



HAL
open science

Nonlinear Convex Control for Robotic Assistive Therapy via LMIs

Juan Carlos Arceo Luzanilla, Jimmy Lauber, Emilie Simoneau-Buessinger,
Sylvain Cremoux

► **To cite this version:**

Juan Carlos Arceo Luzanilla, Jimmy Lauber, Emilie Simoneau-Buessinger, Sylvain Cremoux. Non-linear Convex Control for Robotic Assistive Therapy via LMIs. Workshop on Assistance and Service Robotics in a Human Environment: From Personal Mobility Aids to Rehabilitation-Oriented Robotics in the International Conference on Intelligent Robots 2018 (IROS 2018), Oct 2018, Madrid, Spain. hal-01911178

HAL Id: hal-01911178

<https://hal.science/hal-01911178v1>

Submitted on 2 Nov 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Nonlinear Convex Control for Robotic Assistive Therapy via LMIs

Juan Carlos Arceo¹, Jimmy Lauber¹, Emilie Simoneau¹ and Sylvain Cremoux¹

Abstract—A device for personalized robotic assisted therapy is introduced, the process for system identification and control design for a class of nonlinear discrete-time systems with output disturbance attenuation is presented. The results obtained are in terms of linear matrix inequalities. Simulation and real-time results are shown as well.

I. INTRODUCTION

According to World Health Organization there are 15 million stroke cases each year, 5 million of these people will be disabled. The economic cost for this illness is equivalent to 53.6 billion dollars in USA, more than 4% of the total health care cost in UK and 3% in Netherlands [3].

One of the goals in robotic rehabilitation is restore the motor function of stroke patients and this can be achieved by constraint-induced movement. Therapy must adapt to user's requirements [5] in aspects such as the degree of disability and its economic cost [1], given this context and inspired by [10] the motoBOTTE shown in Fig. 1 is a device designed for robotic assisted therapy with a low production cost.

This system will be coupled with a hybrid brain-computer interface and a force sensor to offer personalized therapy and a first step is track a reference with the ankle (denoted by ϕ). Therefore, this report deals with modeling and control of the motoBOTTE. Convex systems (also named Takagi-Sugeno) [8] combined with Lyapunov's direct method can be used for nonlinear controller design; a common approach is parallel compensated distribution (PDC) [7], which mimics the nonlinearities in the system by means of the interpolation functions, this approach yields to conditions in terms of linear matrix inequalities (LMIs) that can be solved via optimization algorithms in polynomial time with commercially available software [2]; performance parameters like disturbance attenuation, input constraint and decay rate can easily be included in the design [7].

In Section II system identification and nonlinear controller design processes are shown. In Section III simulation and real-time results are exhibited. In Section IV a conclusion based on the results is given.

II. PRELIMINARIES

The system is actuated by a linear piston [6], the angle is limited mechanically $\phi \in [1.273, 2.5212]$ in radians; the sampling time (denoted by T_s) is 0.01 seconds, there is an



Fig. 1. The motoBOTTE.

input constraint of ± 10 Volts and if the input equals zero the system will not move. Based on the given information we can describe the dynamics of the motoBOTTE with:

$$\phi(t+1) = \phi(t) + (b_0 + b_1\phi(t))u(t), \quad (1)$$

where b_0 and b_1 are unknown parameters to be estimated via input and output data and the Parameter Estimation Toolbox in MATLAB. The real-time signals (in black) and the estimated (in blue) are shown in Fig. 2, a squared error cost function was optimized to a value of $ss_{cf} = 1.0475$ via the Levenberg-Marquardt algorithm [4] the parameters values obtained were $b_0 = -0.00203$ and $b_1 = 0.00054$.

Defining the filtered reference as $r_f(t+1) = Ar_f(t) + (1-A)r(t)$, with $|A| < 1$, $r(t)$ is the reference, the error $e(t+1) = \phi(t+1) - r_f(t+1)$, its integral $e_i(t+1) = e_i(t) + T_s e(t)$ and its time derivative $e_d(t+1) = \frac{e(t+1) - e(t)}{T_s}$; using $\Delta_1 = \frac{r}{r_f}$ and $\Delta_2 = \frac{r_f}{\phi}$ to gather the expressions as $Ex(t+1) = A(\Delta_1)x(t) + B(\phi)u(t) + P(\Delta_1, \Delta_2, \phi)$ with

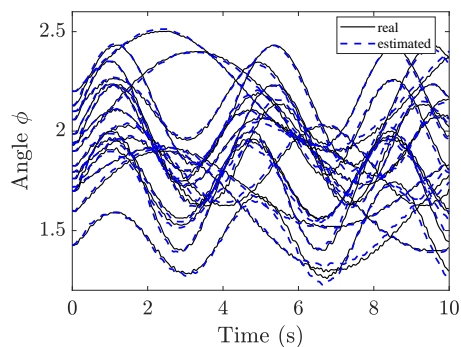


Fig. 2. Estimation process.

*This work was supported by the University of Valenciennes.

¹The authors are with the Univ. Valenciennes, LAMIH UMR CNRS 8201, F-59313 Valenciennes, France. (corresponding author: Jimmy.Lauber@univ-valenciennes.fr)

$x(t) = [e(t) \ e_i(t) \ e_d(t)]^T$. If $r_f = r$, then $\Delta_1 = 1$ and $P(\cdot) = 0$, we will design a controller for this. As there exists E^{-1} , our error system is $x(t+1) = \bar{A}x(t) + \bar{B}(\phi)u(t)$, with $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & T_s \end{bmatrix}$, $A = \begin{bmatrix} 1 & 0 & 0 \\ T_s & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b_0 + b_1\phi(t) \\ 0 \\ 0 \end{bmatrix}$, $\bar{A} = E^{-1}A$ and $\bar{B}(\phi) = E^{-1}B$. The nonconstant term $\phi(t)$ can be rewritten by means of the nonlinear sector approach [8], it is bounded by $\phi \in [1.273, 2.5212]$. Define the interpolation functions as $h_1 = \frac{\phi_{\max} - \phi(t)}{\phi_{\max} - \phi_{\min}}$ and $h_2 = \frac{\phi(t) - \phi_{\min}}{\phi_{\max} - \phi_{\min}}$, notice that $h_1 + h_2 = 1$ and $0 \leq h_i \leq 1$ for $i \in \{1, 2\}$, the convex system is $x(t+1) = \sum_{i=1}^2 h_i (\bar{A}x(t) + \bar{B}_i u(t))$, with $\bar{B}_i = \bar{B}(\phi)|_{h_i=1}$ constant terms $\bar{B}_1 = \begin{bmatrix} -0.0013 \\ 0 \\ -0.1340 \end{bmatrix}$ and $\bar{B}_2 = \begin{bmatrix} -0.0007 \\ 0 \\ -0.0665 \end{bmatrix}$.

For control design with disturbance attenuation consider a disturbance $w(t)$ measured in the output $y(t) = Cx(t) + Dw(t)$. The origin of the system is asymptotically stable with a PDC control law [7] $u(t) = \sum_{j=1}^2 F_j x(t) = F_h x(t)$, decay rate α^2 and γ is minimized subject to $\|\frac{y(t)}{w(t)}\| \leq \gamma$ if there are matrices $X > 0$, $\gamma > 0$, M_j for $(i, j) \in \{1, 2\}$, where $F_i = M_i X^{-1}$ and $2\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0$ [9] holds with :

$$\Upsilon_{ij} = \begin{bmatrix} -\alpha^2 X & X C^T D & X A_i^T + M_j^T B_i^T & X C^T \\ D^T C X & D^T D - \gamma^2 I & 0 & 0 \\ A_i X + B_i M_j & 0 & -X & 0 \\ C X & 0 & 0 & -I \end{bmatrix}.$$

III. RESULTS

The output and disturbance matrices are $C = I_3$ and $D = 0.1 \times [1 \ 0.01 \ 100]^T$, we want a decay rate of $\alpha^2 = 0.935$. A solution was found for the LMIs with $\gamma = 10.0241$, $F_1 = [146.7459 \ 578.6829 \ 0.1757]$ and $F_2 = [285.8307 \ 1127.1404 \ 0.3419]$. The closed-loop response is shown in Fig. 3, where the dashed line is the filtered reference, the solid black line is the output angle in simulation, the blue line is the real-time output angle and the red line is the real-time output angle where the disturbance is a leg pushing in an opposite direction to r_f . The control signal used for each case is illustrated in Fig. 4.

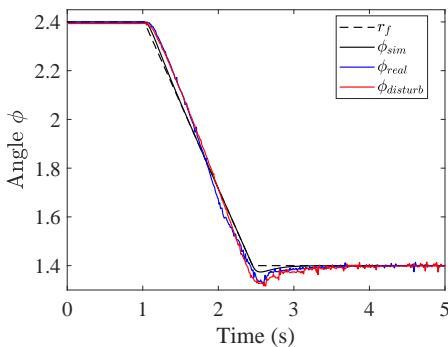


Fig. 3. Simulation and real-time response.

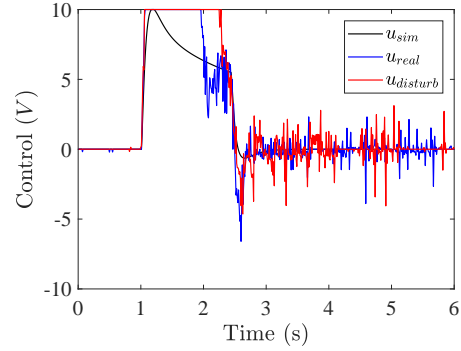


Fig. 4. Simulation and real-time control.

IV. CONCLUSIONS

The mathematical model of a device used for robotic assisted therapy has been obtained, then it was used for non-linear controller design with output disturbance attenuation guaranteed. Conditions in terms of LMI have been provided and results in simulation and real-time have been shown. The performance of the closed-loop system might be improved by combining a feedforward with a feedback control approach considering that the trajectory its known *a priori*.

REFERENCES

- [1] I. Díaz, J. J. Gil, and E. Sánchez, "Lower-limb robotic rehabilitation: literature review and challenges," *Journal of Robotics*, vol. 2011, 2011.
- [2] J. Lofberg, "YALMIP : a toolbox for modeling and optimization in matlab," in *2004 IEEE International Symposium on Computer Aided Control Systems Design*, 2004, pp. 284–289.
- [3] J. Mackay, G. A. Mensah, and K. Greenlund, *The atlas of heart disease and stroke*. World Health Organization, 2004.
- [4] J. J. Moré, "The levenberg-marquardt algorithm: implementation and theory," in *Numerical analysis*. Springer, 1978, pp. 105–116.
- [5] R. Riener, M. Frey, M. Bernhardt, T. Nef, and G. Colombo, "Human-centered rehabilitation robotics," in *Rehabilitation Robotics, 2005. ICORR 2005. 9th International Conference on*. IEEE, 2005, pp. 319–322.
- [6] *CAHB-21 Linear Actuator: Installation, operation and maintenance manual*, SKF Taiwan Co., Ltd, No. 3, Lane 11, Tzu-Chiang St., Tu-Cheng Industrial District, Taipei, Taiwan, 2010.
- [7] K. Tanaka and H. Wang, *Fuzzy Control Systems Design and Analysis: A linear matrix inequality approach*. New York: John Wiley & Sons, 2001.
- [8] T. Taniguchi, K. Tanaka, and H. Wang, "Model construction, rule reduction and robust compensation for generalized form of Takagi-Sugeno fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 2, pp. 525–537, 2001.
- [9] H. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto, "Parameterized linear matrix inequality techniques in fuzzy control system design," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 2, pp. 324–332, 2001.
- [10] R. Xu, N. Jiang, N. Mrachacz-Kersting, C. Lin, G. A. Prieto, J. C. Moreno, J. L. Pons, K. Dremstrup, and D. Farina, "A closed-loop brain-computer interface triggering an active ankle-foot orthosis for inducing cortical neural plasticity," *IEEE Transactions on Biomedical Engineering*, vol. 61, no. 7, pp. 2092–2101, 2014.