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# Nonlinear Convex Control for Robotic Assistive Therapy via LMIs

Juan Carlos Arceo<sup>1</sup>, Jimmy Lauber<sup>1</sup>, Emilie Simoneau<sup>1</sup> and Sylvain Cremoux<sup>1</sup>

**Abstract**—A device for personalized robotic assisted therapy is introduced, the process for system identification and control design for a class of nonlinear discrete-time systems with output disturbance attenuation is presented. The results obtained are in terms of linear matrix inequalities. Simulation and real-time results are shown as well.

## I. INTRODUCTION

According to World Health Organization there are 15 million stroke cases each year, 5 million of these people will be disabled. The economic cost for this illness is equivalent to 53.6 billion dollars in USA, more than 4% of the total health care cost in UK and 3% in Netherlands [3].

One of the goals in robotic rehabilitation is restore the motor function of stroke patients and this can be achieved by constraint-induced movement. Therapy must adapt to user's requirements [5] in aspects such as the degree of disability and its economic cost [1], given this context and inspired by [10] the motoBOTTE shown in Fig. 1 is a device designed for robotic assisted therapy with a low production cost.

This system will be coupled with a hybrid brain-computer interface and a force sensor to offer personalized therapy and a first step is track a reference with the ankle (denoted by  $\phi$ ). Therefore, this report deals with modeling and control of the motoBOTTE. Convex systems (also named Takagi-Sugeno) [8] combined with Lyapunov's direct method can be used for nonlinear controller design; a common approach is parallel compensated distribution (PDC) [7], which mimics the nonlinearities in the system by means of the interpolation functions, this approach yields to conditions in terms of linear matrix inequalities (LMIs) that can be solved via optimization algorithms in polynomial time with commercially available software [2]; performance parameters like disturbance attenuation, input constraint and decay rate can easily be included in the design [7].

In Section II system identification and nonlinear controller design processes are shown. In Section III simulation and real-time results are exhibited. In Section IV a conclusion based on the results is given.

## II. PRELIMINARIES

The system is actuated by a linear piston [6], the angle is limited mechanically  $\phi \in [1.273, 2.5212]$  in radians; the sampling time (denoted by  $T_s$ ) is 0.01 seconds, there is an



Fig. 1. The motoBOTTE.

input constraint of  $\pm 10$  Volts and if the input equals zero the system will not move. Based on the given information we can describe the dynamics of the motoBOTTE with:

$$\phi(t+1) = \phi(t) + (b_0 + b_1 \phi(t)) u(t), \quad (1)$$

where  $b_0$  and  $b_1$  are unknown parameters to be estimated via input and output data and the Parameter Estimation Toolbox in MATLAB. The real-time signals (in black) and the estimated (in blue) are shown in Fig. 2, a squared error cost function was optimized to a value of  $ss_{cf} = 1.0475$  via the Levenberg-Marquardt algorithm [4] the parameters values obtained were  $b_0 = -0.00203$  and  $b_1 = 0.00054$ .

Defining the filtered reference as  $r_f(t+1) = Ar_f(t) + (1-A)r(t)$ , with  $|A| < 1$ ,  $r(t)$  is the reference, the error  $e(t+1) = \phi(t+1) - r_f(t+1)$ , its integral  $e_i(t+1) = e_i(t) + T_s e(t)$  and its time derivative  $e_d(t+1) = \frac{e(t+1) - e(t)}{T_s}$ ; using  $\Delta_1 = \frac{r}{r_f}$  and  $\Delta_2 = \frac{r_f}{\phi}$  to gather the expressions as  $Ex(t+1) = A(\Delta_1)x(t) + B(\phi)u(t) + P(\Delta_1, \Delta_2, \phi)$  with

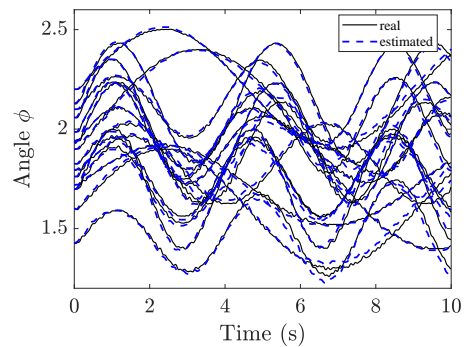


Fig. 2. Estimation process.

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$x(t) = [e(t) \ e_i(t) \ e_d(t)]^T$ . If  $r_f = r$ , then  $\Delta_1 = 1$  and  $P(\cdot) = 0$ , we will design a controller for this. As there exists  $E^{-1}$ , our error system is  $x(t+1) = \bar{A}x(t) + \bar{B}(\phi)u(t)$ , with  $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & T_s \end{bmatrix}$ ,  $A = \begin{bmatrix} 1 & 0 & 0 \\ T_s & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} b_0 + b_1\phi(t) \\ 0 \\ 0 \end{bmatrix}$ ,  $\bar{A} = E^{-1}A$  and  $\bar{B}(\phi) = E^{-1}B$ . The nonconstant term  $\phi(t)$  can be rewritten by means of the nonlinear sector approach [8], it is bounded by  $\phi \in [1.273, 2.5212]$ . Define the interpolation functions as  $h_1 = \frac{\phi_{\max} - \phi(t)}{\phi_{\max} - \phi_{\min}}$  and  $h_2 = \frac{\phi(t) - \phi_{\min}}{\phi_{\max} - \phi_{\min}}$ , notice that  $h_1 + h_2 = 1$  and  $0 \leq h_i \leq 1$  for  $i \in \{1, 2\}$ , the convex system is  $x(t+1) = \sum_{i=1}^2 h_i (\bar{A}x(t) + \bar{B}_i u(t))$ , with  $\bar{B}_i = \bar{B}(\phi)|_{h_i=1}$  constant terms  $\bar{B}_1 = \begin{bmatrix} -0.0013 \\ 0 \\ -0.1340 \end{bmatrix}$  and  $\bar{B}_2 = \begin{bmatrix} -0.0007 \\ 0 \\ -0.0665 \end{bmatrix}$ .

For control design with disturbance attenuation consider a disturbance  $w(t)$  measured in the output  $y(t) = Cx(t) + Dw(t)$ . The origin of the system is asymptotically stable with a PDC control law [7]  $u(t) = \sum_{j=1}^2 F_j x(t) = F_h x(t)$ , decay rate  $\alpha^2$  and  $\gamma$  is minimized subject to  $\| \frac{y(t)}{w(t)} \| \leq \gamma$  if there are matrices  $X > 0$ ,  $\gamma > 0$ ,  $M_j$  for  $(i, j) \in \{1, 2\}$ , where  $F_i = M_i X^{-1}$  and  $2\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0$  [9] holds with :

$$\Upsilon_{ij} = \begin{bmatrix} -\alpha^2 X & X C^T D & X A_i^T + M_j^T B_i^T & X C^T \\ D^T C X & D^T D - \gamma^2 I & 0 & 0 \\ A_i X + B_i M_j & 0 & -X & 0 \\ C X & 0 & 0 & -I \end{bmatrix}.$$

### III. RESULTS

The output and disturbance matrices are  $C = I_3$  and  $D = 0.1 \times [1 \ 0.01 \ 100]^T$ , we want a decay rate of  $\alpha^2 = 0.935$ . A solution was found for the LMIs with  $\gamma = 10.0241$ ,  $F_1 = [146.7459 \ 578.6829 \ 0.1757]$  and  $F_2 = [285.8307 \ 1127.1404 \ 0.3419]$ . The closed-loop response is shown in Fig. 3, where the dashed line is the filtered reference, the solid black line is the output angle in simulation, the blue line is the real-time output angle and the red line is the real-time output angle where the disturbance is a leg pushing in an opposite direction to  $r_f$ . The control signal used for each case is illustrated in Fig. 4.

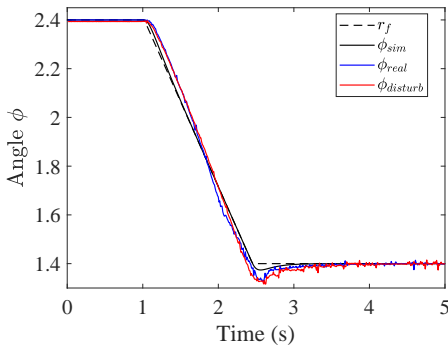


Fig. 3. Simulation and real-time response.

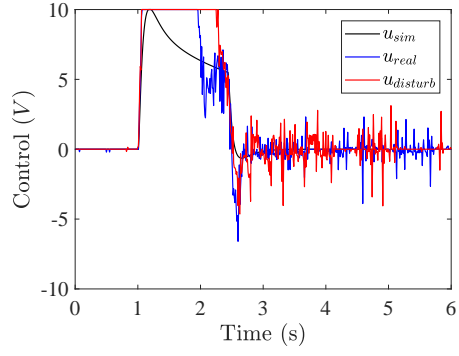


Fig. 4. Simulation and real-time control.

### IV. CONCLUSIONS

The mathematical model of a device used for robotic assisted therapy has been obtained, then it was used for non-linear controller design with output disturbance attenuation guaranteed. Conditions in terms of LMI have been provided and results in simulation and real-time have been shown. The performance of the closed-loop system might be improved by combining a feedforward with a feedback control approach considering that the trajectory its known *a priori*.

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