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### ► To cite this version:

Luc Trouche, Verônica Gitirana, Takeshi Miyakawa, Birgit Pepin, Chongyang Wang. Studying mathematics teachers interactions with curriculum materials through different lenses: Towards a deeper understanding of the processes at stake. *International Journal of Educational Research*, 2019, 93, pp.53-67. 10.1016/j.ijer.2018.09.002 . hal-01907304

**HAL Id: hal-01907304**

**<https://hal.science/hal-01907304>**

Submitted on 29 Oct 2018

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Contents lists available at ScienceDirect

## International Journal of Educational Research

journal homepage: [www.elsevier.com/locate/ijedures](http://www.elsevier.com/locate/ijedures)

# Studying mathematics teachers interactions with curriculum materials through different lenses: Towards a deeper understanding of the processes at stake

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## ARTICLE INFO

## Keywords:

Anthropological theory of the didactic  
Cultural-historical activity theory  
Curriculum ergonomics  
Curriculum material  
Documentational approach to didactics

## ABSTRACT

In this study we explore the interactions between teachers and resources during lesson preparation from different perspectives. We ask in which ways the crossing of perspectives help to develop deeper understandings of teacher interactions with curriculum resources. For doing this, we have chosen the following three theoretical frames: (1) Documentational Approach to Didactics (DAD); (2) Anthropological Theory of the Didactic (ATD); and (3) Cultural-Historical Activity Theory (CHAT). We chose data concerning two experienced mathematics teachers (at lower secondary school level) working together, preparing lessons on the topic of “algorithmics”, a topic newly introduced in grade 6 of the French curriculum, and hence a topic they had never taught before.

## 1. Introduction

The theme of this special issue is concerned with curriculum ergonomics, which the issue editors define as “exploring the interaction between the design and use of curriculum materials”. This would include, amongst others, teacher design of curriculum materials (Pepin et al., 2017a) and how they engage with curriculum materials *re-sourcing* (Adler 2000) their activity. At the same time teacher interaction with curriculum materials, in particular with digital materials (e.g. Pepin et al., 2017b), is at the centre of interest of mathematics teacher education as a scientific field. The processes involved (including design and enactment) are complex, taking place in different environments (e.g. school, home), and with many aspects interacting and influencing them (e.g. national curriculum; textbooks; school facilities for collaborative work). Of course, the theme of teacher/user interaction with curriculum materials, including their design and enactment, is not a new field (e.g. Fan et al., 2018; Pepin et al., 2013; Remillard 2005). However, exploring it from different perspectives, or through different theoretical lenses, is original.

Connecting theories in mathematics education has been the theme of previous research (e.g. Radford 2008; Prediger et al., 2008), and so has been studying phenomena (e.g. mathematics instruction; metacognition in problem solving) through different theoretical lenses (e.g. Charalambous & Praetorius 2018; Drijvers et al., 2013; Rodriguez et al., 2008). The different theoretical frameworks

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<https://doi.org/10.1016/j.ijer.2018.09.002>

Received 31 March 2018; Received in revised form 25 July 2018; Accepted 11 September 2018

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utilized at present in mathematics education offer different ways of approaching teacher interaction with curriculum resources, in order to develop a more nuanced view of the processes at stake. No single framework is able to provide a “full” understanding of the complex phenomena and processes, and comparing their respective insights, and delineating their strengths and weaknesses is far from trivial.

In this study we investigate the interactions between teachers and curriculum resources during lesson preparation from different theoretical perspectives, and subsequently compare the insights gained from each. Our question of research is the following:

In which ways does the analysis of the same situation from three theoretically different perspectives, and the comparison of insights gained, help us to develop deeper understandings of teacher interaction with curriculum resources?

For doing this, we have chosen the following three theoretical frames: (1) Documentational Approach to Didactics (DAD) (e.g. Gueudet, & Trouche 2009; Trouche, Gueudet, & Pepin, to be published); (2) Anthropological Theory of the Didactic (ATD) (e.g. Chevallard 1999; Bosch, & Gascón 2006) and (3) Cultural-Historical Activity Theory (CHAT) (e.g. Leonti'ev 1981; Engeström 2014). We have chosen these frameworks due to the familiarity with each of them for at least one of the authors, but also due to their (perceived) commensurability and complementarity, as we will evidence at the end of Section 2. We hypothesise that each frame will provide particular insights, and that when comparing these, will provide a “fuller” understanding of the phenomenon “teacher interaction with curriculum resources”. This in turn is likely to have implications for teacher learning and developing expertise (Pepin et al., 2016).

After this introductory part (Section 1), we present the three theoretical frames in the second section. In the third section we describe the methodological choices, in the fourth section the context of the study, before presenting our findings in the fifth section. Section six is dedicated to the discussion of the findings and the conclusions.

## 2. Theoretical frames

In this section we present each theoretical frames, highlighting particular features that we consider helpful for the analysis of the chosen data.

### 2.1. Documentational approach to didactics (DAD)

Over the past 10 years, DAD has been developed, principally in the context of mathematics education research (e.g., Trouche, Gueudet, & Pepin, Online first). The aim is to investigate teachers' work and professional learning through the lens of their interactions with resources in/for teaching. This approach is based on a dialectic between what a teacher is working on – the *resource/s* (resource/s gathered for facing a given *situation*); and what a teacher is producing– a *document*, a mixed entity composed of (re)organised resources and a *scheme* for their usage. The notions of *situation* and *scheme* (Vergnaud 2009) are critical: a situation is an issue faced by an agent in his/her regular activity (e.g. for a teacher: preparing a lesson); a *scheme* is the invariant organization of the activity developed by this agent for facing this kind of situation. A scheme has four components: goal/s in facing this situation, rules of actions, inferences, and *operational invariants*, namely the beliefs and knowledge in action.

Working with this dialectic, two concepts are critical in DAD: the concept of *documentational genesis*, i.e. the process leading from a set of resources to a document; and the concept of *resource system*, i.e. the set of resources accumulated and organised (over time) by a teacher in line with his/her regular teaching activity. At the same time, DAD acknowledges the collective aspect of teachers' documentation work (Gueudet & Trouche, 2012b; Pepin et al. 2013): teachers' interactions with resources are essentially social, as they take place, for example, in schools, or via Internet, and often in collectives.

### 2.2. Anthropological theory of the didactic (ATD)

ATD has been developed principally from an epistemological perspective of mathematics education research (Gascón 2003). The objective is to better understand didactic phenomena, with a special focus on investigating the nature of mathematical knowledge that is disseminated or taught in society (e.g., Chevallard 1999; Bosch, & Gascón 2006). Whilst ATD has not been developed specifically for the analysis of teachers' work, we note that the theoretical tools (developed over the past 35 years) within this framework allow us to specify three crucial aspects of teachers' collective work with resources:

- (1) The structure of knowledge and practices designed and effected by the teacher (and also by the students in the classroom): in ATD it is assumed that any human activity can be modelled in terms of a *praxeology* (Chevallard 1999). This is also the case for students' work or teachers' work, which is to be implemented, or has been implemented, in the classroom. A praxeology consists of two blocks: (a) the *practical block* where the practice is explained, and (b) the *theoretical block* where the knowledge underpinning the practice is explained.
- (2) The dynamic process of teachers' work of designing and implementing a task and/or a teaching sequence in the classroom: this process can be seen as an *inquiry*, and described by the notions of (a) *the dialectic of questions and answers*, and (b) *the dialectic of media and milieus* (e.g. as introduced in the context of inquiry-based teaching in ATD – Chevallard 2004). Media means here any means presenting information about the world, and milieu is a system of objects acting as a fragment of “nature”, able to produce feedbacks to the inquirer (the teacher in this case).
- (3) The *ecological* aspect exposing the *conditions* that make teachers' collective work viable and the *constraints* that hinder it: this third aspect – one of the main contributions of ATD – allows us to better understand the conditions and constraints beyond the

classroom or the school, through *the levels of didactic codetermination*: Civilization – society – school – pedagogy – discipline – domain – sector – theme – subject (Chevallard et al., 2002; Bosch, & Gascón 2006).

The analysis of the situation with respect to these three aspects is expected to provide insights into how resources and/or teachers' collective work with resources may affect the development of praxeologies (to be) implemented in the classroom.

### 2.3. Cultural-historical activity theory (CHAT)

Cultural-Historical Activity Theory (CHAT) is a theoretical framework aiming to understand human activity, taking into account its relation to the human mind (e.g. reason and feelings). It has one of its origins in Vygotsky's idea of the dynamic of consciousness that is essentially subjective and shaped historically by social and cultural experiences (Vygotsky 1978). In this study we used third generation CHAT (as outlined by Engeström 2014), with its five principles:

- (1) The collective, artefact-mediated and object-oriented activity system is taken as the prime unit of analysis;
- (2) Activity systems are multi-voiced: the division of labour creates different positions for the participants, and the activity system itself carries multiple layers and strands of history engraved in its artefacts, rules, and conventions;
- (3) Activity systems take shape and get transformed over lengthy periods of time;
- (4) Contradictions have a central role as sources of change and development (Ilöenkov 1977);
- (5) The activity systems have the potential for expansive transformation.

Within CHAT, documents are interpreted as emerging from rules, artefacts, and division of labour, as well as from community feedback. Combining the idea of activity network/s (Ritva Engeström 1995) and the curriculum ergonomics perspective (Choppin 2011), we regard teachers' work as situated within a network of activity systems: the activity system of the teacher in her/his teaching activity; and the activity system of the teacher as a curriculum designer. Whilst DAD and ATD are theoretical frameworks nested within mathematics education research, CHAT originates from an organizational/workplace learning tradition. However, we claim that CHAT is useful for this study, because our situation (analyzed from different perspectives) is a situation where (two) teachers work/learn at their workplace and in a particular organizational structure. Hence, we hoped for a potentially different point of view that might add to developing insights.

These three theoretical frames, DAD, ATD, and CHAT, were chosen due to their *commensurability*: focusing each on actors' work-praxis-activity, taking into account their mediated activity (with resources, media, tools), and the importance of social aspects (collective-institution-community). They were also chosen because of their *complementarity*, that is each seems to highlight an important aspect of teacher interaction with curriculum materials: (1) the mutual adaptation of user and resource through documentary genesis in the DAD; (2) knowledge generation when working with curriculum materials in the ATD approach; and (3) activity processes and the interrelationship of rules, division of labour, community and mediating artefacts in the CHAT approach. In the next section we describe and explain our methodological choices for comparing the three approaches, within and across theoretical frames.

## 3. Methodological choices

We present in this section: (1) the data to be analysed; (2) the analytical-theoretical perspectives (according to the respective theoretical lens) guiding our analysis; and (3) the tools we decided to commonly adopt to facilitate the comparative analysis.

### 3.1. The data to be analysed with respect to three perspectives

We took advantage of a platform, AnA.doc (Alturkmani, Daubias, Loisy, Messaoui, & Trouche to be published), which had been developed for the analysis of teachers' interactions with curriculum resources. With this platform, researchers were encouraged to share their data related to different *situations* of teachers' working with resources (e.g. lesson preparation, lesson implementation, or reflection/revision). The shared data were organized following a given model: teacher self-presentation; presentation of the context of their work (including their educational backgrounds and professional experiences); videos of the situation; and the resources used/designed in this situation. Hence, the methodological choices presented in the following apply to the analyses of existing data.

In order to answer our research question, we chose, amongst the AnA.doc situations, a situation of two teachers (Anna and Cindy) preparing a new lesson (of a topic they had never taught before). In addition to the AnA.doc data, we also included the interviews with these two teachers conducted before the lesson preparation situation. The lesson was related to the topic of "algorithmics", a topic newly introduced in the French curriculum at grade 6 (see § 4 for more details), and Anna and Cindy had never taught this topic before. We hypothesised that the necessity of facing a new topic would stimulate teachers' interactions. Anna and Cindy were two experienced mathematics teachers regularly working together and preparing their lessons together (more details in § 4.2), constituting what Wang (2018) coined *documentation-working mates*. We hypothesised that their interactions when preparing their lesson would provide access to the rationales for their choices, which often remains implicit in the case of teachers' individual work.

### 3.2. Three methodological perspectives

We present here the methodological perspectives associated with DAD, ATD, and CHAT.

DAD has developed a specific methodology, based on *reflective investigation* (Gueudet & Trouche 2012a): following teacher's interactions with resources, beyond the direct observation by the researcher, involving teachers closely in the data collection, and accessing their "thinking" through reflective analysis and stimulated recall. The AnA.doc platform had been set up in this spirit, giving access not only to the video/s of the situation/s at stake and to the associated resources, but also to the teachers' self presentations, providing elements of reflective analysis. By combining these diverse sources, we inferred elements of structure of the teacher's resource system (static point of view), and of schemes developed over the documentational genesis (dynamic point of view). Finally, the situation was not considered as isolated (one hour common documentation work), but as a moment in a extensive documentation process, individual as well as collective, which began a long time ago.

ATD allowed us to investigate the crucial aspects of teachers' collective work with three key conceptual tools: (1) *praxeology*; (2) *dialectics* between (a) *questions and answers*, and (b) *media and milieus*; and (3) *the levels of didactical codetermination*. The analysis of empirical data of the two teachers, Anna and Cindy, required to specify different elements related to these theoretical concepts. In addition, we also adopted selected ideas from previous studies analyzing teachers' work with ATD (Chevallard 1999; Margolinas, & Wozniak 2010). In particular, in the analysis of the dynamic process of lesson preparation we identified, along with the questions and answers of the dialectic, how they dealt with answers.

Regarding CHAT, we analysed the teachers' collective *activity system*, guided by its aims and oriented towards an object, as the prime unit of analysis. We traced the evolutions of teachers' activity systems, starting from teachers self presentation, using the *triangles model*, in a network of activity systems (Ritva Engeström 1995), as a first profile for the analysis of the activity collectively undertaken by the two teachers. Focusing on the main session, we identified stages of the activity by considering the goals defined by the two teachers to reach the aim and produce the object. Their own *division of labour* within the collective work were historically created, and brought into the session. Thus, earlier interviews were our main key for identifying the *mediating artefacts, rules and conventions* which shaped the activity system. We also used the network of activities system to study two different activity systems that the teachers were involved in. This helped us to identify *contradictions and conflicts* when teachers were the designer of their own materials – our focus was to understand their decisions and the evolution of the documents they were producing. Finally, while acting within the activity, we tried to understand the new knowledge they brought to the solution.

### 3.3. Common tools for analysis

Beside analysing the same data from these different methodological perspectives, we also decided to use selected common tools:

- Designing a *script* of the session, determining successive significant/important *stages* of the two teachers' documentation work;
- Distinguishing a number of significant/important *issues* (questions) the teachers were facing, the *answers* they proposed, and the *moments* where these answers occurred;
- Analyzing how these stages, answers and moments could be explained from each theoretical perspective, allowing to arrive at answers for our (common) research question.

For this we set up and provided a timeline model (e.g. Fig. 1), which was to be filled in according to each of our three theoretical perspectives.

In the following section we present the context of Anna and Cindy's work.

## 4. Setting the scene

We present here our case study: the recent French curriculum changes; Anna's and Cindy's profiles and their resource environment; and the context of the session that was the object of our analyses.

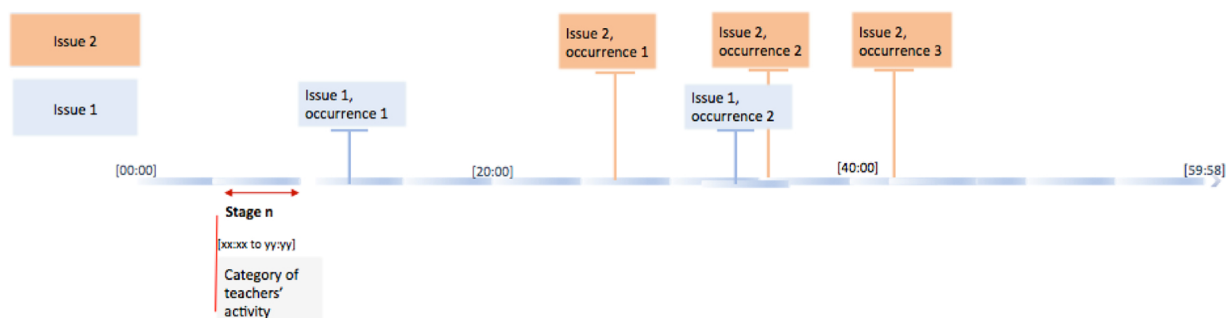


Fig. 1. Model for presenting analyses of teachers' documentation work.

#### 4.1. French curriculum in a time of transition

Since September 2016, major curricular changes occurred in France (Gueudet et al., 2017) in primary and middle schools (from grade 1–9): concerning the structure of teaching, the content, and the resources for supporting teachers.

Regarding the structure, the changes concerned all disciplines: the organization of the curriculum on the basis of three 3-year cycles (i.e. 3 cycles over 9 years); the formulation of teaching objectives in terms of competencies and knowledge to be attained at the end of each cycle; the development of specific settings, i.e. IPT (Interdisciplinary Practical Teaching), leading teachers (from different disciplines) to guide students' projects together (e.g. in mathematics and history). Both the 3-year cycles and the IPT organization were initiated to develop teachers' collective work in schools.

As for the content, a major change was the introduction of 'algorithmics and programming', to be taught by both technology teachers (mainly for the practical part) and mathematics teachers (mainly for the formal part). The National curriculum stipulates what was to be taught:

"notions of variable, instructions, parallel computing or event-driven programming. It does not demand that specific algorithms or programs are taught, but gives examples of situations and activities that can be developed with the students to contribute to learning this general content. Like every discipline of the middle school, computer science has to contribute to the interdisciplinary teaching and examples are proposed in this direction" (Gueudet et al. 2017, p. 60)

As most teachers in middle schools had never learnt how to teach this topic before, the Ministry initiated organized specific training sessions (generally one or two days, monitored by inspectors) for supporting teachers. It provided also teachers, via its website, with institutional resources (MEN 2016), including the official *program* (curriculum– what is to be taught) and the official *accompanying resources* (<http://eduscol.education.fr/>); and other recommended resources, like Scratch (<https://scratch.mit.edu/>) for discovering event-driven programming, and unplugged computing activities for developing algorithmics thinking beyond the use of the tools.

In addition to these institutional suggestions, teachers were also provided with textbooks (produced and developed by publishing companies – about 10 different textbook series offered to schools); resources provided by the network of IREM (Institutes for Research on Mathematics Teaching), bringing together scholars and mathematics teachers for collaborative work (Trouche, 2016); resources developed by teams bringing together teachers and researchers in mathematics education, e.g. Sésames (<http://pegame.ens-lyon.fr/>), with reflections on the teaching of Algebra, seen as a "developing programs of computation for solving a given problem" (Alves et al., 2013); resources developed by mathematics teacher associations, particularly APMEP (the national French public schools mathematics teachers association) and Sésamath (<http://www.sesamath.net>), an association designing and freely providing textbooks at a large scale, which seems to be an emblematic effect of digitalization, supporting both the emerging of online communities, and a large dissemination of their resources (Rocha 2018); and a wide set of open educational resources (Trouche, Gueudet, & Pepin, 2018).

#### 4.2. The teachers chosen: Anna and Cindy

Anna and Cindy were two French experienced mathematics middle school teachers. At the moment of the data collection in 2016, they worked in the same middle school (MS hereafter), in the centre of a large French city, and they regularly worked together. Since the beginning of their careers both teachers had experienced a culture of collective work, as they had worked in schools with difficult social contexts.

Anna had 25 years of teaching experience (last 10 years in MS). She had been involved in Sésames (§ 4.1) for 13 years and had developed, within this environment, specific resources following the same model: the "warming-up" model (*Mise en Train* in French [http://pegame.ens-lyon.fr/theme.php?rubrique=2&id\\_theme=25](http://pegame.ens-lyon.fr/theme.php?rubrique=2&id_theme=25)). "Mise en train" aims at capturing students' attention as soon as they enter the classroom. It has been used to introduce a new notion at the start of the lesson, under the form of a short inquiry question and providing students with room for initiative. After some years in Sésames, Anna also joined IREM and APMEP (§ 4.1). She was now the coordinator of the MS Mathematics teacher team, and she managed the MS "scientific workshop", which was aimed at teaching voluntary pupils 'living scientific activities'.

Cindy had 23 years' teaching experience. Ten years ago she met Anna when she arrived in the MS college. Since then, they have developed a stable cooperation. Cindy joined the Sésames and IREM teams eight years ago. Due to her expertise, she has been recruited as a pre-service teacher educator (part time) by the regional High School for Teacher Education (HSTE – in French: ESPE).

Anna and Cindy chose each other as a documentation-working mate (Wang 2018):

"Anna is always my first choice when I need some resources" (Cindy's self presentation)

They used several digital *interfaces* for sharing resources amongst themselves, and within the different collectives they were working in (e.g. Dropbox, Padlets, or Pearltrees). Due to their long-term common documentation work, they also shared strong beliefs: a critical attitude towards textbooks (e.g. even if their school had chosen the Sésamath book, they felt that its exercises were didactically poor); a wish to collaboratively develop resources differentiated by students' needs, to make them activity-based and based on a deep understanding of the mathematical notion/s at stake. Their beliefs appeared as structuring features of their work, as in-service teacher educators in IREM, as members of regular teacher teams through APMEP (for Anna); and as pre-service teacher educator (e.g. Cindy).



Fig. 2. Anna and Cindy in front of their computers and of a set of resources.

#### 4.3. The context

In May 2016, Anna and Cindy decided to plan together their “algorithmics” teaching for the coming semester, and accepted the presence of a researcher, who would video-record this situation. They had attended short training sessions on this new topic, which were organized by the mathematics inspectors, and mainly dedicated to the use of Scratch.

They were asked to bring with them all the available curriculum materials they could mobilize for preparing this teaching – they finally brought a diversity of textbooks (13 different ones), IREM booklets, institutional requirements, their personal laptop containing their digital resources including the links that they normally used for preparing their lessons. This preparation situation happened in the MS middle school, in a mathematics classroom, where they typically did their regular collective documentation work (Fig. 2).

### 5. One situation, three approaches: results of the analyses

In this section we report on the analyses of our data using the three theoretical lenses.

#### 5.1. The DAD analysis

In this part, we analyse (1) the successive stages of documentation work, critical issues and rules of action; (2) the digital interfaces supporting interactions between teachers’ resource systems; and (3) the fundamental role of teachers’ experience and related operational invariants (§ 2.1).

##### 5.1.1. Stages of documentation work, critical issues and rules of action

We could distinguish seven stages (Fig. 3) over this collective preparation work, with respect to the resources used by Anna and Cindy, evidencing a conceptual deepening of their preparation work:

- Stage 1: structuring their work through a careful analysis of the (national) curriculum;
- Stage 2: visiting the available resources (e.g. mainly textbooks, but also online resources on their laptops and mobile phones);
- Stage 3: selecting the resources expected to stimulate student activity – this reflected their didactical thinking;
- Stage 4: trying to integrate *algorithmics* teaching in their teaching progression – this reflects their mathematical thinking;
- Stage 5: reflecting on the requirements and ideas of the curriculum – this reflects their institutional thinking;
- Stage 6: confronting mathematics and *algorithmics* concepts by making some compromises (e.g. the use of Scratch). This stage could be considered as an epistemological deepening of stage 4;
- Stage 7: deciding practically on an *algorithmics* teaching plan with respect to the curriculum. This stage appeared to be the achievement of the previous stages, particularly stages 1 and 4.

For deepening our analysis of their interactions, we chose two main issues (Fig. 4):

- The occurrence of Scratch (evoked 37 times during this session);
- The use of the Sésames resources (evoked only three times).

Their attitudes towards Scratch changed during their planning work. At the beginning, Anna opposed the institutional suggestion of using Scratch to teach *algorithmics*:

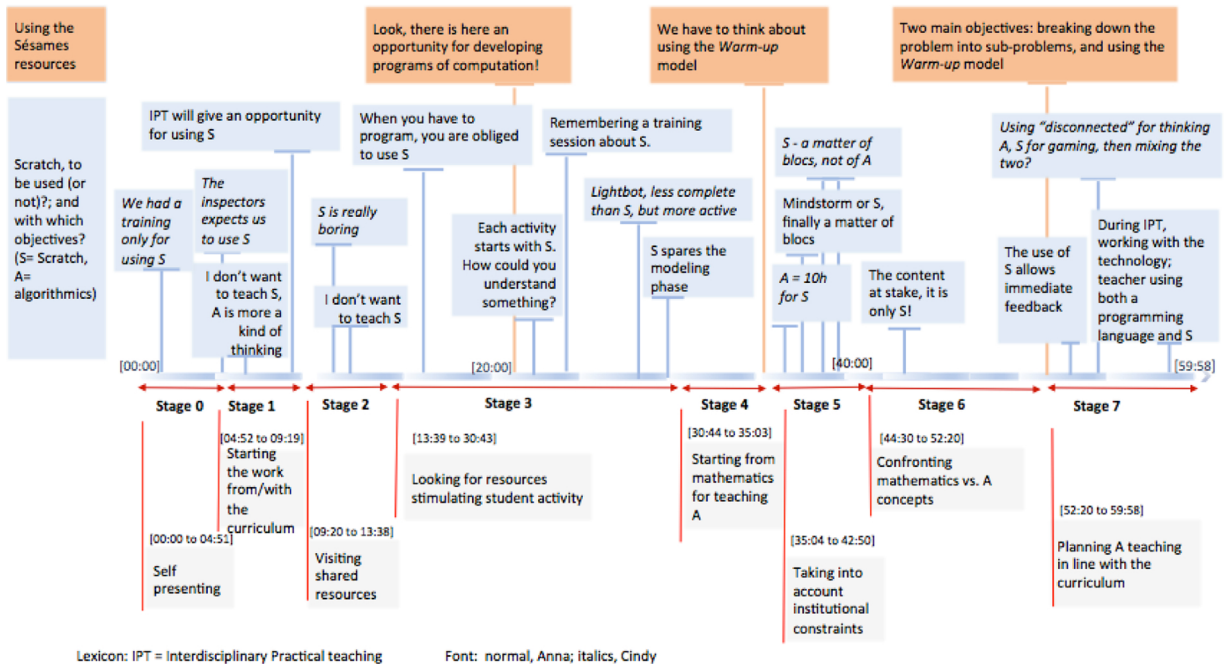


Fig. 3. Stages, issues and moments, through the lens of DAD.

Theme: Algorithmics and programming – official curriculum	
Expected at the end of the cycle: writing, adapting and executing a program	
Knowledge and associated competencies	Examples of situations, activities, and resources for students
Decomposing a problem into sub-problems for structuring a program; recognizing schemas [...]	Games in a labyrinth, game of Nim
Notions of variable in programming [...]	Designing a figure with a programming software [...]
Progressivity benchmark: from grade 5, students discover event-driven programming. Step by step, they develop new competencies, using the notion of variable [...]	
Decomposing a problem into sub-problems	
Recognizing schemas	
Warming-up: what about programs?	
Difficulties:...	
Vocabulary: variables, functions	
Progressivity: needed due to the complexity of the curriculum	
All the language arrives in 7 <sup>th</sup> grade	
<b>Activities</b>	
- Rather beginning by unplugged computing activities	
- Pancakes, Binary, Choreography, Sorting, Pixels, Robots	
Asking time for splitting the class in two groups for using Scratch	

Fig. 4. The algorithmics teaching plan designed by the two teachers during their preparation session.

“Algorithmics, for me, is more a type of thinking than knowing how to use a software” [Anna 7:20].

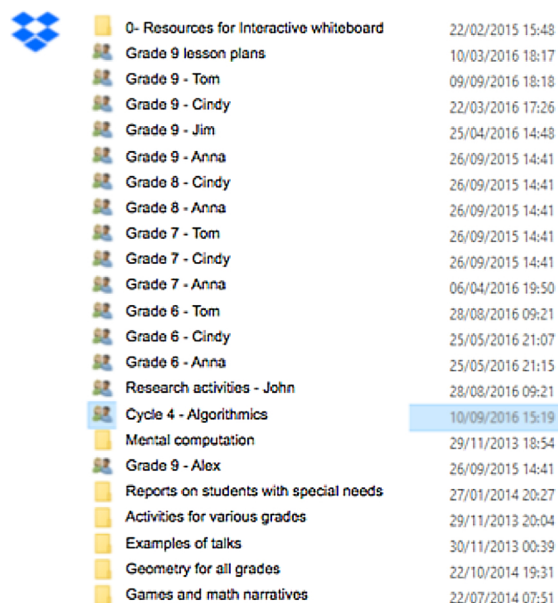
Cindy reminded Anna of the requirements of the curriculum and the suggestions of the inspectors. When starting to check the contents in the textbooks, they changed their attitudes towards Scratch. When Cindy complained that the suggestions of teaching Scratch in the textbooks were boring, Anna changed her account:

“It doesn't matter to use Scratch, but what embarrasses me is spending a whole lesson on it [Anna 12:15] ... [However] it allows an immediate feedback [for the students]” [Anna 51:22].

They finally decided to use it: “we can spend half of a mathematics lesson on teaching Scratch, and the other half in IPT” [35:07]. This process showed their efforts of finding a balance between their beliefs and ideas, and the curricular requirements.

A tacit agreement between Anna and Cindy resulted in their long-term relationship of documentation working mate: they shared not only resources, but also models for designing resources. As they were both members of the Sésames team, they regarded teaching





0- Resources for Interactive whiteboard	22/02/2015 15:48
Grade 9 lesson plans	10/03/2016 18:17
Grade 9 - Tom	09/09/2016 18:18
Grade 9 - Cindy	22/03/2016 17:26
Grade 9 - Jim	25/04/2016 14:48
Grade 9 - Anna	26/09/2015 14:41
Grade 8 - Cindy	26/09/2015 14:41
Grade 8 - Anna	26/09/2015 14:41
Grade 7 - Tom	26/09/2015 14:41
Grade 7 - Cindy	26/09/2015 14:41
Grade 7 - Anna	06/04/2016 19:50
Grade 6 - Tom	28/08/2016 09:21
Grade 6 - Cindy	25/05/2016 21:07
Grade 6 - Anna	25/05/2016 21:15
Research activities - John	28/08/2016 09:21
Cycle 4 - Algorithmics	10/09/2016 15:19
Mental computation	29/11/2013 18:54
Grade 9 - Alex	26/09/2015 14:41
Reports on students with special needs	27/01/2014 20:27
Activities for various grades	29/11/2013 20:04
Examples of talks	30/11/2013 00:39
Geometry for all grades	22/10/2014 19:31
Games and math narratives	22/07/2014 07:51

Fig. 5. Part of the MS mathematics teachers shared Dropbox (captured in 2016, time of this working session).

algebra as ‘teaching programs of computation for problem solving’, and they developed specific resources following the *warming-up* model. Finally, teaching *algorithmics* appeared to be a follow-up of teaching algebra, and the warming-up model a natural *permeating resource* for insuring this follow-up.

### 5.1.2. The interfaces supporting interactions between teachers’ resource systems

During their documentation work Anna and Cindy developed (Fig. 4) a step-by-step ‘*algorithmics* teaching plan’.

In this process, they mobilized their resource systems through different common digital *interfaces*:

(1) Dropbox folder shared by MS mathematics teachers (Fig. 5), and giving an image of their structured documentation work: Anna started to fill the *algorithmics* teaching plan with texts from the curriculum, and saved it in the Dropbox:

“I created a document, and pasted an extract of the curriculum at its top ... I will save it in a jumble in our shared Dropbox folder, till now, we have no folder dedicated to algorithmics... no, I have to create a new folder for this topic.” [Anna, 7:40]

“I think we need to re-organize the folders.” [Cindy, 8:17].

Some “secrets” of teachers’ collective documentation work in MS were revealed in the Dropbox interface (Fig. 5): Anna and Cindy were the only two teachers (in the school) who had taught from grade 6 to grade 9, which indicated that they were more familiar with the curriculum and with the students in the school. This made them the leading figures for planning the teaching. At the same time each teacher shared their own resources with everybody, even with those who did not teach the same grade. All teachers used the same format for storing files, and saved them under “grade + name”. Anna said that “when necessary [she] will check the folder of Cindy to see what can be borrowed.” (Anna’s interview).

(2) Collaborative platforms like Padlet (<https://padlet.com/>) and Viaeduc (<https://www.viaeduc.fr>) were used by Anna and Cindy; here they collected shared online resources. This meant that they had an extra resource sharing space outside MS, where resources were collected which appeared to be used in their roles as teacher educators/professional developers and researchers, for example.

(3) Resources/ideas/experiences retrieved from professional collectives (e.g. IREM of Grenoble: this inspired them to revisit the activity of “the dance” in the document of “Creative Computing”; IREM of Clermont: this reminded Anna the idea of “fraction addition”; IREM of Paris: this inspired them the activity of “robots”).

The *algorithmics* teaching plan (Fig. 4) provided an evolving structure with requirements of the curriculum, *warming-up* model, and a list of activities.

### 5.1.3. The fundamental role of teacher professional experience and related operational invariants

Both Anna and Cindy had many years of professional experience: in 2016 Anna had been a mathematics teacher for 25 years, and Cindy for 23 years, and they both had been working as teacher educators for approximately 10 years. Hence, they had accumulated many years of teaching practice, also as teacher educator. It appeared that they could transfer smoothly between the two roles (teacher and teacher educator), especially when they had to teach a new topic. We could infer some operational invariants grounding their *schemes* (§ 2.1), in particular when facing the situation of curriculum change. For example,

- When Cindy said [11:20] “I think it will be nice to ask the students to work and experience the meaning of the operations, you know, like the primary teachers that I used to educate”, she referred to her own experience as teacher educator;
- When discussing the suggestions in the program about “reviewing the variable”, Anna said: “the problem is that they (the students) can not review it because they have not started learning algebra, let alone the algebraic variable [9:45]”, and “there is really a need to let them manipulate the objects” [10:30] to foster students’ understanding – this could be related to her teaching experience in MS (§ 4.2);
- When both Anna and Cindy mentioned the need for working on the natural language, and for combining different languages, this can be traced back to their joint experiences in *Sésames* (when they collectively worked on the enhancement of their algebra teaching).

Finally, the dynamics of this situation came from the intertwining of the two teachers’ documentation works, favoured by the existence of digital interfaces. Each documentation work is supported by operational invariants, sometimes common, sometimes different for each teacher, each of them anchored in their own professional experiences as a mathematics teacher.

## 5.2. The ATD analysis

In this section, we present results of the data analysis on three aspects of teachers’ collective work.

### 5.2.1. Two kinds of praxeologies: algorithmic, and didactic

Using ATD as an analytical tool, it is possible to describe not only the structure of mathematical knowledge and practices, which are planned for the students – we call this *mathematical praxeology*, or *algorithmic praxeology* in our case –, but also the structure of didactic knowledge and practices to be implemented on the part of the teacher – we call this *didactic praxeology*.

In their collective work, Anna and Cindy explored and looked for the activities and exercises that would allow students to encounter the *algorithmic praxeology*. This praxeology itself was not predetermined beforehand but developed during the discussion. At the beginning of the session Anna and Cindy referred to the techniques mentioned in the official curriculum. Subsequently, they decided to focus on the two techniques in grade 7, “breaking down the problem into sub-problems” and “recognizing the diagram”. In the session Cindy identified the elements of the theoretical block of this praxeology (rules of algorithm) when exploring the *Sésamath* textbook. However, in their discussion Cindy and Anna considered that this was not the object of teaching, and only the activities with which students made use of such techniques were important: “it [algorithmics] should not be a rule, it should be something ... a challenge, or ... in Lightbot, that's it, it's a kind of small challenge.” [14:15]

At the same time the *didactic praxeology* referred to the ways of operationalizing the *algorithmic praxeology* in the classroom. It included several elements: for example, Anna and Cindy discussed in particular the overall structure of the teaching sequences, such as starting with unplugged computing activities, the sequence of one-hour activities, warming-up activities in the beginning of the class, etc. This structure was based on the ideas of *algorithmics* teaching (e.g. “*algorithmics*, for me, it's more a type of thinking than knowing how to use the software” [Anna 7:20]) and mathematics teaching in general, which could be interpreted as a component of the theoretical block of didactic praxeology.

### 5.2.2. Dynamic process of the development of praxeologies to be implemented in the classroom

This process could be described by the questions the teachers dealt with during the collective work, and the answers to these questions, from the theoretical perspective of ATD, especially the dialectic between questions and answers. We segmented the teachers' work into stages (Fig. 6) according to the two teachers’ principal questions, and also to the ways they accessed the *media* (e.g. resources such as textbooks, Internet, etc.), which related to the other important dialectic mentioned earlier, between *media* and *milieus*. The answers (to those principal questions) during the discussion appear above the timeline in Fig. 6. These answers were classified according to the kinds of praxeologies: algorithmic in blue; and didactic praxeologies in orange. It is evident from the diagram that two thirds of the session were predominantly used to explore activities for students that were in line with the target *algorithmic praxeology*; and one third was used for the discussion on the didactic organization with respect to the activities they found.

Looking in more detail, one can detect differences in teachers’ dialectics between *media* and *milieus*, especially how Anna and Cindy, each of them, interacted with the *media* (e.g. textbooks). Throughout the whole session Anna explored the textbooks and other *media* (e.g. *Creative Computing*) much less than Cindy. Moreover, the principal questions they asked during the session were not identical. From the beginning Anna's main concern was to find ‘unplugged’ activities. She quickly browsed possible answers in textbooks and other resources, but did not analyse them deeply: “Well, very few unplugged in this [textbook]” [16:52]. In contrast, Cindy posed several questions, not limited to the ‘unplugged’ activities, and tackled them. Her questions were related to the activities (e.g. programming with Python and Scratch), the exercises, the structure of the textbook, the ways of teaching with Scratch, etc. Moreover, she not only observed the pre-established answers obtained from the *media*, but also analysed and evaluated them, sometimes with Anna.

### 5.2.3. Ecological perspective of teachers’ collective work and the developed praxeologies

This analysis could be applied not only to the *algorithmic* and didactic praxeologies, which were expected to be implemented in the

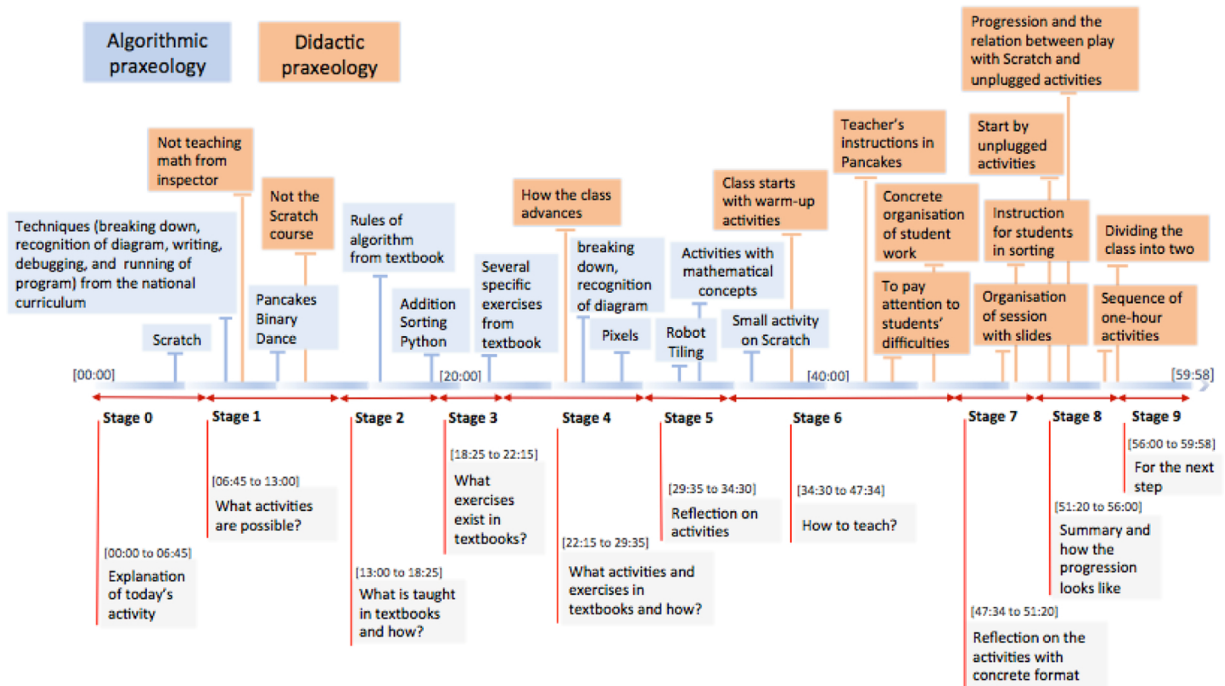


Fig. 6. Stages, issues and moments, through the lens of ATD.

classroom, but also to *paradidactic praxeology* (Miyakawa & Winslow 2013, 2017 Online First), i.e. to the teachers' collective work outside of classroom, over different levels of codetermination. In our data, we identified selected elements that conditioned teachers' collective work, and other elements that conditioned and shaped the *algorithmic* teaching.

Regarding collective work: at the level of society, the curricular reform in France was the principal condition that led Anna and Cindy to prepare the lessons together. It required the teaching of the new topic “algorithmics”, which the two teachers had never taught before (neither experienced in their teacher education). At the school level different elements were mentioned. For example, Cindy referred to the need for making/providing a common teaching progression in the same school (“we get agreement among three [teachers] on the progression” [3:35]), and also the selection of common textbooks for their school (“we are obliged to choose textbooks” [2:00]). These elements supported and promoted their collective work. In addition, the resources such as textbooks and the computer environment and the interfaces it offered (Padlets and Viaeduc), were also institutional conditions that shaped teachers' collective work.

Regarding the conditions and constraints related to the praxeologies to be implemented in the classroom:

- (1) constraints came from the inspector's directions (e.g. "it is not doing math" [10:46]) which affected their teaching of *algorithmics*.
- (2) At society level the national examination requirements (e.g. Brevet – national diploma at the end of lower secondary school) affected their teaching, in the sense that students were expected to use the *algorithmic* thinking in the paper and pencil environment, in addition to the computer environment.
- (3) At the school level we could identify the following constraint: (a) the number of classes (10 h of *algorithmics* teaching), (b) the school equipment (e.g. availability of computer room and materials, such as Mindstorms robot), (c) the use of the multi-disciplinary class (IPT).
- (4) At the pedagogy level the idea of implementing ‘playing activity’ when using the computer, was a constraint that hindered them to fully carry out the *algorithmic* activity in the mathematics class.
- (5) There were also several constraints related to the discipline. Amongst these, one of the biggest constraints was the fact that *algorithmics* was to be taught under the label of mathematics (and not technology). This constraint strongly affected teachers' collective designing of lessons: for example, at Stage 5 Anna and Cindy discussed how the activities they were analysing related to the mathematical concepts.

### 5.3. The CHAT analysis

In this section, our findings using the CHAT analysis are discussed under three sub-sections: (1) teachers' professional profile; (2) the evolution of the aim and objects of their activity; and (3) the conflicts (e.g. between using natural language and/or programming language).

### 5.3.1. Teachers' professional profiles

In this part we set some historically constructed rules and division of labour, by analyzing the two teachers' professional profiles. From the interviews, we identified a network of three activity systems that both teachers were involved in: (1) teaching; (2) designing lessons; and (3) research/collaborative work on curriculum materials.

- (1) In terms of teaching, both teachers started their professional life working in communities with social difficulties. They both complained about their difficulties of dealing with the new context despite the absence of training, but according to Anna the collective work helped her.
- (2) Cindy's teaching activity system was based on a familiar collection of textbooks. When she met Anna, her activity system changed quite drastically to use many more resources from research groups and associations. According to her, this was the time when she really "started to take a distance from the textbooks and to use mainly this kind of resources" [01:08]. Nonetheless, she kept using the available textbook/s in class, mainly for exercises. In terms of CHAT we interpreted this as transforming the roles of textbooks as mediating artefact: from a central resource in class, to a repository of training exercises.
- (3) Regarding designing lessons and student activities, Anna reported to follow the rule: instead of designing isolated lessons, she would integrate them into progressions for a given grade. As a teacher team, they also developed a well-established division of labour. For the *algorithmics* teaching planning, for example, Anna was responsible to organize the materials on the Internet using specific interfaces.
- (4) We call the third activity system reported by Anna 'teacher-researcher activity', because here the teacher was involved in research and development project about her own practice, particularly regarding the design of curriculum materials. This activity system consisted of their work in research groups and teachers' associations, where they learned/applied rules regarding the design of resource. One of them said that a learning resource should not be isolated, but coherently embedded in a teaching progression: "We started to think that they did not work very well, these great activities that were isolated. And we started doing things that were connected, following each other, and we figured we had to do little things very regularly" [02:32].

Both teachers designed resources and also made them available to others via the Internet. This also worked as a way to get appreciation from the community ("Society", or "other teachers").

### 5.3.2. Changes in aims and products of their activity

The two teachers, given the motive of the activity, defined the goals of the activity/ies. Hence, for CHAT we cut the session into ten stages using the teachers' identified goals (see Fig. 7). We regarded the change-of-goal as key-moments for changing the activity.

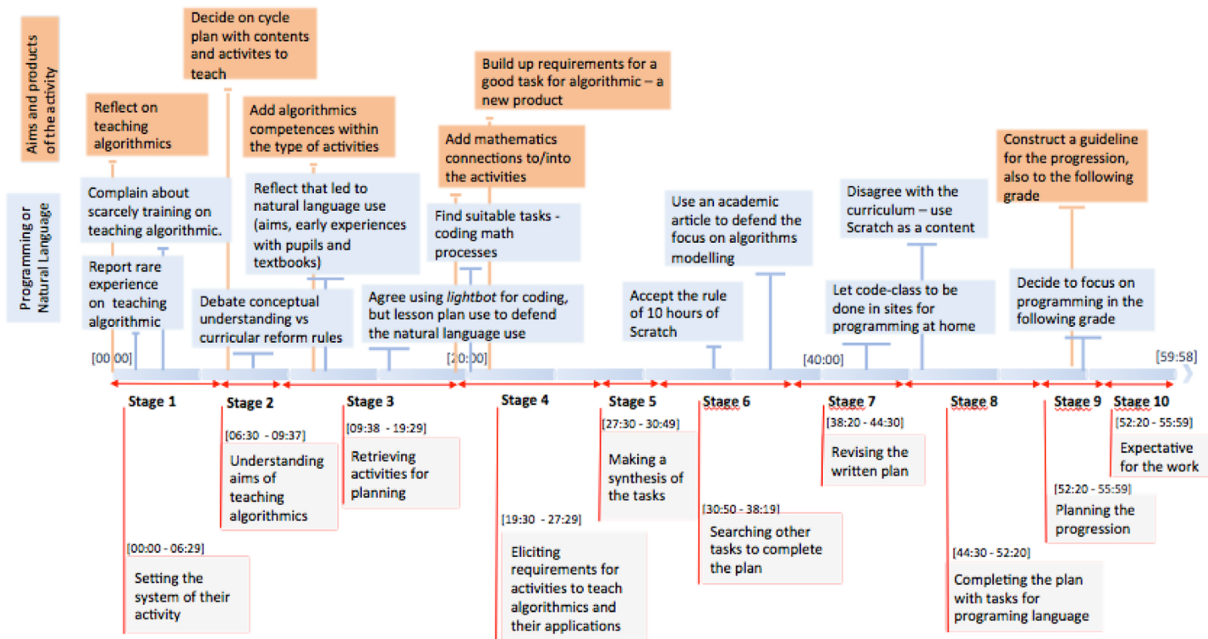


Fig. 7. Stages, issues and moments, through the lens of CHAT.

Anna and Cindy were asked “to plan together the teaching of *algorithmics*” in a one-hour session. This limited their activity goal in terms of time. At the same time they were unfamiliar with the textbook approaches and the new curriculum, and they had not had any professional education on the teaching and learning of the new topic area. This led them to change their activity. Anna stated the reduced product: a reflection on “the introduction of *algorithmics* for next year” [00:06].

Considering the rule “to not treat lessons and resources in isolation”, they changed their activity aim step by step to a draft plan for the year. As soon as the collective work started, Anna proposed to focus on the activities they would work on with pupils [06:30], and Cindy added the type of activities they would use. After reading part of the program, they brainstormed searching for tasks on the Internet and in the Sésamath textbook. They also included the connection between algorithmic knowledge and competence and tasks.

Cindy: [...] I also like the idea of Dance, it's the idea of language, in fact; the crêpes, it's really the idea of breaking down a problem into sub-problems...

Anna: Yes, absolutely. [11:10]

Both Anna and Cindy wanted tasks that connected *algorithmics* thinking to mathematical thinking. Eventually they found suitable tasks in Sésamath, which demanded to write a program in Scratch for the mathematics problems – this also conformed to the Inspectors' demands [19:54]. Reflecting on the writing of programs to control robot movements led them to a new suitable product, requirements for a good task, which are a central product for the Curriculum Ergonomic field [21:14].

According to their rules to work collectively and in sequence, they added “how to make the progression to the other grades” to their plan, as an outcome of the activity [54:10]. Supported by their professional experience, motivated by their conflicts with the curricular reform and unknown materials, bit by bit, they built up a, for them, useful product.

### 5.3.3. Conflicts: using natural or programming language

One of the conflicts appeared to be the use (or not) of a programming language (Fig. 8). Anna clearly voiced her disappointment with the reform regarding *algorithmics*. Cindy supported this, explaining that there was an absence/scarcity of professional training regarding *algorithmics* teaching in mathematics [04:05]. At the end of the session, Anna complained about the use of *Scratch* as obligatory for teaching *algorithmics*.

Anna: [...] well what annoys me ... is that the content we give, it's just the use of Scratch.

Cindy: Well this content here, it is clearly that ... [46:10]

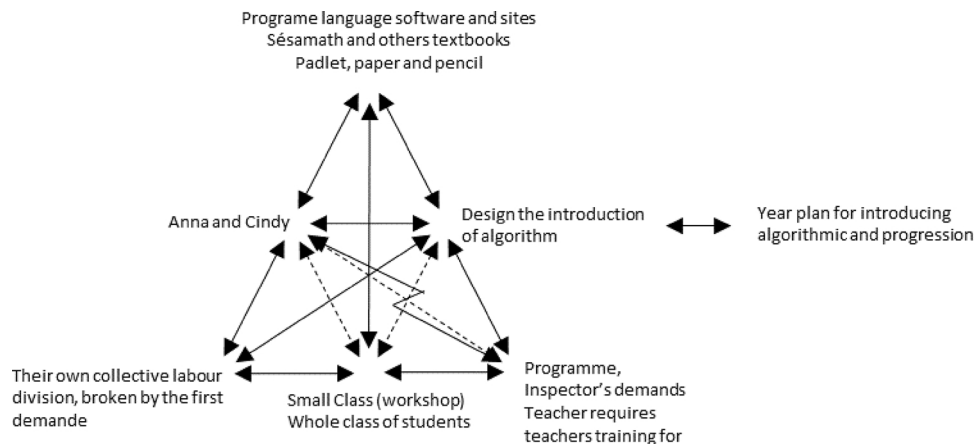


Fig. 8. Initial activity system of programming language use.

As soon as they started, Cindy used the inspector's voice, as a rule, to defend using *Scratch* as well as programming language/s. The scarcity of a proper professional training (concerning the teaching of *algorithmics*) was used as a counter-argument that (together with her viewpoint of conceptual understanding as central teaching aim) led Anna to defend working with natural language [10:24]. It was interesting how Anna's professional experiences in class led her back to natural language, even after having agreed to use an application that exploited coding and algorithm (*lightbot*) [15:24].

Despite having found suitable tasks [20:05], it was really the constraints of official requirements that led Anna to accept to add 10 h of using *Scratch* [34:20]. At a later stage, supported by an academic paper, Anna still argued that what was really important was that pupils reached *algorithmic* thinking, and the language used could be any [36:15].

Finally, the conflicts were overcome by letting pupils use two sites with tasks for programming language. Anna also suggested to indicate to pupils to access these at home and not to spend any class-time [43:15]. Further, they decided to focus programming language in the following grade [54:55].

### 5.3.4. Their interactions with curriculum material – the insights of the CHAT

Interactions between the teachers were undertaken with support of different aspects of their professional profiles, constructed within different systems of activity.

A curricular reform led to some rules to be broken, and so, many conflicts had to be faced. Supported by their general knowledge of curriculum material, such as textbooks and internet materials, they arrived at suitable findings that supported the argumentations about the conflicts created by using or not programming language. Their early constructed division of labour was very clear in the way they worked, and this led them to work flexibly, and even to create new objects of the activity as the requirements to choose a good task for algorithmics.

## 6. Discussion and conclusions

In this section, (1) we relate and compare the results coming from the three approaches to answer our research question; (2) we recognize the potential of each approach (for our situation) by evidencing how each added to our understandings; and (3) we suggest possible contributions of these three theoretical frames to the emerging field of curriculum ergonomics.

### 6.1. Relating and comparing the results from the three approaches

We have analysed, with respect to three theoretical perspectives, a specific documentation working session consisting of planning a whole lesson series on a new topic, *algorithmics* (which did not belong to the mathematics syllabus teachers were used to). The choice of a given perspective led to different cuttings of the session into successive stages, according to the chosen lens: for DAD, the “resource lens”, focusing on the teachers opening up and reorganising their resource systems (Fig. 3); for ATD, the “questions/answers lens”, focusing on teachers articulating two *praxeologies*, a *didactical one and an algorithmics one* (Fig. 6); and for CHAT, the “goals lens”, focusing on teachers’ roles and activity rules (Fig. 7). Within these different cuts, the lenses led to different issues of the two teachers’ collective work:

- From a DAD perspective, the collective work of the two teachers developed from both their own resource systems as well as the interfaces with/links to other resource systems (from other collectives). Interacting with resources worked along two dimensions: a vertical one (leading to a mathematical, didactical, epistemological, curricular reflection), and a horizontal one (switching from a resource from one collective to a resource from another collective). In this reflection, what we coin the term of *permeating resources* – i.e. models incorporating the experience of a collective – like the Warming up, play a critical role;
- From an ATD perspective, this teachers’ work formed an *inquiry*, asking questions, looking for answers in the *media* (textbooks, national curriculum, internet, etc.), obtaining pre-established answers, evaluating them, and developing their own answers. Teachers’ collective work made this inquiry fruitful and promoted a deeper investigation, because the colleague was able to play the role of *media* which provided different information (e.g. questions, answers, data) and at the same time the role of *milieu* which provided antagonistic (Brousseau 1997) feedbacks to the teacher’s actions, ideas, or suggestions.
- From a CHAT perspective, the interactions developed from three activity systems belonging to their professional profiles. Their collective work was in fact historically constructed and that was why it worked effectively to support the construction of the plan, as well as to adapt the demanded activity. Early experiences within different communities, rules and events of broken rules, led them to build up rules for their new activity system when dealing with the same context, the school.

These interactions fed a dialectics between *adhering to* and *rejecting* contextual affordances (inspectors’ demands, role of Scratch; role of playful activities), constrained by the need for the teachers to reach a consensus.

Taking into account teachers’ interactions at large was in line with an ecological perspective (in ATD terms), a historical perspective (in CHAT terms), a documentational genesis perspective (in DAD terms). When interacting, the two teachers “carried within them” their previous experiences: Anna, in charge of the coordination of the MS mathematics teachers, was more inclined to find solutions to institutional requirements (conflicting with their usual practices); Cindy, involved in pre-service teacher education programs, was more inclined to take into account the mathematical *knowledge* in the lesson to be taught. Anna and Cindy are experienced teachers, and we have evidenced the advantage, for them, of working in collectives. Our hypothesis, to be checked in further studies, is that this advantage remains for regular teachers.

### 6.2. The benefits of each approach for deepening our understandings

Prediger, Bikner-Ahsbahs, and Arzarello (2008) distinguished between different ways of working with different theoretical frames

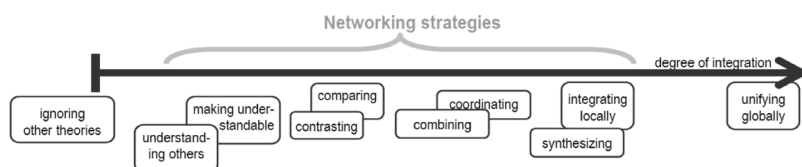


Fig. 9. Different theoretical networking strategies (Prediger, Bikner-Ahsbahs, & Arzarello, p. 170).

(Fig. 9). When writing this paper we went through various stages of: developing *understandings* of each other's viewpoint; *coordinating* the work (e.g. agreeing on common elements of methodology for analyzing the data separately); *bringing together* the different findings/elements, and *comparing* insights. Moreover, we developed another aspect of networking: we profited from this cross analysis in terms of deepening our understanding of each approach.

DAD supported an enrichment of the analysis of teachers' interactions by: 1) drawing attention to the whole *resource environments* of the two teachers, and to their crossing; 2) focusing on the genesis of the document: looking for schemes helped to understand not only the dynamic evolution of the activity, but also the dynamic evolution of the subjects' knowledge.

ATD focused on the structured aspect – the praxeology – of the knowledge (about both didactics and algorithmics) at stake in teachers' interactions. It also directed us to take a hierarchy of contextual constraints into account – the levels of codetermination (e.g. level of school, curriculum, society).

CHAT drew our attention to the structured aspect of teachers' interactions, with the notions of roles, rules, division of labour, and (re)definition of goals of the activity. It also directed us to consider the historicity of the activity (i.e. including a larger time scale).

It can be said that the theoretical perspectives would at times complement each other, and at others they would be incommensurable. Comparing the insights coming from each frame made us aware of missing, or particularly beneficial, aspects of each approach. For example, for DAD we realised that it would be necessary to consider the long-term history of teachers' work, beyond a given documentational genesis, a history that Rocha (2018) coined as *documentational trajectory*. As another example, we recognised the need for a better understanding of the (developing) knowledge at stake, and the structure of such knowledge, connected to the structure of teachers' resource systems.

### 6.3. Contributing to the domain of curriculum ergonomics

We claim that our study is likely to contribute to curriculum ergonomics: for the actors in the field, and for the field itself. For the actors (e.g. developers, researchers, teacher-designers), we retain the following results:

- In terms of *features* of the curricular materials to be developed: their *suitability for integration* into existing teachers' system of resources (DAD); their *compatibility with contextual constraints* coming from different levels of codetermination (ATD); their *potential to support teachers' goals* (CHAT).
- In terms of conditions for curriculum material development: the necessity to take into account the potential of teachers' work (DAD), and moreover the potential of teacher communities (CHAT), for contributing to the design process/es; and the necessity to develop curricular material as components of coherent praxeologies (ATD).

We also hope to have contributed to the field of curriculum ergonomics, from methodological and theoretical points of view. From a methodological point of view, it seems beneficial to analyze (in greater depth) and evidence teachers' documentation work in micro collectives, before going to scale. From a theoretical perspective, it seems important for the field of curriculum ergonomics to take selected critical dimensions (evidenced by the complementarity of our three approaches) into account: the didactical dimension, the collective dimension, the institutional dimension, and the cultural/historical dimension. Moreover, we would argue that in each theoretical perspective the knowledge dimension needs more attention and analytical depth.

### Funding

Japan Society for the Promotion of Science, Grant Numbers KAKENHI (JP15KK0113), Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Grant Numbers EST-SENIOR 88881.120750/2016-01, China Scholarship Council, Grant Numbers 201406140096.

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