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## Durability of structures based on bio-sourced materials against three-dimensional crack problems

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### Résumé :

La durabilité des structures à base de matériaux biosourcés est confrontée à des problèmes de fissurations sous chargements mécaniques complexes associées à des variations climatiques sévères. L'objectif principal de ce travail est de présenter une nouvelle modélisation mathématique pour les problèmes de fissurations tridimensionnelle sous chargement en mode mixte. Le modèle repose sur nouvelle intégrale, intitulée  $M_{\theta}^{3D}$ , dont l'implémentation dans un logiciel par élément finis permet d'obtenir le taux de restitution d'énergie ou la ténacité associée. En prenant le matériaux bois pour exemple, les résultats montrent comment évolue la résistance à la fissuration en fonction de différents taux de mixité.

### 1. Introduction

The complex mechanical loading and high climatic variations on structures implies having a better understanding of their fracture mechanical behavior. It is necessary to consider three-dimensional case in the study of reel crack growth problem. For it the most common approach is the determination of the energy release rate by energetic approach. Civil engineering and mechanical structures are usually submitted to mixed mode loading. As a consequence, a mixed mode crack growth process occurs.

This fact appears again as an important key in the case of three-dimensional medium. 2D idealization case can be applicable to a variety of engineering problems. Nevertheless, some problems in nature cannot be idealized. For example, the case of a non-straight crack front or the case when climatic variations are associated with a mechanical stress field.

Most of the studies carried out deal with two-dimensional case. The studies of the durability of structures based on bio-sourced materials request the development of specific tools for three-dimensional configurations.

In this context, our approach is to rewrite the M-integral formalism, starting from the very beginning with a three-dimensional analysis. The topic of this paper deals with the generalisation of the mixed mode M-integral formalism to a three-dimensional problem and its adaptability of the theta method for a finite element implementation. In order to complete fracture tools, this paper deals with a new 3D integral parameter allowing the fracture mode separation taking into account effect of anti-plane (mode III) and thickness during crack growth process. The first section recalls some integral parameters that we use in calculating the energy release rate for three-dimensional configuration around the crack front line. The development of the three-dimensional M-integral concept, based on a Noether's theorem, is presented and the numerical validation is proposed in the second section. Results are presented in terms of the energy release rate evaluations for mixed-mode configurations.

### 2. Analytical formulation

For plan problem and for static crack Rice (1968) [1] has defined A path independent integral which allows to compute energy release rate around the crack tip. For cracked linear elastic material, Rice [1] have used J-integral to compute energy release rate for curvilinear contour. J-integral takes the following notation:

$$J^{2D} = \int_{\Gamma} (W \cdot n_1 - (\sigma_{ij} \cdot n_j \cdot u_{i,1})) \cdot d\Gamma \tag{1}$$

Where  $W$  denotes the strain energy density,  $\Gamma$  is arbitrary curvilinear contour oriented by its normal vector,  $u_i$  is the displacement component and  $\sigma_{ij}$  is the stress component.

For mixed mode three-dimensional problem, the crack front line is defined as the intersection of two surface (figure 1).

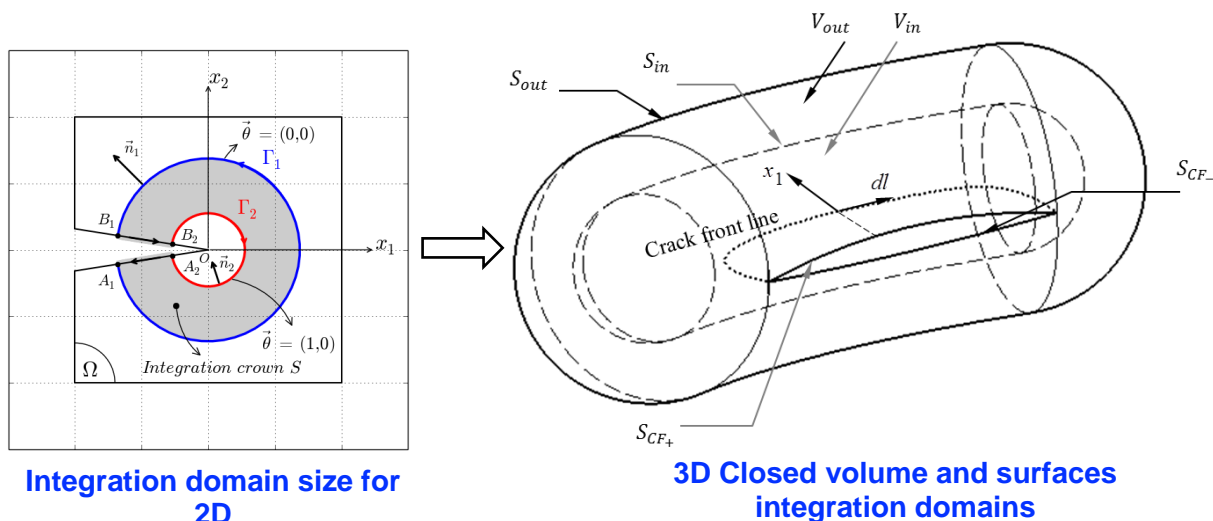


Figure 1. Closed volume and surfaces integration domains for 2D and 3D case

The new mixed mode integral formulation is based on energetic approach using consideration described in figure 1. After some mathematical developments, we obtain:

$$M_{\theta}^{3D} = \frac{1}{2} \int_V (\sigma_{ij}^u \cdot v_{i,k} - \sigma_{ij}^v \cdot u_i) \cdot \theta_{k,j} \cdot dV \tag{2}$$

This integral entitled  $M_{\theta}^{3D}$  is a combination between real and virtual displacement fields which enables computing fracture parameter. This integral designates the classical part used for the determination of the crack growth initiation.

### 3. Numerical validation

The numerical implementation is based on a Mixed Mode Crack Growth specimen (MMCG) loaded in an opening mode, shear mode, and mixed mode (figure 2). The MMCG specimen is adapted to wood material [3, 4 5]. Figure. 2 presents the dimensions in millimetres of the initial specimen. This specimen is adapted to obtain a stable crack growth rate during propagation for mixed mode.

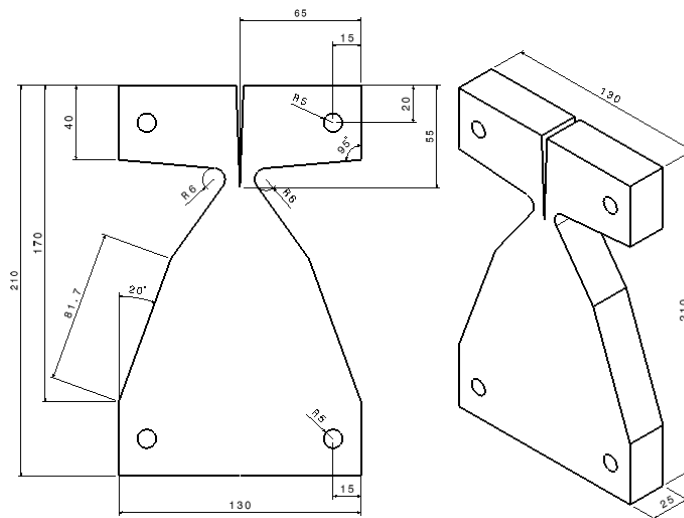


Figure 2. Mixed Mode Crack Growth specimen

In this specimen, four holes are machined in order to fixe the Arcans device (figure 3). In this work, new shapes and dimensions of the Arcans loading device are proposed, see e.g. Figure.3. The system allows considering, during the test, the specimens for the factor  $a = 1/2$  and  $a = 1$ , and different thicknesses for MMCG specimen.

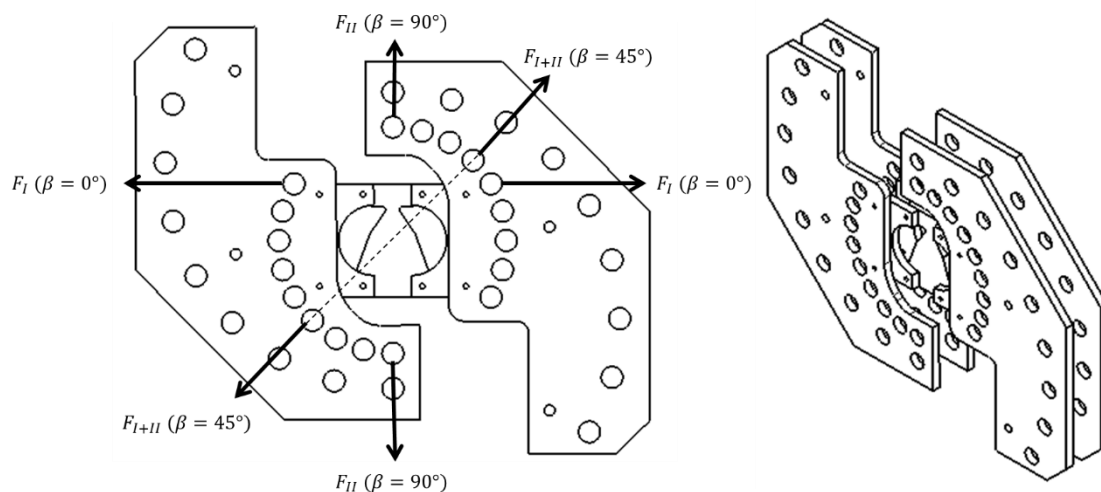


Figure 3. loading device adapted for MMCG and CTS specimen [4]

The finite element computation is realized for an elastic isotropic behavior and has the following elastic characteristics  $E=210.000$  MPa, Poisson ratio  $\nu=0.3$ . The initial crack length is fixed to 79 mm. Finite

element computation is realized with the software Cast3m, produced by French Energy Atomic Commission CEA.

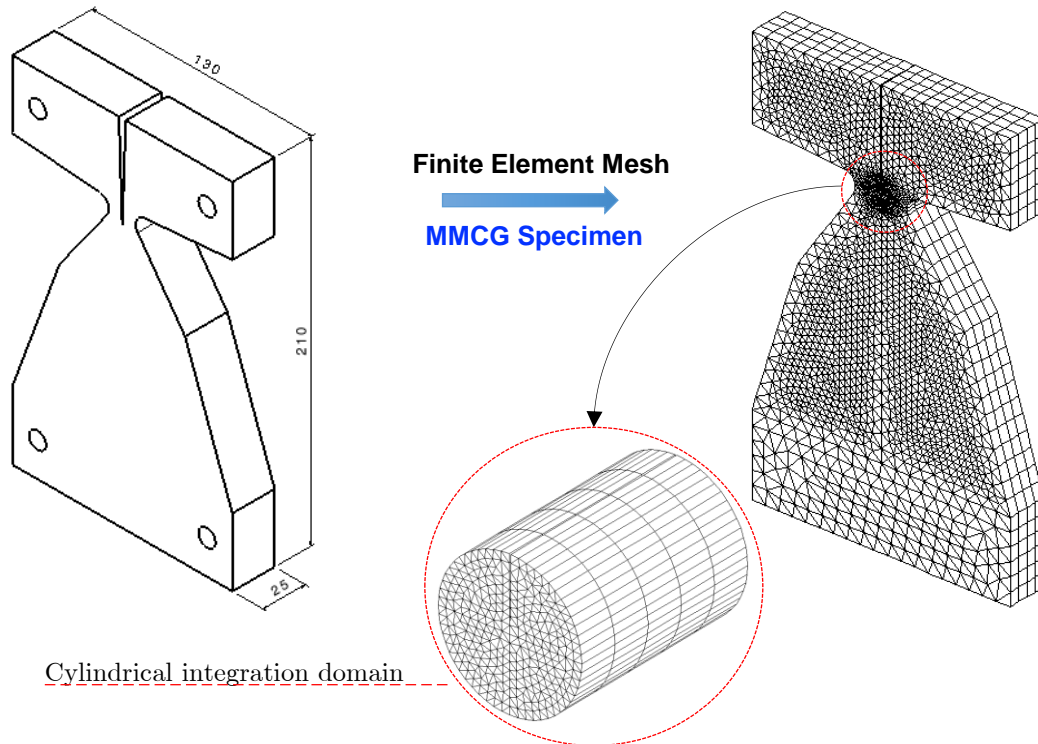


Figure 4. Finite element mesh of MMCG specimen and Integral domain around the crack front line

Integral domain around the crack front line and Finite element mesh of MMCG specimen are shown in Fig. 4. The description of  $\theta$  field around the crack front line is shown in Fig. 5. The  $\theta$  field is equal to zero on outside surface, and 1 on inside surface.

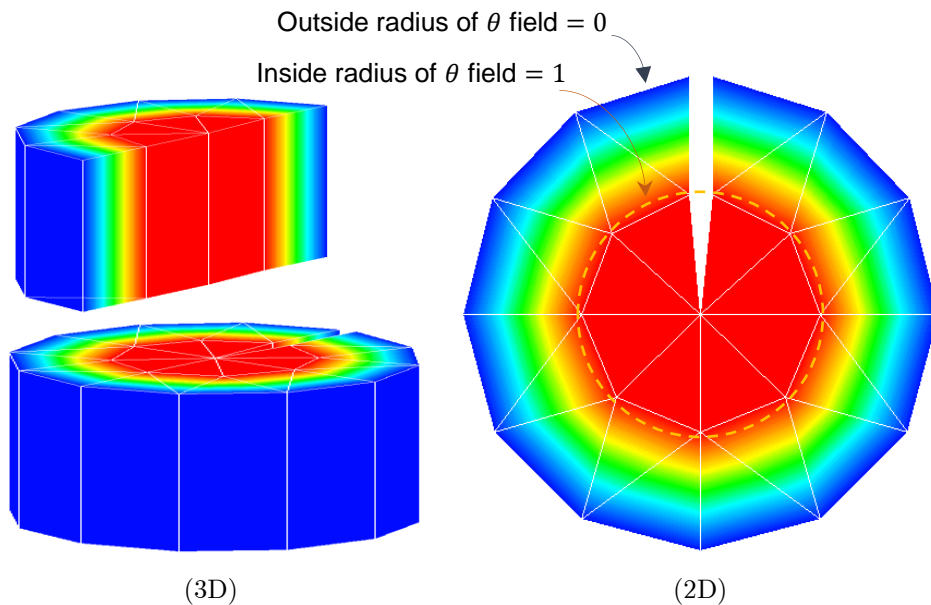


Figure 5.  $\theta$  field around the crack front line

In the follows, the first results of numerical study are exposed in order to observe the non-dependence domain for each fracture mode.

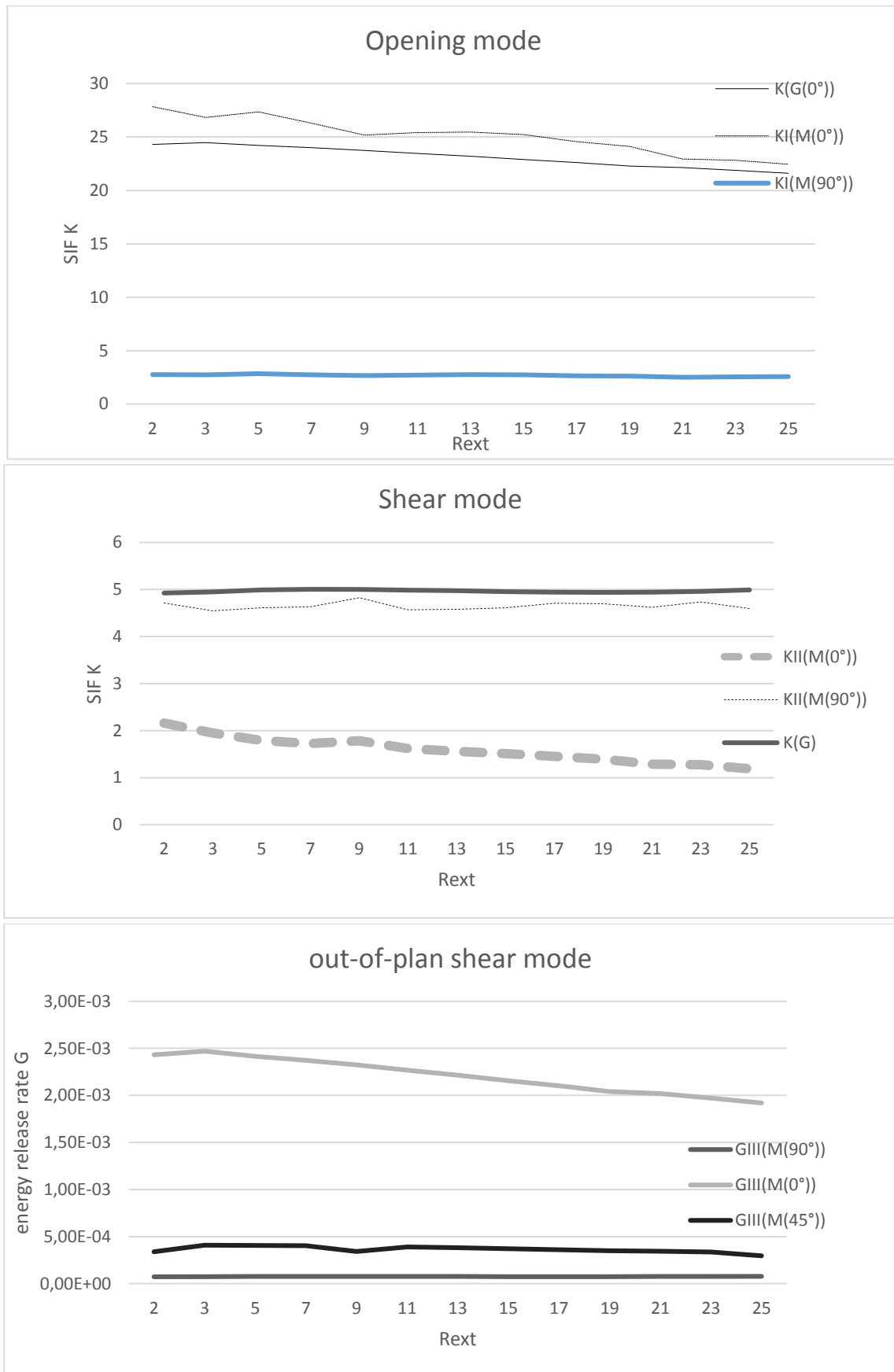


Figure 1. Average energy release rate vs. outside radius of  $\theta$  field  $R_c$

Numerical results validate the non-dependence of the integration domain.

## 4. Conclusion

This paper deals with a new formulation of the M-integral for the study of fracture process in element by taking into account three dimensional effects. A theoretical and numerical approach are established. In this case, the M integral is generalized to three-dimensional approach using the Lagrangian variation.

At this stapes, more numerical investigations are necessary. Also, it will be necessary to extend the M integral in order to introduce hydrological and thermal effects for three dimensional problems [6].

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