



**HAL**  
open science

## Comment on "Growth of Low-Frequency Electrostatic and Electromagnetic Instabilities in a Hall Thruster"

Y Elskens, N Lemoine

► **To cite this version:**

Y Elskens, N Lemoine. Comment on "Growth of Low-Frequency Electrostatic and Electromagnetic Instabilities in a Hall Thruster". 2018. hal-01906635v1

**HAL Id: hal-01906635**

**<https://hal.science/hal-01906635v1>**

Preprint submitted on 21 Nov 2018 (v1), last revised 18 Feb 2021 (v2)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Copyright

# Comment on “Growth of Low-Frequency Electrostatic and Electromagnetic Instabilities in a Hall Thruster”

Y. Elskens<sup>1,\*</sup> and N. Lemoine<sup>2,†</sup>

<sup>1</sup>*Aix-Marseille Université, UMR 7345 CNRS, PIIM,  
équipe turbulence plasma, case 322 campus Saint-Jérôme,  
av. esc. Normandie-Niemen, FR-13397 Marseille cedex 20*

<sup>2</sup>*Institut Jean Lamour, université de Lorraine,  
UMR 7198 CNRS, BP 70239, FR-54506 Vandoeuvre-les-Nancy*

We revisit the electrostatic instability model of Singh and Malik (IEEE Trans. Plasma Sci. 39:10 (2011) 1910–1918) and correct some errors.

*Keywords* : Hall thruster, low-frequency oscillations, resistive instabilities

*PACS numbers* :

52.75.Di Ion and plasma propulsion

52.35.Fp Electrostatic waves and oscillations

## I. INTRODUCTION

Hall thrusters are promising devices for satellite station keeping and propulsion because of their high thrust density, and they are undergoing significant development. Understanding and controlling their instabilities is a major research topic.

In this context, low-frequency instabilities have been investigated in particular by Singh and Malik [1]. We refer to Wei *et al.* [2] and Smolyakov *et al.* [3] for recent reviews.

## II. ASSUMPTIONS AND DISPERSION RELATION FOR ELECTROSTATIC MODES

It is well-known that “[the] full picture of instabilities in Hall plasmas is complex and, for typical experimental conditions, likely involves a number of interacting modes which require numerical simulations” [3]. However, analytical approaches with simplified models help to grasp processes intuitively. This motivates the interest for classical stability analyses near simple reference states of the plasma. Unfortunately, Hall plasmas are often out-of-equilibrium, nonuniform, unsteady systems, challenging elementary descriptions.

Singh and Malik’s model describes the plasma as two fluids in a uniform magnetic field  $\vec{B}_0 = B_0 \vec{e}_z$ , with ions flowing along the  $x$ -axis and electrons along the  $y$ -axis, respectively with velocities  $\vec{v}_{i0} = v_{i0} \vec{e}_x$  (denoted  $\vec{v}_0$  in [1]) and  $\vec{v}_{e0} = v_{e0} \vec{e}_y$  (denoted  $\vec{u}_0$  in [1]). This geometry assimilates the cylindrical chamber to a slab, with  $z$  radial,  $x$  parallel to the chamber axis, and  $y$  azimuthal. Due to the axial electric field in the chamber, electrons have a significant azimuthal drift, while ions move essentially axially, and, due to their large mass ratio, ions

can be considered unmagnetized while electrons are sensitive to the magnetic force. Both species have finite temperatures,  $T_i$  and  $T_e$ , and the small pressure oscillations should be understood to be adiabatic<sup>1</sup> ( $\delta(p_s n_s^{-Y_s}) = 0$  with  $Y_s = 2$  for both species  $s = e, i$  treated as two-dimensional monatomic ideal gases). The model also involves electron collisions with neutrals, with a frequency  $\nu$ .

Dynamics is essentially two-dimensional, in the  $xy$  plane (normal to  $\vec{B}_0$ ). The reference state is taken to be uniform, with density  $n_{i0} = n_{e0} = n_0$ . The effect of collisions involving ions are neglected. The effect of elastic electron-neutral collisions is taken into account. The linearized evolution equations read

$$\partial_t n_{i1} + v_{i0} \partial_x n_{i1} + n_{i0} \nabla \cdot \vec{v}_{i1} = 0 \quad (1)$$

$$\partial_t \vec{v}_{i1} + v_{i0} \partial_x \vec{v}_{i1} = \frac{e}{m_i} \vec{E}_1 - \frac{1}{m_i n_{i0}} \nabla p_i \quad (2)$$

$$\partial_t n_{e1} + v_{e0} \partial_y n_{e1} + n_{e0} \nabla \cdot \vec{v}_{e1} = 0 \quad (3)$$

$$\begin{aligned} \partial_t \vec{v}_{e1} + v_{e0} \partial_y \vec{v}_{e1} = & -\frac{e}{m_e} (\vec{E}_1 + \vec{v}_{e1} \wedge \vec{B}_0 + \vec{v}_{e0} \wedge \vec{B}_1) \\ & - \frac{1}{m_e n_{e0}} \nabla p_e - \nu \vec{v}_{e1} \end{aligned} \quad (4)$$

$$\nabla \cdot \vec{E}_1 = \frac{e}{\epsilon_0} (n_{i1} - n_{e1}) \quad (5)$$

Focusing on electrostatic modes, the contribution of  $\vec{B}_1$  is neglected, while electric field oscillations derive from a potential,  $\vec{E}_1 = -\nabla \phi_1$ . This system is Fourier-transformed with  $\partial_t = i\omega$  and  $\nabla = -i\vec{k}$ , and the Poisson equation reads

$$(k_x^2 + k_y^2) e \phi_1 = \frac{e^2}{\epsilon_0} (n_{i1} - n_{e1}) = \omega_{pi}^2 m_i \frac{n_{i1} - n_{e1}}{n_{i0}} \quad (6)$$

where we highlight the ion plasma frequency which will be used to normalize all frequencies and growth rates.

\* yves.elskens@univ-amu.fr

† nicolas.lemoine@univ-lorraine.fr

<sup>1</sup> Rather than obeying the unusual  $p_s = Y_s n_s T_s$ .

The dispersion relation of system (1)–(5) is then a fifth-degree polynomial in  $\tilde{\omega} := \omega/\omega_{pi}$ . Focusing on low-frequency oscillations leads the authors to a third-degree polynomial, under the assumption that the electron gyrofrequency  $\Omega = |e|B_0/m_e$  be much larger than  $\max(\omega, k_y v_{e0}, \nu)$ . Computing the coefficients in the polynomial is tedious, but straightforward, see [1], leading to the lengthy Eq. (18) in Ref. [1]. Inspection of the coefficient of the  $\omega^2$  term in the polynomial shows that the sum of all three roots has a positive imaginary part (proportional to  $i\nu$ ); actually, there are generally two roots with positive  $\text{Im}\omega$ , corresponding to damped modes. Therefore, the largest  $\text{Im}(-\omega)$  characterizes either the instability (when it is positive) or the least damped stable mode (when it is negative).

The numerical values they report from studying this equation turn out to be erroneous. With their numerical data, we find much smaller growth rates and qualitatively different dependences on physical parameters.

We followed Ref. [1] for all data selections on figures 1–6. For all figures, except for the abscissa, parameters are  $n_0 = 10^{18} \text{ m}^{-3}$ ,  $B_0 = 15 \text{ mT}$ ,  $k_x = 20 \text{ rad/m}$ ,  $k_y = 400 \text{ rad/m}$ ,  $T_e = 10 \text{ eV}$ ,  $T_i = 1 \text{ eV}$ ,  $v_{e0} = 10^6 \text{ m/s}$ ,  $v_{i0} = 5 \cdot 10^4 \text{ m/s}$ ,  $\nu = 10^6 \text{ s}^{-1}$  with xenon ions ( $m_i = 131 \text{ Da}$ ). The ordinate (note factor  $10^6$ ) is the instability growth rate scaled to the ion plasma angular frequency  $\omega_{pi} = 1.15 \cdot 10^8 \text{ rad/s}$ , according to Eq. (18) in Ref. [1].

For Figure 7, we consider another range of electron

drift velocities, showing that a smaller  $v_{e0}$  (outside the thruster regular operation regime) results in a significantly stronger instability, except for a tiny velocity interval around  $k_x v_{i0}/k_y$  (the value for which both ion drift and electron drift generate the same Doppler shift in the system (1)–(4)) where the growth rate decreases and the instability can even be suppressed: all eigenmodes (with the same wavenumbers) are stable for these small  $v_{e0}$ 's.

We leave the analysis of these figures for a later publication, as this discussion requires deeper investigation in the physics of the instability process. Besides, the consistency of the linearized equations may be questioned, as Hall thruster plasmas are often neither uniform nor stationary.

#### ACKNOWLEDGMENT

The authors are pleased to thank anonymous reviewers (who further questioned the foundations of the equations, left aside in this comment) for their comments on an attempt at a sequel to Ref. [1], and H.K. Malik for instructive discussions. This work was initiated thanks to the Indo-French Centre for the Promotion of Advanced Research (IFCPAR/CEFIPRA project 5204-3), New Delhi. Assistance from L. Couédel and discussions with F. Doveil were helpful to YE.

- 
- [1] S. Singh and H.K. Malik, “Growth of low-frequency electrostatic and electromagnetic instabilities in a Hall thruster”, *IEEE Trans. Plasma Sci.* 39, pp. 1910–1918, 2011. doi: 10.1109/TPS.2011.2162652
- [2] Wei Li-Qiu, Han Liang, Yu Da-Ren and Guo Ning, Low-frequency oscillations in Hall thrusters, *Chin. Phys. B* 24, 055201 (10 pp.), 2015. doi: 10.1088/1674-

1056/24/5/055201

- [3] A.I. Smolyakov, O. Chapurin, W. Frias, O. Koshkarov, I. Romadanov, T. Tang, “Fluid theory and simulations of instabilities, turbulent transport and coherent structures in partially-magnetized plasmas of  $\mathbf{E} \times \mathbf{B}$  discharges”, *Plasma Phys. Control. Fusion* 59, 014041 (20 pp.), 2017. doi: 10.1088/0741-3335/59/1/014041



FIG. 1. Growth rate as a function of magnetic field.



FIG. 2. Growth rate as a function of azimuthal wavenumber.



FIG. 3. Growth rate as a function of electron-neutral collision frequency.



FIG. 4. Growth rate as a function of electron drift velocity.



FIG. 5. Growth rate as a function of electron temperature.



FIG. 6. Growth rate as a function of ion temperature.



FIG. 7. Growth rate as a function of electron drift velocity.