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A New Approach for the Identification of Reduced Thermal Models

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Abstract

This paper describes a method aimed at producing reduced order models in state-space form for speeding up the transient thermal simulation of mechatronics products. Designed in a non-intrusive framework, this method is implemented as an offline tool that post-processes the inputs and outputs of a detailed reference model considered as a black box. The reduction algorithm is based on robust linear algebra tools, such as the householder decomposition and the least square approximation. Thanks to the underlying modal representation, the model order, typically equal to the number of observation points, can be also further reduced. A first linear test case illustrates the quality of the results. The investigation of various input signals ensuring the reliability of the identification process while minimizing the simulation time of the detailed model, as well as the derivation of non-linear reduced models, are the ongoing paths of improvement and extension.

1 Introduction

The need for the reduction of simulation model size is not new, as testified by the substructuring approach published by Guyan [6] in 1965, still part of most commercial finite element packages. Since then, simulation models tend to be larger and larger, due to the widespread availability of high computing horsepower and the use of MCAD (Mechanical Computer Aided Design) designs as the main source of geometrical data. The increasing processing of strongly-coupled multiphysics phenomena also contributes to reinforce this evolution. Consequently, extremely detailed models can be built, resulting in the dramatic lengthening of transient simulations, to such an extent that they may even become impossible in practice.

Mechatronics thermal simulations perfectly illustrate these various points. Indeed, most of the time, the related models draw their geometrical data from both MCAD and ECAD (Electronic Computer Aided Design) software, which frequently leads to an excessive amount of details whose manual simplification is extremely time consuming. Then, these models must account for electrical and thermal effects in solids, along with an accurate characterization of the environmental conditions by computational fluid dynamics techniques. In addition, the processing of transient regimes generally involve complex variations with time of the power dissipated by electronic components, which requires the use of very short integration time steps hampering large model simulations over long periods of time, in particular when several power profiles have to be processed. By contrast, almost paradoxically, the set of results to produce, mainly constituted by the junction temperature of critical components, is frequently fairly small compared to the model size.

These facts emphasize why the derivation of reduced models is crucial for performing transient simulations of

mechatronics products. The well-known thermal impedance method [3][4][5][9] can serve that purpose but its limitations calls for a reduction methodology with a broader scope regarding in particular the extensibility to the processing of non linear phenomena. Indeed, material properties dependency with temperature, heat transfer by radiation and above all by natural convection make the behaviour of mechatronics products intrinsically non linear.

In an industrial context, the implementation of a reduction method must also take into account the locked access to the source code of commercial software. Hence, the development framework has been deliberately restricted to non intrusive methods based on the processing of the input data and output results only, the system in-between, i.e. the detailed mechatronics model to be reduced, being considered as a black box.

2 Thermal impedance method

This method is well documented in the technical literature and this section only intends to recall some of its main features for emphasizing the differences with the proposed approach. The thermal impedance method is based on the fact that the knowledge of a system impulse response enables to produce its output for any input, with the help of a convolution product. This assumes that the problem to solve is linear, allowing for the application of the superposition principle and the scaling of power dissipation. In practice, the method use consists in applying *in turn* step power profiles to each dissipation source of the detailed model and in collecting the temperature responses at all observation points. Those latter typically refer to junction temperatures and sensors at critical locations. On completion of all simulations, *performed up to the steady state*, the matrix of thermal impedances defined by $Z_{ij}^{th} = (T_{ij}(t) - T_0) / P_j$ can be built. In this relationship, T_0 is the initial temperature and T_{ij} the response at point i after a

step at point j . This matrix is then derived with respect to time to produce the transfer functions corresponding to impulse signals. Finally, the convolution product

$$T(t) - T_0 = dZ^{th}(t) / dt \otimes P(t) \quad (1)$$

enables to compute the temperature $T(t)$ at observation points for any input power profile defined by the vector $P(t)$.

In addition, an extremely useful feature comes from the possibility to cast the ROM (Reduced Order Model) represented by (1) as a network generally made of Foster ladders that electronic designers can employ on their own in circuit simulators [5].

This method proves to be quite useful when carefully utilized. For instance, any non-zero level of power dissipation could theoretically be used during the detailed model simulations. Actually, these levels must be close to the ones involved by the real product, allowing for the problem linearization around the targeted operating condition.

However, despite these precautions, our experience reveals that this method may lead to significant errors, in particular for systems cooled by natural convection.

3 Modal decomposition

On completion of the spatial discretization, for instance by means of the control volume or finite element method, the heat equation becomes

$$C_d(dT_d / dt) + K_d T_d = Q_d, \quad (2)$$

where the subscript d emphasizes the reference to the detailed model. In (2), $C_d \in \mathbb{R}^{N \times N}$ is the heat capacity matrix, $K_d \in \mathbb{R}^{N \times N}$ the heat conduction matrix, $T_d \in \mathbb{R}^N$ the nodal temperature vector and $Q_d \in \mathbb{R}^N$ the nodal load vector that includes the effect of heat sources and boundary conditions.

Multiplying (2) on the left by C_d^{-1} gives

$$dT_d / dt = A_d T_d + C_d^{-1} Q_d \quad (3)$$

with $A_d = -C_d^{-1} K_d$. The eigenvalue problem associated to (3) is then expressed by the *homogeneous* equation

$$(A_d - \lambda_i I) P_d = 0 \quad (4)$$

where I designates the identity matrix and P_d the matrix made of the orthonormal eigenvectors corresponding to the real eigenvalues λ_i such that A_d is similar to the diagonal matrix $\Lambda_d = P_d^T A_d P_d$. Equation (4) *does not include the load vector*, meaning that the eigenvalues and eigenvectors constitute the *kernel* of the thermal system and that this kernel does not depend on the excitation applied on the system.

Using the modal basis change, the *nodal* temperature vector T_d is transformed into the vector of *modal* temperature Θ_d such that $T_d = P_d \Theta_d$. Substituting this relation in (3) yields

$$P_d(d\Theta_d / dt) = A_d P_d \Theta_d + C_d^{-1} Q_d \quad (5)$$

Premultiplying (5) by P_d^T and noting that P_d is orthonormal ($P_d^T P_d = I$) leads to the system Σ_d expressed by

$$d\Theta_d / dt = \Lambda_d \Theta_d + P_c^T C_c^{-1} Q_d \quad (6)$$

$$T_d = P_d \Theta_d \quad (7)$$

Thus, the detailed model (2) is transformed into the LTI (linear time invariant) system Σ_d whose state matrix Λ_d is *diagonal*.

The opposite reciprocals of the Λ_d diagonal entries are the *time constants* τ_i of the thermal system. In an intrusive framework, the inspection of these time constants enables to perform a *mode truncation* for reducing the size of Σ_d [8]. By contrast, in the non-intrusive context considered in this paper, the matrices of the LTI system to produce are not directly computed from those of the detailed model, but are identified from the system inputs and output temperatures instead.

4 Method description

The current approach aims at producing a LTI system conventionally denoted as $\Sigma = (A, B, C, D)$ such that

$$dX / dt = AX + BU \quad (8)$$

$$Y = CX + DU \quad (9)$$

The data and results coming from the simulation of the detailed model give the input vector $U \in \mathbb{R}^{N_{ex} \times N_{snap}}$ and observation vector $Y \in \mathbb{R}^{N_{obs} \times N_{snap}}$, where N_{snap} designates the number of snapshots, i.e. the number of time samples. Hence, with n standing for the order of Σ , the state, command, observation and static gain matrices to derive are such that $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times N_{ex}}$, $C \in \mathbb{R}^{N_{obs} \times n}$ and $D \in \mathbb{R}^{N_{obs} \times N_{ex}}$. With respect to Σ_d in (6)–(7) the inclusion of D brings a static correction to the ROM approximation.

With C^\dagger denoting the pseudo inverse of C , the extraction of the state vector $X \in \mathbb{R}^n$ from (9) and the derivation of (8) with respect to time give

$$X = C^\dagger(Y - DU) \quad (10)$$

$$dY / dt = C(dX / dt) + D(dU / dt) \quad (11)$$

Substituting dX / dt by its value coming from (8) changes (11) into

$$dY / dt = C(AX + BU) + D(dU / dt) \quad (12)$$

Then, after (10)

$$dY / dt = ZY + WU + D(dU / dt) \quad (13)$$

with

$$Z = CAC^\dagger \quad (14)$$

$$W = C(B - AC^\dagger D)U \quad (15)$$

Equation (13) is the reduced first order linear system of order N_{obs} to be solved, in which the unknown operators are

the matrices $Z \in \mathbb{R}^{N_{obs} \times N_{obs}}$, $W \in \mathbb{R}^{N_{obs} \times N_{ex}}$ and $D \in \mathbb{R}^{N_{obs} \times N_{ex}}$. The solution Y of (13) consists of two parts. The first comes from the homogeneous solution driven by the kernel of the thermal system, as shown by the expression of the operator Z independent of the input U , and the second comes from the particular solution related to the excitation applied on this system.

To compute these operators, the main two features of the proposed methods are the followings. First, (13) is projected onto U^\perp , the orthogonal space to the row space of the input data U . This operation, such that the projection U of onto U^\perp is zero, enables the computation of Z by *virtually suppressing the influence of the input*. Then, the matrix Z is assumed to be symmetric. With this assumption, Z is diagonalizable and (14) becomes $Z = CAC^{-1}$, where A is diagonal and C an orthonormal square matrix in $\mathbb{R}^{N_{obs} \times N_{obs}}$ whose columns are the eigenvectors of Z . Consequently, the order of the reduced model is $n = N_{obs}$.

The previous features lead in practice to produce the reduced model according to the main following steps:

1. Collect the input U and observations Y from the simulation of the detailed reference model;
2. Compute the time derivatives dU/dt and dY/dt by finite differences;
3. Compute the orthogonal projections of dY/dt , Y and dU/dt involved in (13) onto U^\perp by means of a Householder decomposition [2] [7];
4. From the projected equation (13), compute the matrices Z and D by a least square approximation;
5. Diagonalize Z to produce C and A ;
6. Compute B as detailed below.

The matrices A , C and D being known, the computation of B is carried out by substituting (14) and (15) in (13) with $C^\dagger = C^{-1}$, which leads to

$$BU = C^{-1}((dY/dt) - CAC^{-1}Y + CAC^{-1}D(dU/dt))$$

This last equation is first projected onto $(dU/dt)^\perp$ i.e. the orthogonal space to the row space of dU/dt , before being solved by a least square approximation in which the projection onto $(dU/dt)^\perp$ minimizes the number of parameters.

Additional steps are carried out on the algorithm completion, such as the verification of the LTI system observability and controllability [1], followed by the resolution of system Σ expressed by (8)–(9) to assess the quality of the identification through the comparison of the results respectively produced by the detailed and reduced models. Also, as mentioned in the modal decomposition section, Σ can be *further reduced* by inspection of the dominant eigenvalues providing the time constants of the thermal system. To this end, the contribution of the eigenvalues in A is evaluated by inspection of the entries in the vector $\text{diag}(X(t)X(t)^T) = \{\sigma_1, \dots, \sigma_n\}$. The eigenvalues are then sorted according to the decreasing order of the σ_i

values and the model order is decreased from n to k after the k greatest values σ_i .

5 Results

The developed method is applied to a test problem made of nine volume sources partially embedded in a parallelepiped (Fig. 1). Newton boundary conditions are applied on the upper and lower faces of it. Despite its simplicity, this test is representative of the thermal behaviour of a PCB (Printed Circuit Board) substrate on which dissipating components are mounted. The simulation is carried out with a finite element model having 20236 nodes. The observation points are located at the centre of each volume source and at the lower left corner of the PCB, on its upper side.

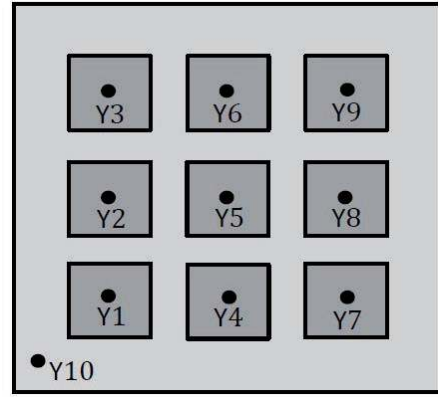


Figure 1: Test problem

By contrast to the thermal impedance method, the proposed approach does not require to perform nine simulations where each heat source is switched on in turn. The identification is based on a single simulation of the detailed model. However, the input signals applied on the heat sources play a significant role for the quality of the identification and they consequently must be carefully selected. In particular, two conditions have to be fulfilled: the Shannon theorem and the persistently exciting condition. The former states that the sampling frequency used during the identification must be at least two times larger than the maximum frequency of the input signal applied to the thermal system. The latter means that the input signals must be linearly independent along the time samples.

Two types of excitation have been applied on the detailed model:

- A. Staggered step signals applied on the heat sources (Fig. 2);
- B. Uncorrelated zero-mean random signals applied on the heat sources (Figs. 3 and 4).

In this paper, we restrict the presentation of results to an identification process based on signal A, but signal B was successfully used as well.

The quality of the results is quantified by the comparison between the temperatures Y_i^d and Y_i^r respectively produced

by the detailed and reduced model. For that purpose, the relative error is computed by the following relationship that takes into account all observations points and time samples

$$\varepsilon = \frac{1}{N_{obs}} \sum_{i=1}^{N_{obs}} \left[\frac{\sum_{j=1}^{N_{snap}} (Y_i^d(t_j) - Y_i^r(t_j))^2}{\sum_{j=1}^{N_{snap}} Y_i^d(t_j)^2} \right]^{1/2} \quad (16)$$

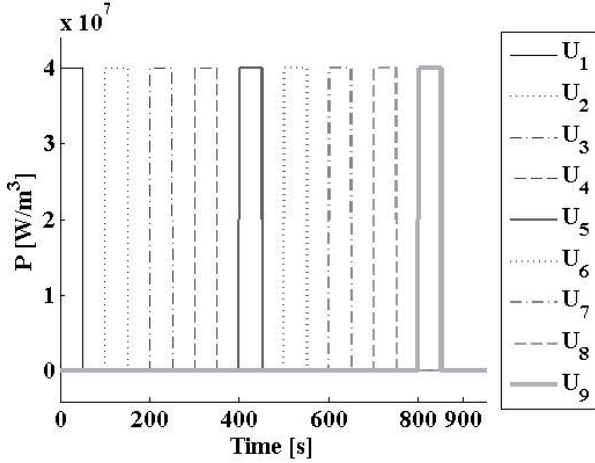


Figure 2: Staggered step signals

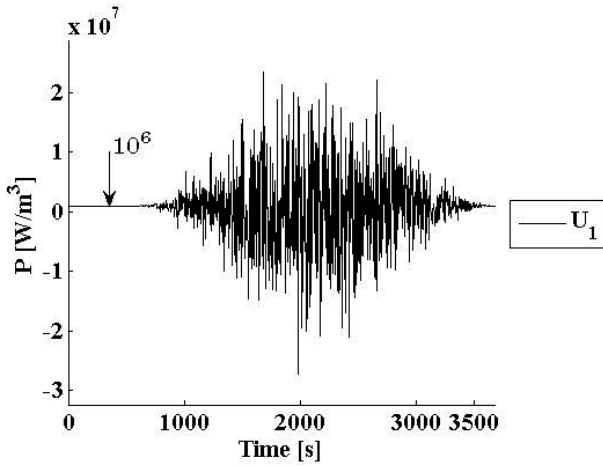


Figure 3: Random signal at source 1

The *identification step* consists in checking the behaviour of the reduced model by applying the same signals as the one used for the identification (type A). Thus, Figs. 5 and 6 compare the evolution with time of the temperature produced by the detailed and reduced model at observation points 1 and 7 when the order of this latter is the default, i.e. the number of observation points ($n = 10$). The error (16) does not exceed 1.3% at point 1 and 2.0% at point 7.

In addition, Fig. 7 shows the evolution of the relative error when the model order is decreased. Even for order 1, the error lower than 10% could be considered as acceptable, depending on the targeted application. Order 1 corresponds to the case where the largest time constant only is retained.

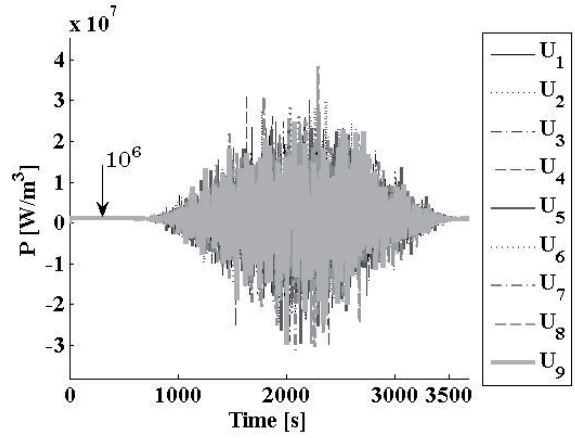


Figure 4: Random signal at all sources

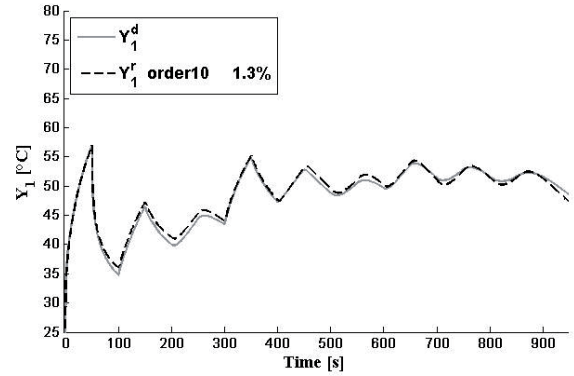


Figure 5: Identification step. Signal A. Temperature at point 1

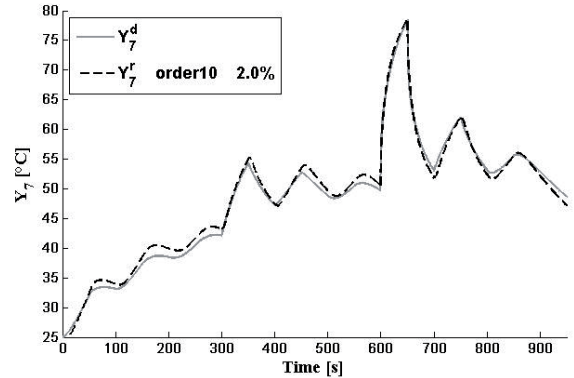


Figure 6: Identification step. Signal A. Temperature at point 7

Following the identification step, the *validation step* consists in simulating both the detailed and reduced model with an excitation other than the one used for the identification. In that respect, for an excitation of type B, Figs 8 and 9 give the error evolution with time at points 1 and 7 for the model of order $n = 10$. The error (16) does not exceed 3.3% at point 1 and 1.6% at point 7.

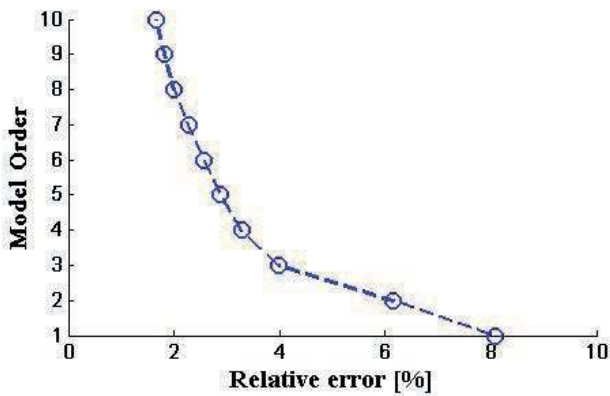


Figure 7: Signal A. Evolution of the relative error

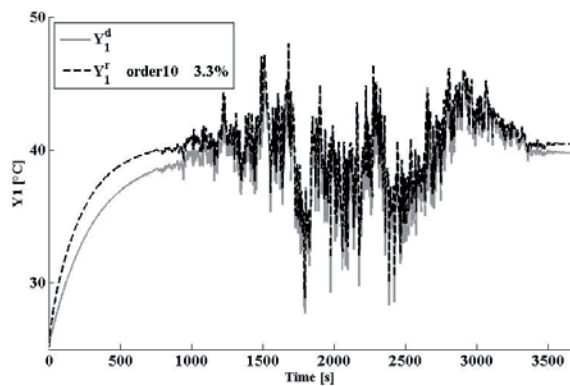


Figure 8: Validation step. Signal B. Temperature at point 1

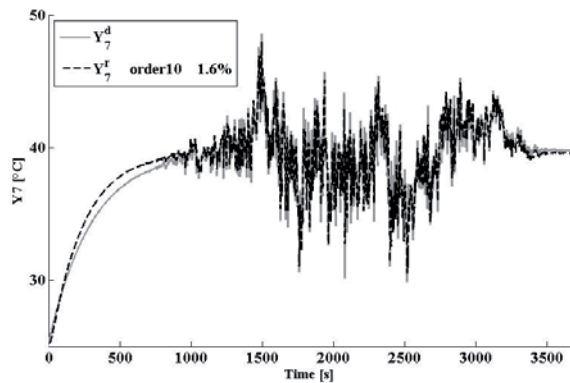


Figure 9: Validation step. Signal B. Temperature at point 7

6 Conclusion

The thermal simulation of mechatronics hardware emphasizes a strong unbalance between the ever increasing size of the involved models and the relatively reduced set of awaited results, consisting typically of the junction temperatures reached by the most critical electronic components. Consequently, the derivation of reduced order models is a crucial need to perform transient simulations involving short integration time steps to account for the

complex variations of the power dissipated by components with time.

For that purpose, this paper describes a new approach based on the processing of the input data and output results produced by a detailed model considered as a black box. With the help of robust linear algebra tools, this approach produces a reduced model in state-space form, whose order is by default equal to the number of observation points. The non-intrusive framework of the proposed method is aimed at allowing its implementation independently of the software involved for the simulation of the detailed model.

The results obtained on a first linear test case are very satisfactory, with a low relative error both during the identification step based on staggered step signals and the validation step based on zero-mean uncorrelated random signals. This test case is also used to illustrate that the underlying modal representation provides the ability to reduce further the model order, at the cost of the error degradation. To this end, a criterion based on the entries of the state vector is used.

Two parallel tasks are currently ongoing for the improvement and extension of the proposed method. The first one consists in investigating various input signals ensuring the reliability of the identification process while minimizing the simulation time of the detailed model. The second task aims at extending the current method for the production of non-linear reduced models.

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