# Achievable Goals in Bayesian Multi-Objective Optimization

#### David Gaudrie

Groupe PSA, École des Mines de Saint-Étienne, France

### Rodolphe le Riche

CNRS LIMOS, École des Mines de Saint-Étienne, France

#### Victor Picheny

Prowler.io, Royaume-Uni

**Mots-clefs**: Bayesian Optimization, Computer Experiments, Multi-Objective Optimization We consider multi-objective optimization problems,  $\min_{\mathbf{x} \in \mathbb{R}^d} (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))$ , where the functions are expensive to evaluate. In such a context, Bayesian methods relying on Gaussian Processes (GP) [1], adapted to multi-objective problems [2] have allowed to approximate Pareto fronts in a limited number of iterations.

In the current work, we assume that the Pareto front center has already been attained (typically with the approach described in [3]) and that a computational budget remains. The goal is to uncover of a broader central part of the Pareto front: the intersection of it with some region to target,  $\mathcal{I}_{\mathbf{R}}$  (see Fig. 1).  $\mathcal{I}_{\mathbf{R}}$  has however to be defined carefully: choosing it too wide, i.e. too ambitious with regard to the remaining budget, will lead to a non converged approximation front. Conversely, a suboptimal diversity of Pareto optimal solutions will be obtained if choosing a too narrow area.

The GPs allow to forecast the future behavior of the algorithm: they are used in lieu of the true functions to anticipate which inputs/outputs will be obtained when targeting growing parts of the Pareto front. Virtual final Pareto fronts corresponding to a possible version of the approximation front at the depletion of the budget are produced for each  $\mathcal{I}_{\mathbf{R}}$ . A measure of uncertainty is defined and applied to all of them to determine the optimal improvement region  $\mathcal{I}_{\mathbf{R}^*}$ , balancing the size of the approximation front and the convergence to the Pareto front.

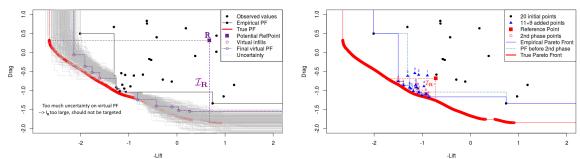


Figure 1: Left: virtual Pareto front (dashed, purple) obtained if targeting  $\mathcal{I}_{\mathbf{R}}$  for the 9 remaining iterations. The uncertainty (grey) at the end of the optimization is forecasted to be too large because  $\mathcal{I}_{\mathbf{R}}$  is too wide. The optimal improvement region  $\mathcal{I}_{\mathbf{R}^*}$  is shown on the right. Good convergence is obtained in this region.

## Références

- [1] D.R. Jones, M. Schonlau, W.J. Welch, Efficient Global Optimization of expensive black-box functions, Journal of Global Optimization, 13(4), 455-492.
- [2] M. Emmerich, K.C. Giannakoglou, B. Naujoks, Single-and multiobjective evolutionary optimization assisted by Gaussian random field metamodels, Springer Series in Operations Research and Financial Engineering. IEEE Transactions on Evolutionary Computation, 10(4), 421-439.
- [3] D. GAUDRIE, R. LE RICHE, V. PICHENY, B. ENAUX, V. HERBERT, Budgeted Multi-Objective Optimization with a Focus on the Central Part of the Pareto Front Extended Version, arXiv pre-print 1809.10482v1.