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The Scale of Quantum Gravity in the Presence of Sinusoidal Variation of Gravitational Constant

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Abstract. In this paper, a non-metric extension of general relativity is developed, based on a Maxwellian approach to gravity. Harmonic quantities such as generalised Riemann curvature tensor and scalar are constructed for a combined state of space-time forms by from the metric and non-metric space-time phases. We find that the sinusoidal time variation of the gravitational constant G_N can be derived from the Lagrangian of the proposed model. Furthermore, we show that the generalised harmonic curvature scalar that are responsible for the variation of the gravitational constant shift the scale of Quantum gravity at 100TeV. Quantum gravity effects are occurring at energy scales significantly beyond that of the LHC. Quantum gravity can be investigated by a 100 TeV Proton-Proton Collider.

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1. Introduction

Newton's gravitational constant, G , has been measured about a dozen times over the last 40 years. Recently, John D. Anderson and coauthors [1] found that the measured G values oscillate over time like a sine wave with a period of 5.9 years. They propose that this oscillation of measured G values does not register variation of G itself, but rather the effect of unknown factors on the measurements [64]. Furthermore, C.S.Unnikrishnan [65] provides a possible explanation to the 5.9-year period oscillation of G values by the gravitational link between the 5.9 year period in the length of the day of the Earth and the 11.86 year orbit of Jupiter around the Sun.

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Klein (2015[2]) suggests that the observed discrepancies between G values determined in different experiments may be associated with a differential interpretation of Modified Newtonian Dynamics (MOND) theory applied to the galaxy rotation curves. Recent quantitative analysis (Lorenzo Iorio, 2015[3]) rules out the possibility that the harmonic pattern observed in laboratory-measured values of G is due to some long-range modification of the currently accepted laws of gravitational interaction. This analytical approach may direct future investigations of the systematic uncertainties that plague measurements of G .

Based on Symbolic Gauge Theory (SGT), a formalism applied to General Relativity (GR) by R. Mignani, E. Pessa and G. Resconi [4,5] and further developed by I. Licata and G. Resconi, M.E. Rodrigues and E.Koorambas [8,9,10], the authors recently proposed a Non-Conservative Theory of Gravity (NCTG) which can explain the observed variations of G at a 5.9-year scale [9].

The strength of the gravitational force depends on the scale at which the gravitational force is measured by Cavendish-type experiments, where two masses (one of which is a test mass) are precisely known, or by (equivalent in principle) gravitational scattering experiments [11]. At laboratory scales, the strength of gravity is characterized by the reduced Planck mass $M_{pl} = 2.435 \times 10^{18}$ GeV, which determines Newton's constant $G_N = M_{pl}^{-2}$. Conventionally, the Planck scale M_{pl} is interpreted as the fundamental scale at which quantum gravitational effects become important in nature. Like all other interactions in nature, nevertheless, the effective strength of gravity is affected by quantum corrections. This effect depends on the characteristic energy of the process probing gravitational interactions (see [12,13] for reviews of an effective theory of gravity). Potential problems of running gravitational couplings by focusing only on physically observable quantities (e.g. amplitudes, cross sections) are discussed in [14,15]. New approaches to the physics of particles with masses greater than 1TeV could offer insights to the problem of the variation of measured G_N values. In such models there is no hierarchy problem [16], whereas quantum gravity can be assessed through experiments at TeV energy levels. That this can be the case in extra-dimensional models is already established [17,18]. Is such modification of gravity also possible in four dimensions [19,39] Current data from the Large Hadron Collider (LHC) experiments at the European Laboratory for Particle Physics (CERN) do not confirm that gravity becomes stronger around 1 TeV [40-44].

Recently, E.Koorambas suggested that if the observed harmonic pattern of the laboratory-measured values of G is due to some environmental or theoretical errors, these errors must also affect the true value of momentum k transferred by the graviton in scattering experiments at the LHC [45]. Furthermore, environmental or theoretical errors could shift the scale of Quantum gravity at 100TeV. Quantum gravity can be investigated by a 100 TeV Proton-Proton Collider as long as environmental or theoretical errors are present. This proposition may explain the current null results for black holes production at the LHC [45].

M.Carmeli (1999) [47] proposed another approach to quantise the gravitational field. It is based on the observation that the quantum character of matter becomes more significant as one gets closer to the Big Bang. As the metric loses its meaning, it makes sense to consider Schrodinger's [48] three generic types of manifolds –unconnected differentiable, affinely connected and metrically connected – as a temporal sequence following the Big Bang. More recently, a non-metric approach of NCTG has been consider by the authors as dark energy candidate [8].

In this paper, based on a Maxwellian approach of gravity, a non-metric extension of general relativity is developed. Harmonic quantities, such as generalised Riemann curvature tensor and scalar, are constructed

for a combined state of space-time forms by the metric and non-metric space-time phases. We find that the sinusoidal time variation of the gravitational constant G_N originates in the Lagrangian of the propose model. Furthermore, we show that the generalised harmonic curvature scalar that is responsible for the variation of the gravitational constant shift the scale of Quantum gravity at 100TeV. Quantum gravity effects are at energy scales significantly beyond that of the LHC. Quantum gravity can be investigated by of a 100 TeV Proton-Proton Collider.

2.The sinusoidal variations of Newton's coupling constant

Measurements of the gravitational constant (G) are notoriously difficult due to the gravitational force being by far the weakest of the four known forces. Recent advances, making use of electronically controlled torsion strip balances at the *Bureau International des Poids et Mesures* (BIPM) in the last 15 years, have improved the accuracy of G measurements (see [2] for details on experimental methods). These recent measurements have also revealed a peculiar type of oscillatory variation, seemingly following a 5.9 years cycle akin to the so called Length-of-Day (LOD) [1]. Although we recognize that the correlation between G measurements and the 5.9 year LOD cycle could be fortuitous, we think that this is unlikely, given the striking match between these two (Fig. 1).

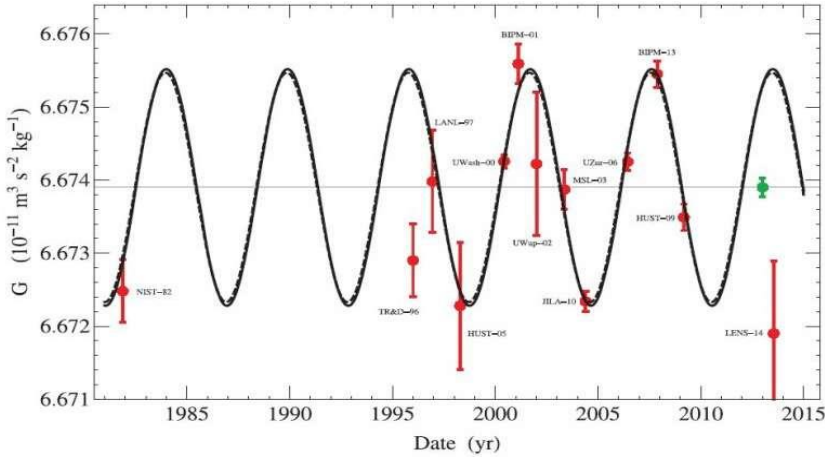


Fig. 1: Comparison of the CODATA set of G measurements) with a fitted sine wave (solid curve) and the 5.9 year oscillation in LOD daily measurements (dashed curve), scaled in amplitude to match the fitted G sine wave. Acronyms for the measurements follow the CODATA convention. Also included are a relatively recent BIPM result from Quinn *et al.* [3] and measurement LENS-14 from the MAGIA collaboration [4] that uses a new technique of laser-cooled atoms and quantum interferometry, rather than the macroscopic masses of all the other experiments. The green filled circle represents the weighted mean of the included measurements, along with its one-sigma error bar, determined by minimizing the L1 norm for all 13 points and taking into account the periodic variation.

Importantly, the observed correlation cannot be due to centrifugal force acting on the experimental apparatus, since changes in LOD are too small by a factor of about 10^5 to explain changes in G . This is because the Earth's angular velocity ω_E is by definition

$$\omega_E = \omega_0 (1 - LOD), \quad (1)$$

where ω_0 is an adopted sidereal frequency equal to 72921151.467064 prad s^{-1} and the LOD is in ms d^{-1} (www.iers.org). The total centrifugal acceleration is given by

$$a_c = r_s \omega_0^2 \left[1 - 2A \sin \left(\frac{2\pi}{P} (t - t_0) \right) \right], \quad (2)$$

where A is the amplitude of the 5.9 year sinusoidal LOD variation ($= 0.000150/86400$) and r_s is the distance of the apparatus from the Earth's spin axis. The maximum percentage variation of the LOD term is 3.47×10^{-9} of the steady-state acceleration, while dG/G is 2.4×10^{-4} . Even the full effect of the acceleration with no experimental compensation changes G by only 10^{-5} of the amplitude shown in Fig. 1.

3. The Maxwellian approach to gravity

Recently, G. Resconi, I. Licata and C. Gorda proposed a generalization of gauge theories based on the analysis of the structural characteristics of Maxwell theory, which can be considered as the 'prototype' of such theories Maxwellian theory [6]. Theories of this type are based on A small number of principles related to different orders of commutators between covariant derivatives. We know that the Maxwell equations in the tensor form are

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= \frac{4\pi}{c} J^\nu \\ \partial^\alpha F^{\mu\nu} + \partial^\mu F^{\nu\alpha} + \partial^\nu F^{\alpha\mu} &= 0 \end{aligned} \quad (3)$$

where the contravariant four-vector which combines electric current density and electric charge density. $J^\nu = (cp, J_x, J_y, J_z)$ is the four-current, the electromagnetic tensor is $F^{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu$. The four-potential $A^\mu = (\Phi, A_x, A_y, A_z)$ contains the electric potential and vector potential.

Using the covariant derivative notation defined by

$$D_\mu = \partial_\mu - ieA_\mu \quad (4)$$

we have the classic relation

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu = -ieF_{\mu\nu} \quad (5)$$

Thus, we have

$$F_{\mu\nu} = -F_{\nu\mu} \quad (6)$$

In order to write the extension of the Maxwell equations, we define the general commutators as in eq. (5) where $F_{\mu\nu}$ is the general form for any field generated by the transformation. Using eq. (5), the equation

$$[D_\mu, [D_\eta, D_\nu]]\psi + [D_\eta, [D_\nu, D_\mu]]\psi + [D_\nu, [D_\mu, D_\eta]]\psi = 0 \quad (7)$$

can be rewritten as

$$[D_\mu, F_{\eta\nu}]\psi + [D_\eta, F_{\nu\mu}]\psi + [D_\nu, F_{\mu\eta}]\psi = 0 \quad (8)$$

We also have the equation

$$[D_\gamma, [D_\alpha, D_\beta]] = [D_\gamma, F_{\alpha\beta}]\psi = \chi J_{\gamma\alpha\beta}\psi \quad (9)$$

In conclusion, the Maxwell scheme for a general gauge transformation is

$$\begin{aligned} [D_\gamma, F_{\alpha\beta}]\psi + [D_\alpha, F_{\beta\gamma}]\psi + [D_\beta, F_{\gamma\alpha}]\psi &= 0 \\ [D_\gamma, F_{\alpha\beta}]\psi &= \chi J_{\gamma\alpha\beta}\psi \end{aligned} \quad (10)$$

where $J_{\gamma\alpha\beta}$ are the currents of the particles that generate the gauge field F. Such currents have the conservation rule

$$J_{\alpha\beta/\gamma} + J_{\beta\gamma/\alpha} + J_{\alpha\gamma/\beta} = 0 \quad (11)$$

A modification of GR by Camenzind [10] was constructed as an analogy to the Yang-Mills theory, with the symmetry group SO (3.1). In the same vein, inspired by the foundations of gauge theory, Resconi et al. [7-9] constructed a modified theory of gravity in which the algebra of the covariant derivatives operators adds a term to the equations of motion. In the framework of this Maxwellian approach to gravity, we can obtain the general equation of motion by applying the commutator of ∇_μ with the commutator of ∇_α with ∇_β to a vector field K_ν . This is achieved through the following steps:

First, it has been shown in [7] that

$$[\nabla_\mu, \nabla_\nu]V_\alpha = -R_{\alpha\mu\nu}^\lambda V_\lambda \quad (12)$$

where the Riemann tensor is

$$R_{\alpha\mu\nu}^\lambda = \frac{\partial \Gamma_{\nu\alpha}^\lambda}{\partial x^\mu} - \frac{\partial \Gamma_{\mu\alpha}^\lambda}{\partial x^\nu} + \Gamma_{\mu\alpha}^\lambda \Gamma_{\nu\sigma}^\lambda - \Gamma_{\nu\alpha}^\lambda \Gamma_{\mu\sigma}^\lambda \quad (13)$$

With application of the double commutator we have the dynamic equation

$$[\nabla_\mu, [\nabla_\alpha, \nabla_\beta]]K_\nu = (\nabla_\mu [\nabla_\alpha, \nabla_\beta])K_\nu - [\nabla_\alpha, \nabla_\beta](\nabla_\mu K_\nu) = -R_{\alpha\mu\nu}^\lambda V_\lambda \quad (14)$$

where ∇_k is the covariant derivative and K_ν is the vacuum field. Now we connect the commutator with the gravity current in this way:

$$\chi J_{\mu\alpha\beta} K_\nu = (\nabla_\mu R_{\nu\alpha\beta}^\lambda)K_\lambda + R_{\mu\alpha\beta}^\lambda (\nabla_\lambda K_\nu) \quad (15)$$

Then, for the conservation of the current we have after contractions the of motion equation

$$\nabla_\mu \left[R^{\mu\nu} + k \left(T^{\mu\nu} + \frac{1}{2} g^{\mu\nu} T \right) \right] K_\nu + R^{\mu\nu} (\nabla_\mu K_\nu) = 0 \quad [7] \quad (16)$$

In the particular case where $\nabla_\mu K_\nu = 0$, considering the validity of Einstein's equations, we re-obtain the second Bianchi identity for a non-zero vector field K_ν [6].

The dynamical equation (16) can be represented by a wave equation with particular source where the variables are symmetric and anti-symmetric gravitational connections (including torsion $T_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda$ in one geometric picture). The dynamic equation for *non-conservative gravity* can be obtained by the SGT as follows [5-9]:

$$\left(\frac{\partial^2 \Gamma_{\nu,\alpha}^\lambda(x,t)}{\partial x^2} - \frac{\partial^2 \Gamma_{\nu,\alpha}^\lambda(x,t)}{\partial t^2} \right) V_\lambda = R_{\nu\alpha} - J_{\nu\alpha} \quad [5,6] \quad (17)$$

We thus obtain the interesting result that the derivative of the gravitational connection $\Gamma_{\nu,\alpha}^\lambda$ has wave behavior. The non-linear reaction of the self-coherent system produces a current that justifies the complexity of the gravitational field and non-linear properties of the non-metric gravitational waves. In analogy with non-linear optic, therefore, we can model gravitational phenomena as optic of tensor potentials. Equations (17) in the free field of the medium contain the Proca terms, $\Gamma_\rho \Gamma_\lambda$, the Chern-Simons terms, $(\partial_\nu \Gamma_\rho) \Gamma_\lambda$, and the Maxwell-like terms $(\partial_\nu \Gamma_\rho) (\partial_\mu \Gamma_\lambda)$. In other words, we have the mass terms, the topologic terms and the electromagnetic field-like terms [13.14].

Recently, it has been proved by the authors that in the limit of vanish torsion the interactions between harmonic gravitational connections are independent of metric [10]. The topological properties of these interactions are represented by knots and links. The size, exact shape, location etc of these knots and links are not of immediate concern for the problem at hand (interaction between dark mater/energy and gravitons) [10].

4.The gravity in a non-metric extension of space-time

Following Moshe Carmeli (1999) [47], we consider the following three generic phases leading to metric space-times [48]:

- (1) General differentiable manifold (unconnected);
- (2) Affinely connected manifold;
- (3) Metrically connected manifold.

In addition, there is an intermediate phase between (2) and (3): space-time with a rudimentary metric structure (STRMS) in metrically connected space-time:

$$\Gamma_{\alpha\beta}^\beta(y) = \left(\ln \sqrt{-g} \right)_{,\alpha} = \partial_\alpha \Phi(y) = \frac{\partial \Phi(y)}{\partial y^\alpha}, \quad (18)$$

where

$$\partial_\alpha = \frac{\partial}{\partial y^\alpha}, \quad (19)$$

are the derivatives.

It follows that, for the STRMS phase, it is possible to define the harmonic scalar $\Phi(y)$ as the potential of the harmonic gravitational connections $\Gamma_{\alpha\beta}^\beta(y)$ and let $e^{\Phi(y)}$ play the role of $\sqrt{-g}$. Equation (18) determines the volume element up to an overall multiplicative factor. STRMS is metric only in the sense of having a volume element; ds^2 is not defined in it [47]. The latter leads to the existence of non-metric regions of space-time:

$$U_{(y)} = \{y^\alpha = (\tau, y^\gamma) \mid \tau_1 \leq \tau \leq \tau_2, -\infty < y^1, y^2, y^3 < +\infty, \alpha = 0, 1, 2, 3, \gamma = 1, 2, 3\}, \quad (20)$$

where $y^\alpha = (\tau, y^\gamma)$, $\alpha = 0, 1, 2, 3, \gamma = 1, 2, 3$ the coordinate frame where ds^2 is not defined in it. Here, τ and y^γ , $\gamma = 1, 2, 3$ are time and space states in which metric time and metric space lose their meaning. Covariant, contravariant and mixed tensors, tensor densities and spinors can be defined in the U_y regions. However, the usual relationship between covariant and contravariant components does not hold: there is no metric tensor to connect them [47,48].

In a non-metric region U_y of space-time, the wave equation (17) for the harmonic gravitational connections becomes:

$$\square \Gamma_{\alpha\beta}^\beta(y) = \square \left(\ln \sqrt{-g} \right)_{,\alpha} = \square \partial_\alpha \Phi(y) = \partial_\alpha \square \Phi(y) = 0, \quad (21)$$

where $\Phi(y)$ is a harmonic scalar by definition:

$$\square \Phi(y) = \partial^\alpha \partial_\alpha \Phi(y) = 0, \quad (22)$$

$$\square = \partial^\alpha \partial_\alpha = \frac{\partial^2}{\partial \tau^2} - \nabla_\alpha^2 = \frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial y^{(1)2}} - \frac{\partial^2}{\partial y^{(2)2}} - \frac{\partial^2}{\partial y^{(3)2}}, \quad (23)$$

is the wave operator in a non-metric region U_y of space-time. The solution of the equation (22) is given by

$$\Phi(y) = 2\Phi_0 \cos(k_\alpha y^\alpha + \varphi) \quad (24)$$

where $k_\alpha = (\omega_G, k_\gamma)$, ω_G is the frequency of the oscillation, k_γ is the wave vector, Φ_0 is the amplitude of the oscillation. Substituting equation (24) to (18), we obtain the solution of the wave equation (21):

$$\Gamma_{\alpha\beta}^\beta(y) = \partial_\alpha \Phi(y) = 4\Phi_0 k_\alpha \sin(k_\alpha y^\alpha + \varphi). \quad (25)$$

Now, by using equation (25) we calculate the Proca terms, the Chern-Simons terms, and the Maxwell-like terms in equation (17):

$$\begin{aligned}\Gamma_{\alpha\beta}^\beta \Gamma_{\beta\alpha}^\beta &= 4\Phi_0 k_\alpha \sin(k_\alpha y^\alpha + \varphi) 4\Phi_0 k_\beta \sin(k_\beta y^\beta + \varphi) \\ &= 16\Phi_0^2 k_\alpha k_\beta \left(\frac{1}{2} \cos[(k_\alpha y^\alpha - k_\beta y^\beta)] + \frac{1}{2} \cos[(k_\alpha y^\alpha + k_\beta y^\beta + 2\varphi)] \right),\end{aligned}$$

where

$$\sin(k_\alpha y^\alpha + \varphi) 4\Phi_0 k_\beta \sin(k_\beta y^\beta + \varphi) = \left(\frac{1}{2} \cos[(k_\alpha y^\alpha - k_\beta y^\beta)] - \frac{1}{2} \cos[(k_\alpha y^\alpha + k_\beta y^\beta + 2\varphi)] \right),$$

$$\begin{aligned}(\partial_\beta \Gamma_{\alpha\beta}^\beta) \Gamma_{\gamma\alpha}^\alpha &= 4\Phi_0 k_\beta k_\alpha \cos(k_\alpha y^\alpha + \varphi) 4\Phi_0 k_\gamma \sin(k_\gamma y^\gamma + \varphi) \\ &= 16\Phi_0^2 k_\beta k_\alpha k_\gamma \left(\frac{1}{2} \sin[(k_\alpha y^\alpha - k_\gamma y^\gamma)] - \frac{1}{2} \sin[(k_\alpha y^\alpha + k_\gamma y^\gamma + 2\varphi)] \right),\end{aligned}$$

where

$$\cos(k_\alpha y^\alpha + \varphi) 4\Phi_0 k_\gamma \sin(k_\gamma y^\gamma + \varphi) = \left(\frac{1}{2} \sin[(k_\alpha y^\alpha - k_\gamma y^\gamma)] - \frac{1}{2} \sin[(k_\alpha y^\alpha + k_\gamma y^\gamma + 2\varphi)] \right),$$

$$\begin{aligned}\partial_\alpha \Gamma_{\rho\beta}^\beta \partial_\beta \Gamma_\lambda^\beta &= 4\Phi_0 k_\rho \partial_\alpha \sin(k_\rho y^\rho + \varphi) 4\Phi_0 k_\lambda \partial_\beta \sin(k_\lambda y^\lambda + \varphi) \\ &= 16\Phi_0^2 k_\rho k_\lambda k_\alpha k_\beta \cos(k_\rho y^\rho + \varphi) \cos(k_\lambda y^\lambda + \varphi) \\ &= \left(\frac{1}{2} \cos[(k_\rho y^\rho - k_\lambda y^\lambda)] + \frac{1}{2} \cos[(k_\rho y^\rho + k_\lambda y^\lambda + 2\varphi)] \right),\end{aligned}$$

where

$$\cos(k_\rho y^\rho + \varphi) 4\Phi_0 k_\beta \cos(k_\lambda y^\lambda + \varphi) = \left(\frac{1}{2} \cos[(k_\rho y^\rho - k_\lambda y^\lambda)] + \frac{1}{2} \cos[(k_\rho y^\rho + k_\lambda y^\lambda + 2\varphi)] \right).$$

For equation (25) we observe that the gravitational connections are a pure gauge in the non-metric region U_y of space-time. Following S.Weinberg (1996 [61]), gauge field is called pure gauge field if there exists a gauge transformation which makes it vanish everywhere. It is not difficult to show that the condition that the gravitational field-strength:

$$R_{\beta\alpha}(y) = \partial_\beta \Gamma_{\alpha\beta}^\beta(y) - \partial_\alpha \Gamma_{\beta\alpha}^\alpha(y) \quad (26)$$

should vanish everywhere is not necessary but also sufficient for the gravitational connection $\Gamma_{\alpha\beta}^\beta(y)$ to be expressible in any simply non-metric connected region U_y as pure gauge field.

Let a -metric region U_x of space-time be defined as follows:

$$U_{(x)} = \{x^\nu = (t, x^\kappa) \mid t_1 \leq t \leq t_2, -\infty < x^1, x^2, x^3 < +\infty, \nu = 0, 1, 2, 3, \kappa = 1, 2, 3\}, \quad (27)$$

where $(x) = x^\nu = (t, x^\kappa)$, $\nu = 0, 1, 2, 3, \kappa = 1, 2, 3$ is the coordinate frame where in ds^2 is defined. Here, t is metric time and x^κ , $\kappa = 1, 2, 3$ is the metric space.

The affine connection $\Gamma_{\nu\lambda}^\mu(x)$ defined by the metric $g_{\alpha\gamma}(x)$ tensor in such a region is given by:

$$\Gamma_{\nu\lambda}^\mu(x) = \frac{1}{2} g^{\alpha\gamma}(x) (g_{\alpha\beta,\mu}(x) + g_{\alpha\mu,\beta}(x) - g_{\beta\mu,\alpha}(x)). \quad (28)$$

From the derivatives of the connection (28) we construct the quantity:

$$R_{\mu\nu\kappa}^\lambda(x) = \frac{\partial \Gamma_{\mu\nu}^\lambda(x)}{\partial x^\kappa} - \frac{\partial \Gamma_{\mu\kappa}^\lambda(x)}{\partial x^\nu} + \Gamma_{\mu\nu}^\eta(x) \Gamma_{\kappa\eta}^\lambda(x) - \Gamma_{\mu\kappa}^\eta(x) \Gamma_{\nu\eta}^\lambda(x) \quad [61], \quad (29)$$

which transforms as a tensor, the Riemann-Christoffel curvature tensor (equation.(13)), of the metric region U_x of space-time. The commutator of two derivatives ∂_α and ∂_β may be expressed in terms of the gravitational field strength tensor given by equation (26) in the non-metric region U_y of space-time.

Similarly, the commutator of two covariant derivatives with respect x^ν and x^κ may be expressed in terms of the curvature:

$$T_{\mu\dots;\nu;\kappa}^{\lambda\dots} - T_{\mu\dots;\kappa;\nu}^{\lambda\dots} = R_{\sigma\nu\kappa}^\lambda T_{\mu\dots}^{\sigma\dots} + \dots - R_{\mu\nu\kappa}^\sigma T_{\sigma\dots}^{\lambda\dots} - \dots, \quad (30)$$

The necessary and sufficient condition for the existence of a gauge in which the gravitational connection $\Gamma_{\alpha\beta}^\beta(y)$ vanishes in a finite simply connected non-metric region U_y is the vanishing of the gravitational field strength tensor. The necessary and sufficient condition for the existence of a coordinate system in which the affine connection $\Gamma_{\nu\lambda}^\mu(x)$ vanishes in a finite simply connected metric region U_x is the vanishing of the Riemann-Christoffel curvature tensor [61].

In General Relativity, the affine connection $\Gamma_{\nu\lambda}^{\mu}(x)$ is itself constructed by the first derivatives of the metric tensor equation (29). In the non-metric region of space-time, the harmonic connection $\Gamma_{\alpha\beta}^{\beta}(y)$ is constructed by the first derivatives of the harmonic scalar $\Phi(y)$ equation (25).

Let a combined state of space-time formed by the metric and non-metric regions. In this combined state the metric $\{x\}$ equation (27) and non-metric $\{y\}$ equation (20) coordinates are linearly independent if $a\{x\} + b\{y\} = 0$ when $a = b = 0$. Therefore, the common space-time of the combined state has 8-dimensions [62,63] but 4-dimensional Riemannian-metric structure $G = G_{\mu\nu} du^{\mu} \otimes du^{\nu}$ []. From the physical point of view, the $\{y\}$ coordinates are just the non-metric description of the time and space and not additional space-time dimensions as in extra dimensional models or multiple time dimensions models [e.g. 17,18,51-60]. The generalized gravitational connections in a common space-time are defined as follows:

$$\tilde{\Gamma}_{\nu\lambda}^{\mu}(x, y) = \Gamma_{\nu\lambda}^{\mu}(x) \delta^{\alpha} \Gamma_{\alpha\beta}^{\beta}(y), \quad (31)$$

$\Gamma_{\nu\lambda}^{\mu}(x)$ is the affine connection (29) and $\Gamma_{\alpha\beta}^{\beta}(y)$ harmonic connection (25); δ^{α} , $\alpha = 0, 1, 2, 3$ is a constant vector field. From the derivatives of the generalize connection (31) with respects x^{κ} and x^{ν} we construct the quantity:

$$\tilde{R}_{\mu\nu\kappa}^{\lambda}(x, y) = \frac{\partial \tilde{\Gamma}_{\mu\nu}^{\lambda}(x, y)}{\partial x^{\kappa}} - \frac{\partial \tilde{\Gamma}_{\mu\kappa}^{\lambda}(x, y)}{\partial x^{\nu}} + \tilde{\Gamma}_{\mu\nu}^{\eta}(x, y) \tilde{\Gamma}_{\kappa\eta}^{\lambda}(x, y) - \tilde{\Gamma}_{\mu\kappa}^{\eta}(x, y) \tilde{\Gamma}_{\nu\eta}^{\lambda}(x, y), \quad (32)$$

which transforms as a tensor: the generalised curvature tensor in common space-time. Using equations (31), (24), (25) the generalized curvature tensor (32) becomes:

$$\begin{aligned} \tilde{R}_{\mu\nu\kappa}^{\lambda}(x, y) &= \delta^{\alpha} \Gamma_{\alpha\beta}^{\beta}(y) \left(\frac{\partial \Gamma_{\mu\nu}^{\lambda}(x)}{\partial x^{\kappa}} - \frac{\partial \Gamma_{\mu\kappa}^{\lambda}(x)}{\partial x^{\nu}} \right) + O^2 \left(\delta^{\alpha} \Gamma_{\alpha\beta}^{\beta}(y) \right) (\Gamma_{\mu\nu}^{\eta}(x) \Gamma_{\kappa\eta}^{\lambda}(x) \\ &\quad - \Gamma_{\mu\kappa}^{\eta}(x) \Gamma_{\nu\eta}^{\lambda}(x)) = \delta^{\alpha} \Gamma_{\alpha\beta}^{\beta}(y) R_{\mu\nu\kappa}^{\lambda}(x) |_P = \delta^{\alpha} \partial_{\alpha} \Phi(y) R_{\mu\nu\kappa}^{\lambda}(x) |_P, \\ &= 4\Phi_0 R_{\mu\nu\kappa}^{\lambda}(x) |_P \delta^{\alpha} k_{\alpha} \sin(k_{\alpha} y^{\alpha} + \varphi) = 4A_G R_{\mu\nu\kappa}^{\lambda}(x) |_P \sin(k_{\alpha} y^{\alpha} + \varphi) = R_{\mu\nu\kappa}^{\lambda}(x) |_P \Omega(y) \end{aligned} \quad (33)$$

where:

$$R_{\mu\nu\kappa}^{\lambda}(x) |_P = \left(\frac{\partial \Gamma_{\mu\nu}^{\lambda}(x)}{\partial x^{\kappa}} - \frac{\partial \Gamma_{\mu\kappa}^{\lambda}(x)}{\partial x^{\nu}} \right), \quad (34)$$

$$\Omega(y) = 2A_G \sin(k_{\alpha} y^{\alpha} + \varphi), \quad (35)$$

$$A_G = \Phi_0 \delta^{\alpha} k_{\alpha}, \quad (36)$$

$\Omega(y)$ is a harmonic function in the (y) coordinate frame; $R_{\mu\nu\kappa}^{\lambda}(x)|_p$ is the Riemann-Christoffel curvature tensor at the point P where the Christoffel symbols are all equal to zero; $A_G = \Phi_0 \delta^\alpha k_\alpha$ is a constant.

The generalised curvature tensor satisfies the wave equation in the (y) coordinate frame. Therefore, the generalised curvature tensor $\tilde{R}_{\mu\nu\kappa}^{\lambda}(x, y)$ is harmonic in common space-time. The generalised harmonic Ricci tensor and the curvature scalar are constructed in common space-time by the metric and equation (36), as follows:

$$\tilde{R}_{\mu\nu}(x, y) = g^{\kappa\lambda} \tilde{R}_{\mu\nu\kappa\lambda}(x, y) = g^{\kappa\lambda} R_{\mu\nu\kappa\lambda}(x) \Omega(y) = R_{\mu\nu} \Omega(y), \quad (37)$$

where:

$$R_{\mu\nu} = g^{\kappa\lambda} R_{\mu\nu\kappa\lambda}(x), \quad (38)$$

is the usual Ricci tensor and $\Omega(y)$ is given by equation (35).

$$\tilde{R}(y) = g^{\mu\nu} \tilde{R}_{\mu\nu}(x, y) = g^{\mu\nu} R_{\mu\nu} \Omega(y) = R \Omega(y), \quad (39)$$

Where:

$$R = g^{\mu\nu} R_{\mu\nu} \quad (40)$$

is the usual curvature scalar. The latter yields to the relation:

$$\frac{\tilde{R}(y)}{\Omega(y)^2} = \frac{R}{\Omega(y)}, \quad \Omega(y) \neq 0 \quad (41)$$

Using equation (41), the Lagrangian for the gravitational field is given by:

$$\mathfrak{L}(y) = \frac{\sqrt{-g} \tilde{R}(y)}{16\pi G \Omega(y)^2} = \frac{\sqrt{-g} R}{16\pi G_N \Omega(y)} = \frac{\mathfrak{L}_{EH}}{\Omega(y)}, \quad \Omega(y) \neq 0 \quad (42)$$

where the Einstein-Hilbert Lagrangian density is:

$$\mathfrak{L}_{EH} = \frac{\sqrt{-g} R}{16\pi G_N}, \quad (43)$$

R is the Ricci scalar and G_N is Newton's gravitational constant.

In analogy with a closed systems in mechanics, where the Lagrangian does not depend explicitly on time, the mathematical expression of a closed system is the absence of any express of the Lagrangian in terms

of coordinates.. Hence the gravitational Lagrangian (42) corresponds to the open sector, while the Lagrangian (43) to the closed sector.

Following our previous work [10], for the time variation of gravitational constant G_N [1], the gravitational Lagrangian (42) can be written as follows:

$$\mathfrak{I}(y) = \frac{\sqrt{-g} R}{16\pi\delta G_N(y)} \quad , \quad (44)$$

where:

$$\delta G_N(y) = G_N \Omega(y) = 2A_G G_N \sin(k_\alpha y^\alpha + \varphi) \quad , \quad (45)$$

and $\Omega(y)$ is given by equation (35). Thus, we have

$$G'_N(y) = G_N + 4A_G G_N \sin(k_\alpha y^\alpha + \varphi) \quad , \quad (46)$$

Equation. (46) is analogous to the model proposed in our previous paper [10], a non-conservative theory of gravity with an effective scale of 5 years. From a general expansion of multiple commutators, it is possible to extract second-order perturbation terms. The latter lead to corrections of source terms that reveal a characteristic harmonic oscillation term. As Fig. 2 shows, the proposed non-conservative theory of gravity can explain the observed variations of G at 5.9-year scale (for details see Ref [10]).

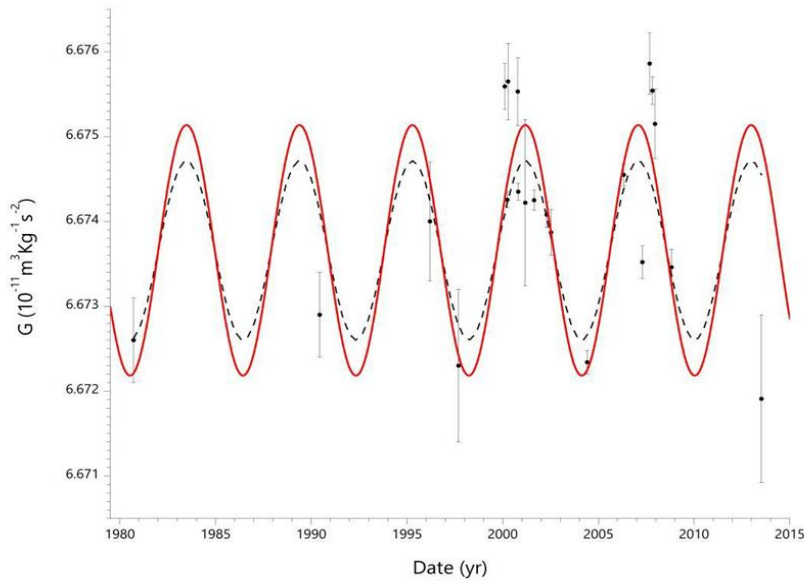


Figure. 2 Comparison of the set of G_N measurements with a fitted sine wave predicted by our model (46) with the 5.9 year LOD oscillation cycle. The black dots are the experimental measurements of G_N , the red curve is the theoretical variation of G_N predicted by model (46). The black dust curve is the set of G_N measurements fitted by the sine wave predicted by model (46). (Ref [10]).

A thermodynamic interpretation of $\Omega(y)$ (equation.(35)) supported by the Ref [66], [67] in multidimensional entropic space is under investigation [68].

6. The scale of quantum gravity in the presence of the variations of the Newton constant

It has become a convention to interpret the Planck scale M_P as a fundamental scale of nature: the scale at which quantum gravitational effects become important. However, Newton's constant $G_N = M_{pl}^{-2}$ in natural units $\hbar = c = 1$ is measured in very low-energy experiments, and its connection to physics at short distances – in particular, quantum gravity - is tenuous. If the strength of gravitational interactions is scale-dependent, the scale μ_* at which quantum gravity effects are significant is one at which:

$$G(\mu_*) \approx \mu_*^{-2} \quad (47)$$

This condition implies that at length scales μ_*^{-1} gravity will be unsuppressed. Below we show that condition (47) can be satisfied in models with μ_* as small as a 100TeV ([45]).

To propose gravity, we consider a coupled scalar field S and adopt the following notation:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{16\pi\delta G_N(y)} R + \frac{1}{2} g^{\mu\nu} \partial_\mu S \partial_\nu S \right), \quad (48)$$

where the variation of the gravitational constant $\delta G_N(y)$ is given by equation.(45)

Consider the gravitational potential between two heavy, non-relativistic sources, which arises through graviton exchange (Fig. 3. [19]).

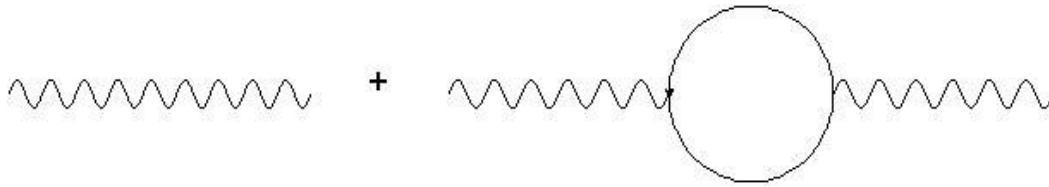


Fig. 3: Contributions to the running of Newton's constant.

The leading term in the gravitational Lagrangian (48) is:

$$\delta G(y)^{-1} R = \delta G(y)^{-1} h \left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) h, \quad (49)$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (50)$$

We can interpret quantum corrections to the massless gravitons propagators from the loop (similar to those given in Ref [19]) as a renormalization of $\delta G(y)$. Ignoring the index structure, the massless graviton propagator with one-loop correction is:

$$D(k)_c \approx \frac{i\delta G(y)}{k^2} + \frac{i\delta G(y)}{k^2} \Sigma \frac{i\delta G(y)}{k^2} + \dots, \quad (51)$$

where k is the momentum carried by the graviton in this model. The term in Σ proportional to k^2 can be interpreted as a renormalization of $\delta G(y)$, and is easily estimated from the Feynman diagram (see Fig. 2):

$$\Sigma \approx -ik^2 \int^\Lambda d^4 p D(p)^2 p^2 + \dots, \quad (52)$$

where $D(p)$ is the propagator of the particle in the loop. In the case of a scalar field, the loop integral is quadratically divergent. By absorbing equation (52) into a redefinition of G in the usual way, one obtains an equation of the form:

$$\delta M_{Pl}^2(t) = M_{Pl}^2(t) - M_{Pl}^2 = \frac{1}{G(t)_{ren}^{shift}} - \frac{1}{G_{ren}} = \frac{1}{2A_G G_{ren} \sin(a_G t + \varphi)} = \frac{1}{\delta G(t)_{ren}^{Error}}, \quad (53)$$

where

$$\frac{1}{G_{ren}} = \frac{1}{G_{bar}} + c\Lambda^2 [19], \quad (54)$$

Λ is the ultraviolet UV cutoff of the loop and $c = 1/16\pi^2$. G_{ren} is the renormalized Newton constant measured in low-energy experiments, for time scales $t \ll \tau$. Fermions contribute with the same sign to the running of Newton's constant, whereas gauge bosons contribute with the opposite sign to scalars.

Taking $\Lambda_{QG} = \mu_*$ (so that the loop cutoff coincides with the onset of quantum gravity) gives $G_{bare} = G(\Lambda_{QG}) = \mu_*^{-2}$. The requirement that $G_{ren} = M_{Pl}^{-2}$ implies that μ_* cannot be very different from the Planck scale M_{Pl} , unless c is very large. For instance, to have $\mu_* = 1 \text{ TeV}$ requires $c = 10^{32}$: 10^{32} ordinary scalars or fermions with masses below 1 TeV (which can run in the loop). This observation has already been made by Dvali et al. [23, 24, 25], although in [23] the argument is expressed in terms of a consistency condition from black hole evaporation rather than as a renormalization of group behaviour

[19,39]. The number of new degrees of freedom that we are required to introduce may seem high, but it is of the same order as in models with large extra-dimensions [17,18].

For equation (53), the square of reduced Planck mass $M_{Pl}^2(t)$ on Earth is given as follows:

$$M_{Pl}^2(t) = \frac{1}{G(t)_{ren}^{shift}} = \frac{1}{G_{ren}} \left(1 + \frac{1}{2A_G \sin(a_G t + \varphi)} \right), \quad (55)$$

Substituting equation (54) to equation (55) we obtain;

$$M_{Pl}^2(t) = \frac{1}{G(t)_{ren}^{shift}} = \frac{1}{G(t)_{bare}^{shift}} + N \frac{\Lambda(t)_{shift-QG}^2}{12\pi}, \quad (56)$$

where

$$\Lambda_{QG}^{shift}(t) = \Lambda_{QG}^{predicted} \sqrt{1 + \frac{1}{2A_G \sin(a_G t + \varphi)}} \quad (57)$$

is the time-dependent shift of quantum gravity scale for the theoretically predicted value $\Lambda_{QG}^{predicted}$ in TeV theories of gravity on Earth due to the generalised harmonic curvature scalar $\tilde{R}(y)$ (39).

For equations (53) and (55), we observe that the possible $\tilde{R}(y)$ that shifts the true value of renormalized gravitational constant G_{ren} in Cavendish-type experiments at low energy scales is also responsible for the shift of the predicted value of the bare gravitational constant $G_{bare}=1/k_{bare}^2$ in gravitational scattering experiments at high energy scales. Here, k_{bare} is the bare momentum carried by the graviton. From equation (57), we find that the shift of quantum gravity for predicted value of 1TeV has the global minima:

$$\Lambda_{QG}^{shift}(t)_{min} = \Lambda_{QG}^{predicted} \min \left\{ \sqrt{1 + \frac{1}{2A_G \sin \theta}} \right\} = 100 \Lambda_{QG}^{predicted} = 100 TeV, \quad (58)$$

$$\text{at } \theta = a_G t + \varphi = 2\pi n + \frac{\pi}{2}, \text{ for integer } n, \quad (59)$$

where $A_G = 10^{-4}$ (Anderson et al. 2015a) [1], and $\Lambda_{QG}^{predicted} = 1TeV$ [17,18,19].

The generalised harmonic curvature scalar $\tilde{R}(y)$ that is responsible for the variation of the gravitational constant shifts the scale of Quantum gravity at 100TeV. Quantum gravity effects are at energy scales significantly beyond that of the LHC [30]. These effects can thus be investigated by a 100 TeV Proton-Proton Collider [31-33].

7. Conclusion

In this paper, we follow a Maxwellian approach to gravity to develop a non-metric extension of general relativity. Harmonic quantities, namely generalised Riemann curvature tensor and scalar, are constructed emerge for a combined state of space-time forms by the metric and non-metric space-time phases. We find that the sinusoidal time variation of the gravitational constant G_N originates the Lagrangian of the propose model. Furthermore, we show that the generalised harmonic curvature scalar that is responsible for the variation of the gravitational constant shifts the scale of Quantum gravity at 100TeV. Quantum gravity effects are at energy scales significantly beyond that of the LHC. These effects can thus be investigated by a 100 TeV Proton-Proton Collider.

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