a methodological framework to test the performance of DA with respect  
 18 to the observation model. Figure 1 provides a flowchart of this framework. It starts with the  
 19 generation of a Ground Truth (GT) with a dynamic traffic flow model. Then an observation model  
 20 is defined with the number and the location of sensors on the network. Observed data are then  
 21 derived: flows and speeds collected every 60s at each loop sensor. Based on this observation  
 22 model, the data assimilation model is applied (see (12) and (5) for more details). It should be  
 23 noted that the demand-supply scenario of the data assimilation model contains errors, so that the  
 24 TSE performance relies mainly on the observation model definition.

## 1 **Ground truth generation**

2 The ground is generated based on a mesoscopic LWR traffic flow model, which provide traffic  
 3 states at any point of the network. Traffic states consist in passing times of every vehicle at ev-  
 4 ery node of the network, which allows to derive easily traffic data classically measured by loop  
 5 sensors (flow, speed). Based on the ground truth scenario, any observation model can be defined.  
 6 This flexibility allows to test any combination of loop sensors, both in number and position. The  
 7 resulting data is considered as an input of a DA model.

## 8 **Data assimilation model**

9 The DA sequential framework used in the paper is identical to the one recently proposed in (12, 5).  
 10 Each sequence can be divided into 4 stages: (1) Collect the observed states from sensors and  
 11 forecast states from the dynamic traffic flow model. (2) Estimate the most likely traffic states  
 12 through a data fusion model. (3) Update the dynamic traffic flow model accordingly. (4) Run the  
 13 model to obtain forecasts over the next sequence. The output of the DA framework also provides  
 14 passing times of every vehicle at every node of the network. And the performance of the DA  
 15 mainly relies on the observation model, which provides information on the current traffic state and  
 16 update the DA model accordingly.

## 17 **Performance indicators**

18 To assess the performance of the observation model, traffic states estimated from the DA model  
 19 are compared with traffic states from the GT. Two Performance Indicators (PI) are defined.

20

- The first PI,  $I_1$ , is the root mean square error of travel time:

$$I_1(O) = \sqrt{\frac{1}{N} \sum_{i=t_1}^{t_N} (TT_i^{GT} - \widehat{TT}_i^{DA}(O))^2} \quad (1)$$

21 where  $N$  is the number time periods (60 s) during the scenario,  $TT_i(O)$  the travel time for the  
 22 period  $i$  from GT and  $\widehat{TT}_i^{DA}$  the travel time obtained from the DA model with the observation  
 23 model  $O$ .

- The second PI is based on the total distance travelled, *i.e.* the distance travelled by all vehicles during the simulation (equation 2).  $I_2$  measures the difference in total distance between the GT and DA model.

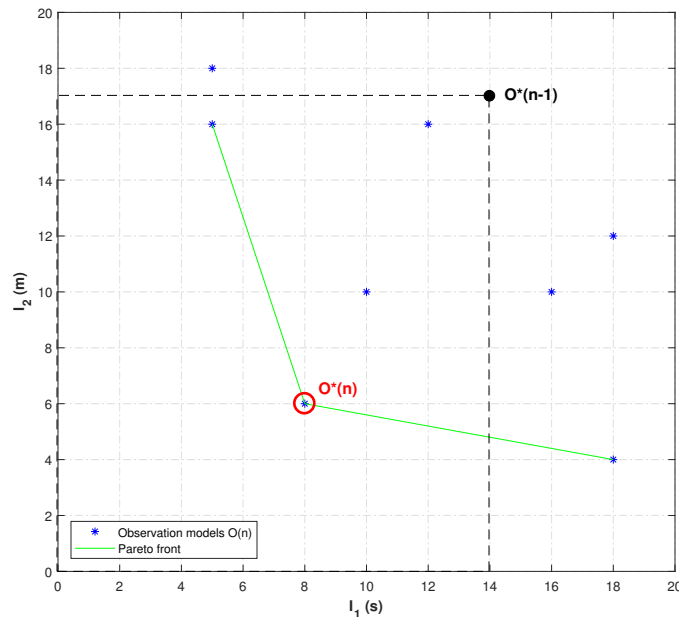
$$I_2(O) = \sum_{n=1}^N (D_n^{GT} - D_n^{DA}(O)) \quad (2)$$

24 where  $D_n$  the distance travelled by vehicle  $n$ , and  $N$  is the total number of vehicles during the  
 25 simulation.

## 26 **Pareto optimum**

27 Two criteria have been defined to assess the performance of the observation model. For a given  
 28 model with  $n$  loop sensors, both indicators will likely not provide the same optimal solution.  
 29 Consequently, the Pareto optimality concept is used to find the optimal observation model  $O^*(n)$ .  
 30 To do so, the Pareto front is estimated (27) to obtain a set of observation models that are Pareto  
 31 efficient.

1 The optimal observation model  $O^*(n)$  is selected by assigning a priority to the maximal  
 2 marginal benefit of both criteria. The process is illustrated in Figure 2, for eight different obser-  
 3 vation models with  $n$  sensors. Three of them compose the Pareto front (green line) and two are in  
 4 the area where a benefice for the both criteria (black dotted line) exists. Finally one is maximizing  
 5 marginal benefit (red circle).



**FIGURE 2 Pareto front**

## 6 Algorithm

7 The methodological framework presented above has been implemented to assess the optimum  
 8 number and the position of loop sensors on toy networks. To assess the marginal gain of an addi-  
 9 tional sensor to an existing observation model, the algorithm has been defined as follows. First, it  
 10 tests a set of possible positions for  $n = 1$  loop sensor on the network, and estimates the optimal  
 11 observation model  $O^*(1)$ . Then, the position of the first loop sensor is set according to  $O^*(1)$   
 12 and the process is repeated for an additional loop sensor on the network. The iterative process is  
 13 repeated until  $n = N$  loop sensors on the network. The different steps are described in Algorithm  
 14 1.

15

## 16 RESULTS

17 Two toy networks are used to assess the performance of the TSE with respect to the observation  
 18 model.

- 19 • Network 1 represents a homogeneous road stretch
- 20 • Network 2 represents a freeway corridor with one merge and one diverge

21 Several demand-supply scenarios are defined for each networks.

**Algorithm 1** Optimum number and locations of loop sensors

---

```

1: procedure INITIALIZATION
2:   set the maximum number of loop sensors  $N$ 
3:   set the initial number of loop sensor  $n = 0$ 
4: end procedure
5: procedure FIND OPTIMUM LOCATION FOR  $n$  SENSORS
6:   while  $n < N$  do
7:     Set  $n = n + 1$ 
8:     Set the position of  $n - 1$  loop sensors according to optimal observation model  $O^*(n - 1)$ 
9:     Define  $S$  the set of possible positions for the loop sensor  $n$ 
10:
11:    for all  $s$  in  $S$  do
12:      Set the observation model  $O(n, s)$ 
13:      Run the DA model with the observation model  $O(n, s)$ 
14:      Calculate indicators  $I_1(O(n, s))$  and  $I_2(O(n, s))$ 
15:    end for
16:    Run the Pareto algorithm and find optimal observation model  $O^*(n)$ 
17:  end while
18: end procedure

```

---

**1 Network 1**

2 Network 1 aims to identify the optimal number and the locations of loop sensors on a single link.

*3 Scenario definition*

4 Network 1 is a homogeneous road stretch (length = 1000m, 3 lanes). A ground truth has been  
5 generated with the following scenario: the upstream demand is piecewise constant: 10% until  
6  $t = 600s$ , then 75% until  $t = 1200s$ , and 10% until  $t = 3600s$ ; the downstream supply (exit) is  
7 constant for the entire duration of the simulation : 50%. Here, both the demand and the supply have  
8 been normalized with respect to the road capacity. For the ground truth scenario, the congestion is  
9 triggered starting from  $t = 1200s$ . Then the congestion propagates backward (until  $x = 750m$ ) and  
10 gradually decreases. The free-flow situation is recovered at the time  $t = 1700s$ .

11 The demand scenario of the DA model has been intentionally changed (constant, 42.5%)  
12 for the DA model. No congestion occurs, which render the traffic states estimated by the DA model  
13 inaccurate. The impact of the observation model on the DA performance is illustrated in Figure 3.

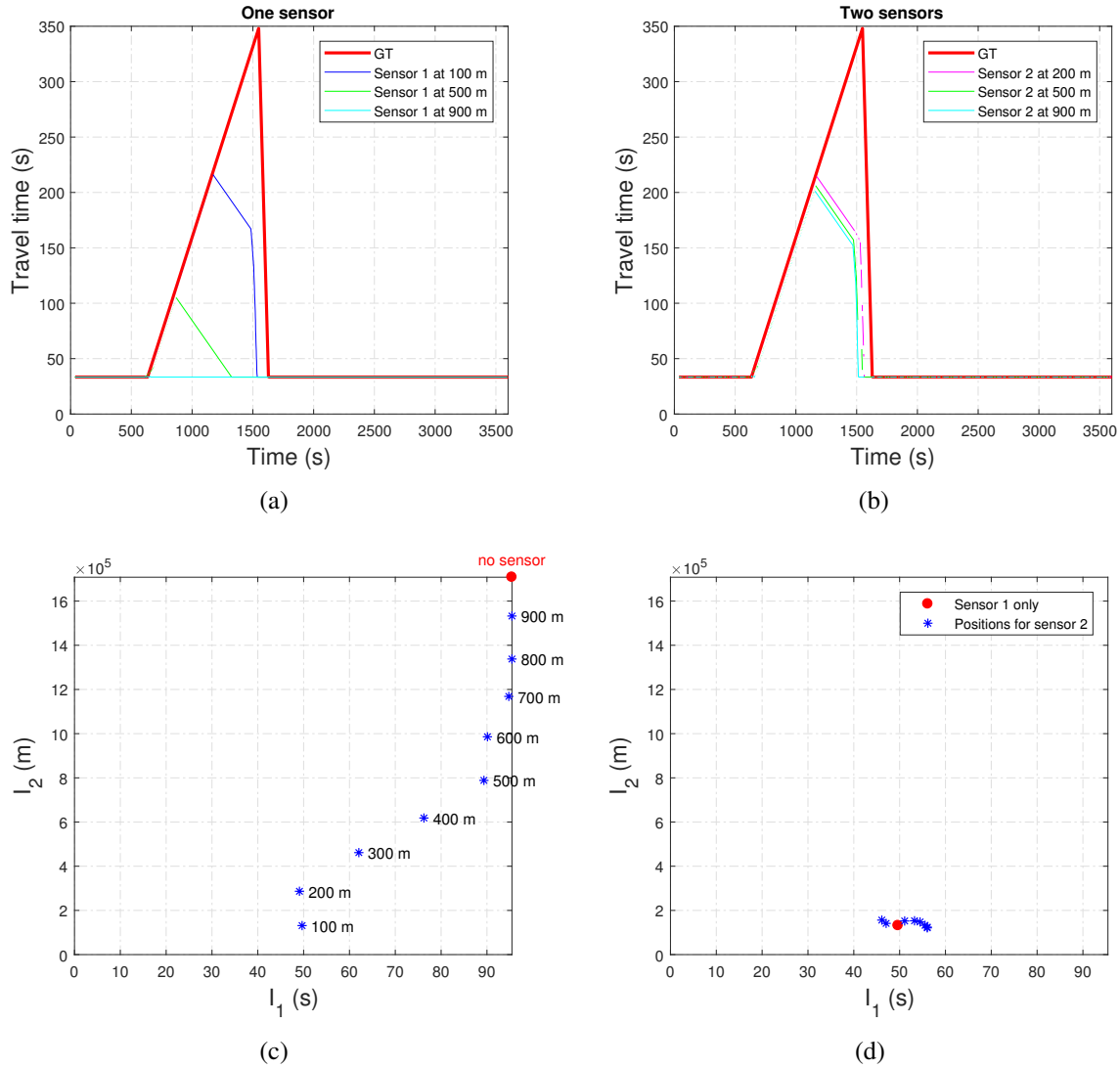
*14 Results*

15 Figure 3 presents the performance of DA with respect to the observation model, both in number  
16 and in position.

17

18 **One loop sensor** On the left of Figure 3, the observation model is composed of a single loop  
19 sensor. The performance of the DA model is illustrated for a different loop sensor position between  
20  $x = 100m$  and  $x = 900m$ . Figure 3(a) shows that the travel time estimation is improved for all  
21 the tested positions. It also shows that the best performance is obtained when the loop sensor is





**FIGURE 3 Performance of DA on Network 1: One sensor (left) and two sensors (right), Travel time estimation (top) and Performance Indicators (down)**

1 positioned close to the entry.

2 The results are confirmed in Figure 3(c), where both indicators  $I_1$  and  $I_2$  are plotted with  
 3 respect to the position of the loop sensor.  $I_1$ , which is related to the travel time estimation, shows  
 4 that the benefits in travel time reconstruction are optimal for  $x = 100m$  and  $x = 200m$ . Both  
 5 optimal positions are located upstream of the most upstream location of the congestion. When  
 6  $x \geq 300m$ , the travel time reconstruction becomes worse as the sensor moves away from the entry.  
 7  $I_2$ , which is related to the total travelled distance, decreases linearly with respect to the distance  
 8 to the entry. The Pareto efficiency constraint gives that the optimal position for sensor 1 is  $x =$   
 9  $100m$ . We conclude on an intuitive result: when the demand of the DA model is inaccurate, states  
 10 observed close to the entry of the network improve TSE. The conclusion is similar for inaccurate  
 11 supplies: the TSE is improved when the observation model has provided a sensor located close to

1 the head of the congestion.  
 2 Looking beyond this intuition, it also confirms that  $I_1$  and  $I_2$  are appropriate indicators to measure  
 3 the performance of the DA model with respect to the observation model.

4 **Two loop sensors** Here, the observation model is composed of two sensors. Sensor 1 is set at  
 5 its optimal position  $x = 100m$  (see above), and all the possible positions are tested for sensor  
 6 2, between  $200m$  and  $900m$ . The performance of the DA model is illustrated on the right of 3  
 7 The travel time estimation slightly improved for all the tested second observations (for a better  
 8 readability, only 3 positions are plotted). The result is confirmed in Figure 3(d). Blue stars show  
 9 the results for different positions for sensor 2, and the red circle shows the result with a single  
 10 loop sensor. The cluster of blue stars is centered around the red circle, which means that the  
 11 marginal benefit of the second sensor is poor. Again, it confirms an intuitive result: the inaccurate  
 12 demand of the DA model is already adjusted from sensor 1 observation. Consequently, when the  
 13 benefit of sensor 2 is marginal, we conclude that a single sensor should be sufficient to provide  
 14 satisfying TSE. It should also be noted that when both the demand at entry and the supply at exit  
 15 are inaccurate, the marginal benefit of sensor 2 is high.

16 From Network 1, we conclude that loop sensor deployment depends on the network topol-  
 17 ogy because sensors must be located close to network discontinuities. The optimal number and  
 18 location of sensors is also related to traffic conditions and traffic model performance with no ob-  
 19 servation model. Indeed, every additional sensor will help the model to rectify an inaccuracy  
 20 related to the demand, the supply, or to a calibration defect.

## 21 Network 2

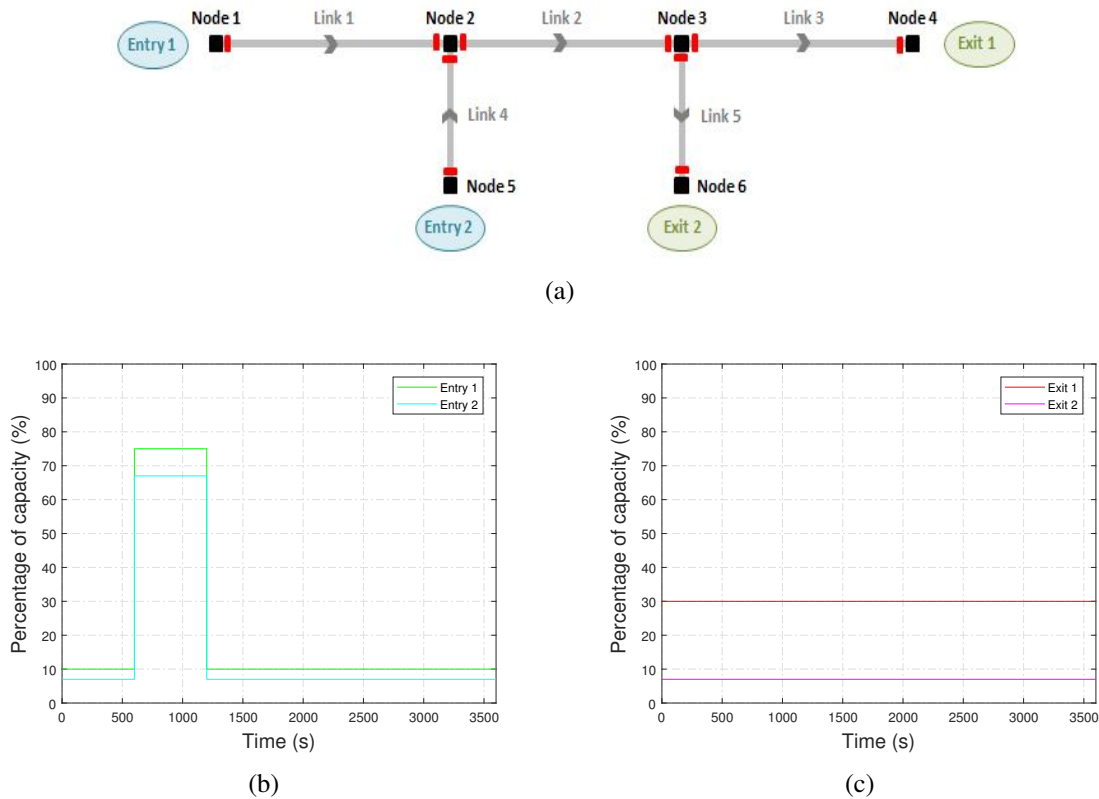
22 The objective is to understand the causal relationship between the observation model and the per-  
 23 formance of the TSE for a more complex network, with on- and off- ramps. The network is  
 24 illustrated in Figure 4(a). Links 1-2-3 share the same properties in length ( $10km$ ), number of lanes  
 25 (3). Links 4 and 5 are shorter ( $5km$ ) with two lanes only.

### 26 Scenario definition

27 We use the same methodology as for network 1. First, a ground truth has been generated with  
 28 a traffic flow model. The demand-supply scenario has been defined over one hour, see Figure  
 29 4(b) The upstream demand at entry 1 is piecewise constant:  $10\% \rightarrow 75\% \rightarrow 10\%$ . The upstream  
 30 demand on entry 2 is also piecewise constant:  $7\% \rightarrow 67\% \rightarrow 7\%$ . The downstream supplies are  
 31 constant:  $30\%$  for exit 1 and  $7\%$  for exit 2 (Figure 4(c)). Here again, demand and supply have  
 32 been normalized with respect to the capacity of a 3-lane carriageway. At the diverge pint (node  
 33 3), the demand is fairly distributed among the two directions. The simulation results show that  
 34 three congestions are triggered: at node 2, propagating backwards along links 1 and 4; at node 4,  
 35 propagating along link 3; at node 6, propagating along link 5.

36 The demand scenario for the DA model has been intentionnaly changed as follows. At  
 37 entries 1 and 2, they have been set to be constant at  $42.5\%$ , and the splitting ratio at node 3 has  
 38 been skewed:  $90/10$  instead of  $50/50$ . Moreover, the capacities at exits have been increased.  
 39 Considering this inaccurate demand-supply scenario, the DA model with no observation returns  
 40 free-flowing conditions on the network.

41 In the following, ten possible locations for optimal loop detectors are used according the network  
 42 1 results (in red rectangle on Figure 4(a)).



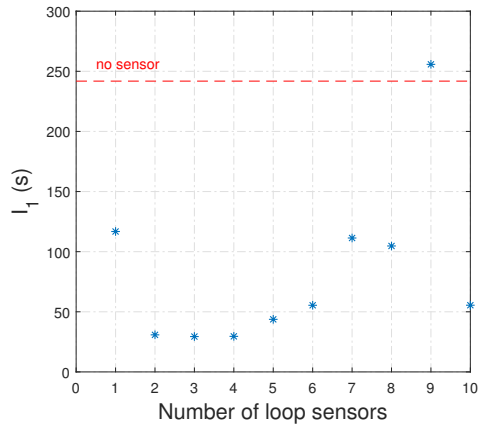
**FIGURE 4** Network 2: (a) network, (b) demand at entries, (c) supply at exits

### 1 Results

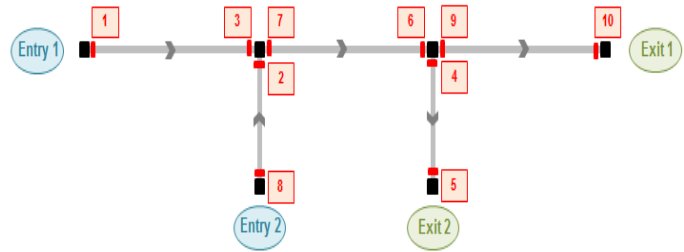
2 Three optimization processes has been tested : minimizing  $I_1$  only, minimizing  $I_2$  only, and mini-  
 3 mizing both using the Pareto optimality concept.

4 Figure 5 presents the performance of DA with respect to the observation model, both in  
 5 marginal benefit and in optimal position. On this figure, the left column represents the perfor-  
 6 mances with respect to the number of loop sensors and the right column show the corresponding  
 7 positions on the network.

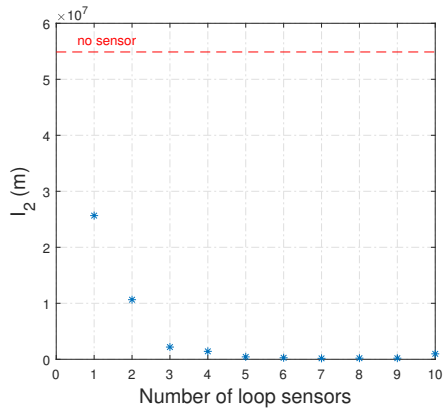
8 **Minimizing  $I_1$  (only)** Figures 5(a) and 5(b) show the optimal results based on  $I_1$  only. As a  
 9 reminder,  $I_1$  is based on travel time, leading to a sensor deployment that gives a priority to the  
 10 quality of travel time estimation on the network. The top priority sensor (1) is located upstream  
 11 node 2, where the major traffic jam is triggered. By doing so, the observation model provides  
 12 decisive information to update the demand upstream the network. The second priority sensors (2  
 13 and 3) are located immediatly upstream the node 2. The traffic data observed at these points play  
 14 an important role to properly model the capacity at this node and update traffic volume from link 4  
 15 accordingly. It should be noticed that the sensors 4 is located immediatly downstream the diverge.  
 16 It helps to rectify the splitting ratio according to observed traffic volumes mesured immediatly  
 17 downstream node 3. Additional sensors (5 or more) deteriorate the travel time estimation. It can  
 18 be explained has follow : additional sensors are located close to already deployed sensors, which  
 19 lead to unconstitencies in traffic data due to agregation procedure (classicaly between 1 and 5 min).



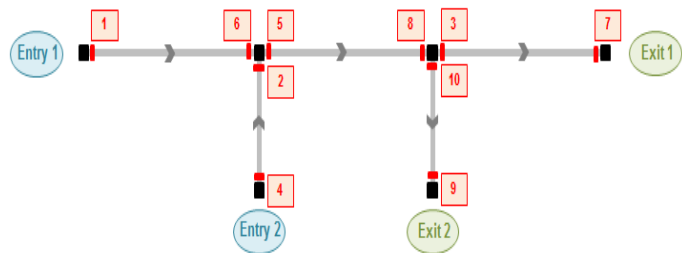
(a)



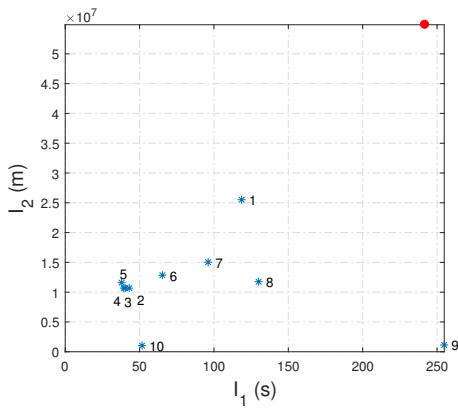
(b)



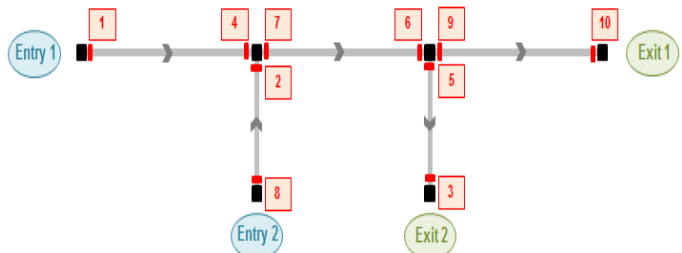
(c)



(d)



(e)



(f)

**FIGURE 5 Performance of DA on Network 2: marginal benefit values (left) and optimal position (right),  $I_1$  (top),  $I_2$  (second line) and Pareto optimum (down)**

1 Consequently, based on  $I_1$  only, the optimal observation model offers a better representation of the

1 imbalance between demand and supply, which size the traffic jam on the network and provide a  
2 better travel time estimation. However, free-flowing situations are not well-adressed, which may  
3 lead to traffic volume errors when the network is free-flowing.

4 **Minimizing  $I_2$  (only)** Figures 5(c) and 5(d) show the optimal results based on  $I_2$  only. As a  
5 reminder,  $I_2$  is based on total travelled distance, leading to a sensor deployment that gives a  
6 priority to the quality of traffic volumes on the network. Only 4 sensors present a major importance  
7 according to  $I_2$ . Sensors 1 and 4 aim to offer a better representation of the traffic volume at  
8 entries of the network. Sensors 2 and 3 both improve the model performance near internal nodes.  
9 The results also show that the benefit of additional observation sensors (5 or more) is negligible.  
10 Consequently, based on  $I_2$  only, the optimal observation model offers a better representation of the  
11 total traffic volume on the network. But the optimal solution does not guaranty that traffic jam and  
12 travel times are properly estimated.

13 **Pareto optimality concept** The optimality of both previous indicators is combined, using the  
14 Pareto optimality concept (Figures 5(c) and 5(d)) Sensors 1 and 2 are identical to the previous  
15 optimality conditions. Sensor 3 is located at immediatly upstream an exit, and sensors 4 and 5  
16 are positionned at internal nodes (upstream merge, downstream diverge). The results also confirm  
17 that additional sensors deteriorate the performance of the Traffic State Estimation, especially travel  
18 time.

19  
20 From Network 2, we conclude that loop sensor deployment must be based on:

- 21 - the topology of the network: near entries and exits for a better representation of total traffic  
22 volumes, upstream merge to properly update the model in case of congestion, and downstream  
23 diverge to rectify errors in splitting ratios.
- 24 - the performance of the traffic model: the optimal number of sensors is 5, which also correspond  
25 to the number of variables that have been intentionnaly changed (two demands at entries, two  
26 capacities at exits, one split ratio).

## 27 CONCLUSION

### 28 Main findings

29 The paper presents a simulation-based methodology to optimize the number and the positions of  
30 loop sensors, in order to improve Traffic State Estimated from loop sensor in the context of Data  
31 Assimilation The method has evaluated the performance of the DA model regarding its ability to  
32 estimate both the travel times and total travelled distance on the network. The results confirms  
33 the importance of the network discontinuities, where congestion are often trigered. It also confirm  
34 that the number of sensors to be deployed should be based on the network topology and the traffic  
35 model performance with no observation model. Based on these principles, both travel times and  
36 total travelled distance can be greatly improved, which demonstrates the ability of the method to  
37 be applied for free-flowing and congested networks. On this basis, sensor removal or addition  
38 strategies can be built for an optimal traffic state estimation.

### 39 Further research

40 The proposed methodology has been applied on toy networks, for inaccurate demand-supply sce-  
41 narios. Further research is still needed to confirm its practical use with real network.

1           • What is the importance of the calibration of the traffic model, and how to design the  
2 observation model in order to rectify calibration errors?

3           • In the paper, the observation model is only composed of loop sensors. Floating Car Data  
4 are more and more popular and the question of observability can be posed similarly : can the  
5 propose methodology be replicated to size the FCD in the context of DA?

6           • In the paper, the authors have used the DA framework proposed in (12, 5). Further  
7 research may be needed to generalise the conclusions for other traffic model (micro, macro) and  
8 other DA frameworks.

9 And finally, the paper proposes to built the optimum observations model based on two indicators,  
10 and the optimum is determined based on Pareto optimality concept. Other optimization techniques  
11 could be explored, for instance bioinspired methods as genetic algorithms.

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