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Comment on

“Growth of Low-Frequency Electrostatic and Electromagnetic Instabilities in a Hall Thruster”

by Singh and Malik

(IEEE Trans. Plasma Sci. 39:10 (2011) 1910–1918)

Yves Elskens and Nicolas Lemoine

Abstract—We revisit the electrostatic instability model of Singh and Malik (IEEE Trans. Plasma Sci. 39:10 (2011) 1910–1918) and correct qualitative and quantitative errors.

Index Terms—Hall thruster, low-frequency oscillations, resistive instabilities, ion and plasma propulsion, electrostatic waves and oscillations

I. INTRODUCTION

HALL THRUSTERS are promising devices for satellite station keeping and propulsion because of their high thrust density, and they are undergoing significant development. Understanding and controlling their instabilities is a major research topic.

In this context, low-frequency instabilities have been investigated in particular by Singh and Malik [1]. We refer to Wei *et al.* [2] and Smolyakov *et al.* [3] for recent reviews.

II. ASSUMPTIONS AND DISPERSION RELATION FOR ELECTROSTATIC MODES

It is well-known that “[the] full picture of instabilities in Hall plasmas is complex and, for typical experimental conditions, likely involves a number of interacting modes which require numerical simulations” [3]. However, analytical approaches with simplified models help to grasp processes intuitively. This motivates the interest for classical stability analyses near simple references states of the plasma. Unfortunately, Hall plasmas are often out-of-equilibrium, nonuniform, unsteady systems, challenging elementary descriptions.

Singh and Malik’s model describes the plasma as two fluids in a uniform magnetic field $\vec{B}_0 = B_0 \vec{e}_z$, with ions flowing along the x -axis and electrons along the y -axis, respectively with velocities $\vec{v}_{i0} = v_{i0} \vec{e}_x$ (denoted \vec{v}_0 in [1]) and $\vec{v}_{e0} = v_{e0} \vec{e}_y$ (denoted \vec{u}_0 in [1]). This geometry assimilates the cylindrical chamber to a slab, with z radial, x parallel to the chamber axis, and y azimuthal. Due to the axial electric field in the

chamber, electrons have a significant azimuthal drift, while ions move essentially axially, and, due to their large mass ratio, ions can be considered unmagnetized while electrons are sensitive to the magnetic force. Both species have finite temperatures, T_i and T_e , and the small pressure oscillations should be understood to be adiabatic¹ ($\delta(p_s n_s^{-Y_s}) = 0$ with $Y_s = 2$ for both species $s = e, i$ treated as two-dimensional monatomic ideal gases). The model also involves electron collisions with neutrals, with a frequency ν .

Dynamics is essentially two-dimensional, in the xy plane (normal to \vec{B}_0). The reference state is taken to be uniform, with density $n_{i0} = n_{e0} = n_0$. The effect of collisions involving ions is neglected. The effect of elastic electron-neutral collisions is taken into account. The linearized evolution equations read

$$\partial_t n_{i1} + v_{i0} \partial_x n_{i1} + n_{i0} \nabla \cdot \vec{v}_{i1} = 0 \quad (1)$$

$$\partial_t \vec{v}_{i1} + v_{i0} \partial_x \vec{v}_{i1} = \frac{e}{m_i} \vec{E}_1 - \frac{1}{m_i n_{i0}} \nabla p_i \quad (2)$$

$$\partial_t n_{e1} + v_{e0} \partial_y n_{e1} + n_{e0} \nabla \cdot \vec{v}_{e1} = 0 \quad (3)$$

$$\begin{aligned} \partial_t \vec{v}_{e1} + v_{e0} \partial_y \vec{v}_{e1} = & -\frac{e}{m_e} (\vec{E}_1 + \vec{v}_{e1} \wedge \vec{B}_0 + \vec{v}_{e0} \wedge \vec{B}_1) \\ & - \frac{1}{m_e n_{e0}} \nabla p_e - \nu \vec{v}_{e1} \end{aligned} \quad (4)$$

$$\nabla \cdot \vec{E}_1 = \frac{e}{\epsilon_0} (n_{i1} - n_{e1}) \quad (5)$$

Focusing on electrostatic modes, the contribution of \vec{B}_1 is neglected, while electric field oscillations derive from a potential, $\vec{E}_1 = -\nabla \phi_1$. This system is Fourier-transformed with $\partial_t = i\omega$ and $\nabla = -i\vec{k}$, and the Poisson equation reads

$$(k_x^2 + k_y^2) e \phi_1 = \frac{e^2}{\epsilon_0} (n_{i1} - n_{e1}) = \omega_{pi}^2 m_i \frac{n_{i1} - n_{e1}}{n_{i0}} \quad (6)$$

where we highlight the ion plasma frequency which will be used to normalize all frequencies and growth rates.

The dispersion relation of system (1)–(5) is then a fifth-degree polynomial in $\tilde{\omega} := \omega/\omega_{pi}$. Focusing on low-frequency oscillations leads the authors to a third-degree polynomial, under the assumption that the electron gyrofrequency $\Omega = |e|B_0/m_e$ be much larger than $\max(\omega, k_y v_{e0}, \nu)$. Computing the polynomial coefficients is tedious and unilluminating, but

¹Rather than obeying Ref. [1]’s unconventional $p_s = Y_s n_s T_s$.

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straightforward, leading to the lengthy Eq. (18) in Ref. [1]. Inspection of the coefficient of the ω^2 term in the polynomial shows that the sum of all three roots has a positive imaginary part (proportional to $i\nu$); actually, there are generally two roots with positive $\text{Im}\omega$, corresponding to damped modes. Therefore, the largest $\text{Im}(-\omega)$ characterizes either the instability (when it is positive) or the least damped stable mode (when it is negative).

With the authors' hypotheses, we agree on their algebraic Eq. (18). However, the numerical values reported in this paper from studying this equation turn out to be erroneous. With the given numerical data, we find much smaller growth rates and qualitatively different dependences on physical parameters.

III. THE LITVAK-FISCH APPROXIMATION

The inconsistency of the graphs (and their interpretation) is manifest when considering the Litvak-Fisch [4] estimate for the growth rate, which reads in dimensionless form

$$\gamma/\omega_{\text{pi}} \approx \nu(2k_y v_{e0})^{-1}(\Omega/\omega_{\text{pe}})(1 + \Omega^2/\omega_{\text{pe}}^2)^{-3/2}. \quad (7)$$

Ref. [1] recovers this expression analytically as the limit of Eq. (18)'s solution for $T_e = T_i = 0$ and $\omega_{\text{pe}} \gg \max(\omega_{\text{pi}}, \Omega)$. With Ref. [1]'s typical parameter values $n_0 = 10^{18} \text{m}^{-3}$, $B_0 = 15 \text{mT}$, $k_x = 20 \text{rad/m}$, $k_y = 400 \text{rad/m}$, $T_e = 10 \text{eV}$, $T_i = 1 \text{eV}$, $v_{e0} = 10^6 \text{m/s}$, $v_{i0} = 5 \cdot 10^4 \text{m/s}$, $\nu = 10^6 \text{s}^{-1}$ and xenon ions ($m_i = 131 \text{Da}$), one finds $\omega_{\text{pi}} \approx 1.15 \cdot 10^8 \text{rad/s}$, $\omega_{\text{pe}}/\omega_{\text{pi}} = \sqrt{m_i/m_e} \approx 490$ and $\Omega/\omega_{\text{pi}} \approx 23$, leading to the Litvak-Fisch estimate $\gamma/\omega_{\text{pi}} \approx 6 \cdot 10^{-5}$, three orders of magnitude smaller than Ref. [1]'s typical values.

The relevance of Litvak and Fisch's approximation $T_e = T_i = 0$ in estimating coefficients in Eq. (18) of Ref. [1] can be assessed by noting that no coefficient of the cubic polynomial involves the temperatures alone. Rather, the electron temperature appears only through the sums $\omega_{\text{pe}}^2 + k^2 Y_e T_e/m_e$ and $\Omega^2 + k^2 Y_e T_e/m_e$, so that $k\sqrt{2T_e/m_e} \approx 7.5 \cdot 10^8 \text{rad/s} \approx 6.5 \omega_{\text{pi}}$ brings a minor correction (see also Fig. 5).

The effect of ion temperature in the cubic equation may seem more dramatic at a first reading, as it occurs in the combination $k_x^2 v_{i0}^2 - k^2 Y_i T_i/m_i$, with $k_x v_{i0} = 10^6 \text{rad/s}$ and $k\sqrt{2T_i/m_i} \approx 0.5 \cdot 10^6 \text{rad/s}$. But other terms in the coefficients always overwhelm the ion temperature contribution, as Fig. 6 confirms numerically.

IV. PLOTS

To ease comparison with the original figures, we follow Ref. [1] for all data selections on figures 1–6, and use the same figure numbering. Our plots display a single curve and ordinate axis, as we consider only electrostatic waves.

For all figures, except for the abscissa, parameters have the typical values listed above. The ordinate (note factor 10^6) is the instability growth rate scaled to the ion plasma angular frequency $\omega_{\text{pi}} = 1.15 \cdot 10^8 \text{rad/s}$, according to Eq. (18) in Ref. [1].

For Figure 7 (which has no counterpart in Ref. [1]), we consider another range of electron drift velocities, showing that a smaller v_{e0} (outside the thruster regular operation regime) results in a significantly stronger instability, except for a tiny

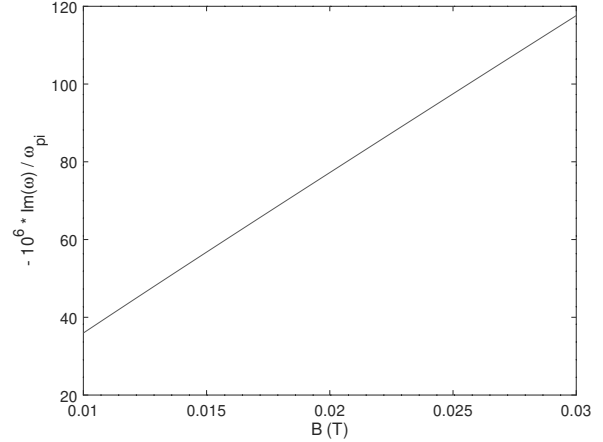


Fig. 1. Growth rate as a function of magnetic field.

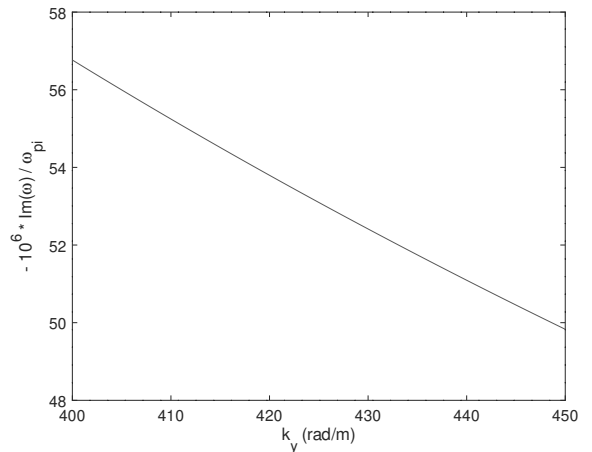


Fig. 2. Growth rate as a function of azimuthal wavenumber.

velocity interval around $k_x v_{i0}/k_y$ (the value for which both ion drift and electron drift generate the same Doppler shift in the system (1)-(4)) where the growth rate decreases and the instability can even be suppressed: all eigenmodes (with the same wavenumbers) are stable for these small v_{e0} 's.

We leave the analysis of these figures for a later endeavour, as this discussion requires deeper investigation in the physics of the instability process. The present comment aims at providing the correct figures implied by the equations in Ref. [1] for electrostatic modes.

It is worth noting that the salient behaviours (linear in Figs 1 and 3, hyperbolic in Figs 2 and 4) are already captured by the Litvak-Fisch estimate (7), to which Eq. (18) of Ref. [1] brings only moderate corrections, so that the need for computing the heavy coefficients and solving Eq. (18) of Ref. [1] is unclear. Besides, the consistency of the linearized equations may be questioned, as Hall thruster plasmas are often neither uniform nor stationary. In particular, the wavelength of the instability is not small compared to the gradient length of the mean density profile, which should in consequence be taken into account.

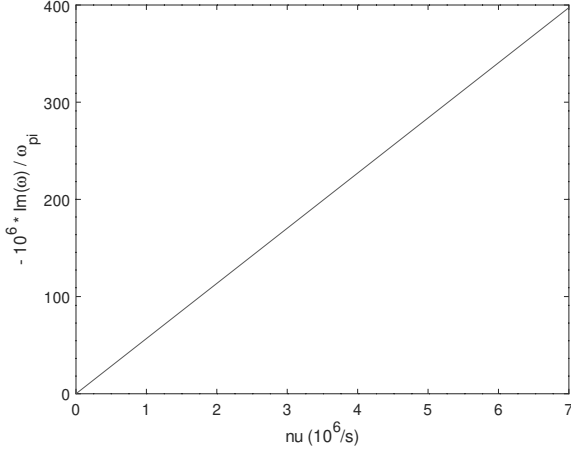


Fig. 3. Growth rate as a function of electron-neutral collision frequency.

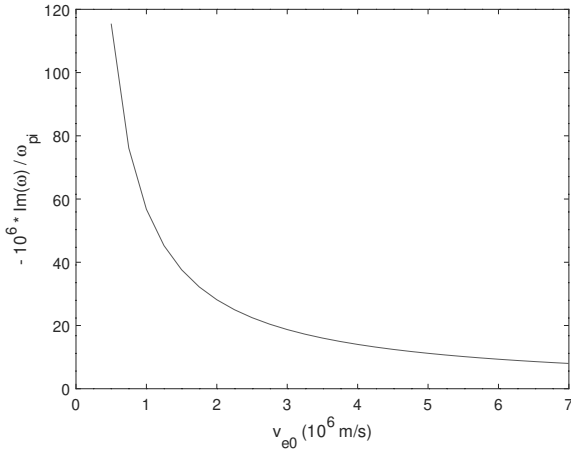


Fig. 4. Growth rate as a function of electron drift velocity.

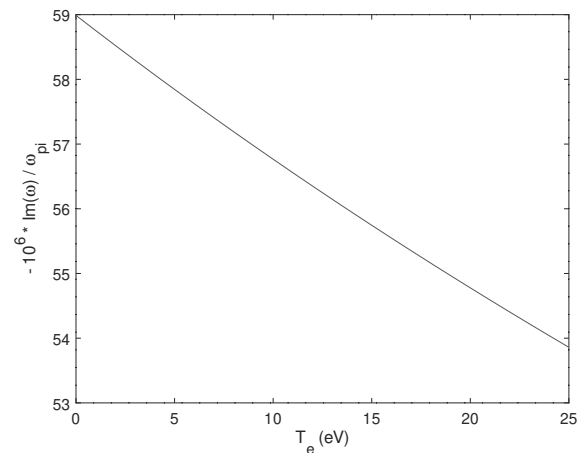


Fig. 5. Growth rate as a function of electron temperature.

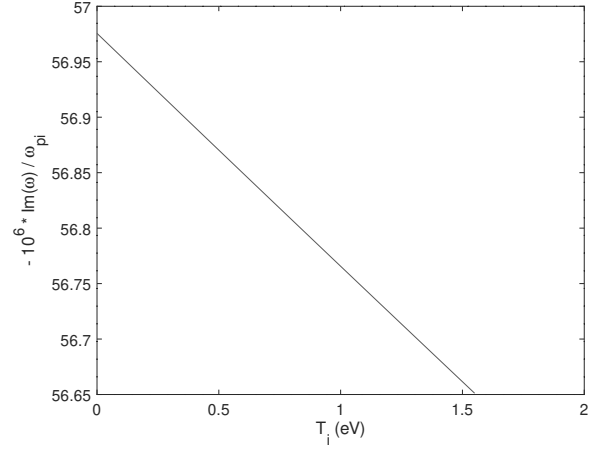


Fig. 6. Growth rate as a function of ion temperature.

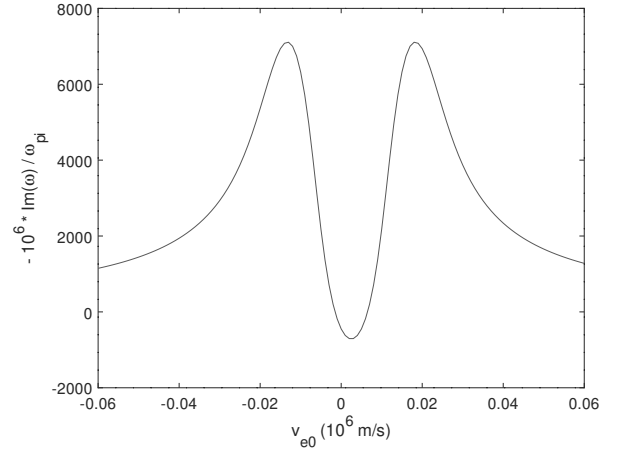


Fig. 7. Growth rate as a function of electron drift velocity.

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