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Electromagnetic proximity effect in planar superconductor-ferromagnet structures

S. Mironov,¹ A. S. Mel'nikov,^{1,2} and A. Buzdin^{3,4}

¹*Institute for Physics of Microstructures, Russian Academy of Sciences, 603950 Nizhny Novgorod, GSP-105, Russia*

²*Lobachevsky State University of Nizhny Novgorod, 23 Gagarina, 603950 Nizhny Novgorod, Russia*

³*University Bordeaux, LOMA UMR-CNRS 5798, F-33405 Talence Cedex, France*

⁴*Department of Materials Science and Metallurgy, University of Cambridge, CB3 0FS, Cambridge, United Kingdom*

The spread of the Cooper pairs into the ferromagnet in proximity coupled superconductor - ferromagnet (SF) structures is shown to cause a strong inverse electromagnetic phenomenon, namely, the long-range transfer of the magnetic field from the ferromagnet to the superconductor. Contrary to the previously investigated inverse proximity effect resulting from the spin polarization of superconducting surface layer, the characteristic length of the above inverse electrodynamic effect is of the order of the London penetration depth, which usually much larger than the superconducting coherence length. The corresponding spontaneous currents appear even in the absence of the stray field of the ferromagnet and are generated by the vector-potential of magnetization near the S/F interface and they should be taken into account at the design of the nanoscale S/F devices. Similarly to the well-known Aharonov-Bohm effect, the discussed phenomenon can be viewed as a manifestation of the role of vector potential in quantum physics.

The proximity phenomena in condensed matter physics are known to include the interface effects which are usually associated with the exchange of electrons between the contacting materials. This exchange is responsible for the mutual transfer of different particle-related qualities through the interface such as superconducting correlations, spin ordering etc.¹⁻³ The goal of the present work is to show that this spread of particle-related qualities in some cases should be supplemented by the long-range spread of the electromagnetic fields. As an example, we demonstrate that such electromagnetic proximity effect can strongly affect the physics of superconductor - ferromagnet (SF) systems which are widely discussed as building blocks of superconducting spintronics^{4,5}.

To elucidate our key observation we can consider an exemplary bilayer system consisting of a superconducting (S) film placed in contact with a ferromagnetic (F) layer with the magnetic moment parallel to the layer plane (see Fig. 1). Considering the S and F subsystems to be isolated we get a perfect example of complete separation of the regions with a nonzero concentration of Cooper pairs and magnetic field. The latter is completely trapped inside the ferromagnet. As we allow the electron transfer between the subsystems the Cooper pairs immediately penetrate the ferromagnet inducing the pair electron correlations there. This process which is usually referred as a standard proximity effect leads to the series of fascinating transport phenomena^{1,6}. The inverse proximity effect namely the transfer of the magnetic moment from the ferromagnetic to the superconducting subsystem is also possible and has been recently studied in a number of theoretical and experimental works⁷⁻¹⁸. This inverse proximity effect is related to the spin polarization of electrons forming the Cooper pair near the S/F interface and results in the small magnetization of the superconducting surface layer at the depth of the or-

der of the Cooper pair size, i.e., the superconducting coherence length $\xi_0 \sim 1 - 10 \text{ nm}$. Experimentally, it has been observed with the help of muon spin rotation techniques^{13,14}, nuclear magnetic resonance¹⁵ and neutron scattering measurements¹⁶⁻¹⁸.

It is commonly believed that neglecting this short range inverse proximity phenomenon we can assume the magnetic field to remain completely trapped inside the F film even when the electron transfer between the films is possible. This conclusion is based on the obvious observation that an isolated infinite single-domain ferromagnetic film with the in-plane magnetization does not produce the stray magnetic field in the outside. In this paper we show that, contrary to this belief, the direct proximity effect is always responsible for the exciting the supercurrents flowing inside the ferromagnet itself and, thus, for the appearance of the compensating Meissner supercurrents in the superconducting subsystem. The appearance of these currents is accompanied by the generation of the magnetic field in the S film which decays at the distances of the order of the London penetration depth λ which can well exceed the coherence length in type-II superconductors (see Fig. 1). From the experimental point of view, this electromagnetic proximity phenomenon reminds the Aharonov-Bohm effect¹⁹ since the current inside the attached superconductor is induced by the ferromagnetic layer which does not create the magnetic field outside the layer in the absence of such superconducting environment. At the same time, the true physical key point is that the wave function penetrating the ferromagnet is responsible for this effect, when the vector-potential of the F magnetization unavoidably generates current in quantum system described by a common Cooper pair wave function (superconductor and a region near S/F interface). Quite surprisingly this long-range electromagnetic (orbital) contribution to the inverse proximity effect has

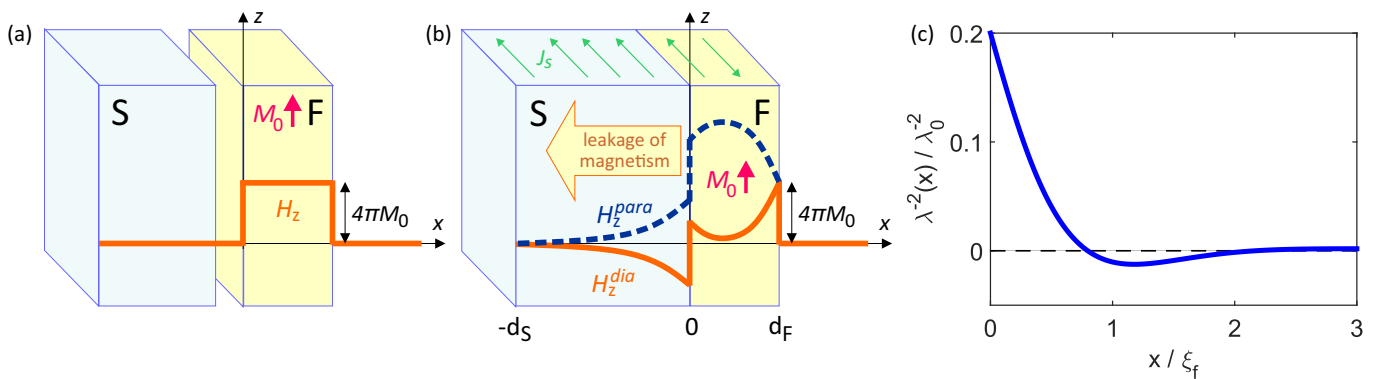


FIG. 1. The sketch of the superconductor/ferromagnet bilayer. (a) When the layers are separated from each other the magnetic field exists only inside the F layer. (b) In contrast, when the layers are put in contact the magnetization inside the ferromagnet becomes the source of the long-range magnetic field in the superconductor. In both panels the orange solid (blue dashed) curves schematically represent the profile of the magnetic field when the total current inside the F-layer is diamagnetic (paramagnetic). (c) The spatial profile of the screening parameter $\lambda^{-2}(x)$ inside the ferromagnetic layer in the dirty limit. We take $\sigma_s = 10\sigma_f$.

been overlooked in all previous studies devoted to this subject while the relevant magnetic fields strongly exceed the ones induced by the spin polarization discussed in^{8,9} and should dominate in the experiments. Indeed, in^{14,16} the induced magnetic field was observed in S/F hybrid systems at distances much larger than all relevant superconducting coherence lengths. The existing theories of proximity effect fail to provide the interpretation of these results, while our approach provides a natural explanation of these phenomena. Thus, the current-carrying states appear to be unavoidable for the SF systems and should be taken into account in the design of the devices of superconducting spintronics^{4,5}.

Further consideration illustrating the above qualitative arguments is organized in two steps: (i) first, we consider a simple model assuming a phenomenological form of the relation between the supercurrent \mathbf{j} and vector potential \mathbf{A} in conditions of the long range inverse proximity effect; (ii) second, we present the results of microscopic calculations of the $\mathbf{j}(\mathbf{A})$ relation which support and justify the phenomenological findings. Accounting for the proximity effect we write the Maxwell equation in the form:

$$\text{rot rot } \mathbf{A} = \frac{4\pi}{c} (\mathbf{j}_s + \mathbf{j}_m), \quad (1)$$

where \mathbf{j}_s is the Meissner current and $\mathbf{j}_m = c \text{ rot } \mathbf{M}$ is the current associated with the magnetization \mathbf{M} .

For the sake of definiteness we consider a bilayer consisting of the superconductor of the thickness $d_s \gg \lambda$ and a ferromagnet of the thickness $d_f \ll \lambda$ with the uniform magnetization $\mathbf{M}_0 = M_0 \mathbf{e}_z$. We choose the x -axis perpendicular to the layers with $x = 0$ at the S/F interface (see Fig. 1). First, we assume the local London relation which is relevant, e.g., for dirty S/F sandwiches:

$$\mathbf{j}_s(x) = -\frac{c}{4\pi} \frac{1}{\lambda^2(x)} \mathbf{A}(x), \quad (2)$$

where the London screening parameter λ^{-2} becomes dependent on x . The penetration of Cooper pairs into the F

film gives rise to the supercurrent there and also slightly modifies the Meissner response of the S layer in the small region of the thickness of the order of the superconducting coherence length ξ from the S/F interface. The latter modification can even vanish if the conductivity of the ferromagnet is much smaller than the normal conductivity of the S layer. So, let us, first, neglect the changes of λ inside the superconductor and assume it to be equal to the constant value λ_0 . Then the solution of Eq. (1) in the S layer reads $A_y(x) = A_0 \exp(x/\lambda_0)$ where A_0 is a constant and inside the F layer $A_y(x) = A_0 + 4\pi M_0 x$. Here we neglect the spatial variations of the vector potential in the ferromagnet related to the weak London screening since the typical scale of such variation is of the order of $\lambda \gg d_f$. Thus, the exact solution of Eq. (1) is redundant for most realistic structure and material parameters and we restrict ourselves to the approximate calculation procedure described below. To find A_0 we integrate Eq. (1) over the width of the ferromagnet accounting the relation $B_z = \partial A_y / \partial x$. The magnetic current $\mathbf{j}_m = c \text{ rot } \mathbf{M}$ integrated over the sample thickness is zero and, thus, the relation between the magnetic field $B_z(d_f)$ outside the sample and the field $B_z(0) = A_0/\lambda_0$ inside the superconductor close to the S/F interface takes the form

$$B_z(d_f) - B_z(0) = A_0 \int_0^{d_f} \frac{dx'}{\lambda^2(x')} + 4\pi M_0 \int_0^{d_f} \frac{x' dx'}{\lambda^2(x')}. \quad (3)$$

The first term in the r.h.s. of Eq. (3) can be neglected since it is of the order of $(d_f/\lambda)B_z(0) \ll B_z(0)$. Note that for the same reason one can neglect the contribution coming from the renormalisation of λ^{-2} in the region of the width $\sim \xi$ inside the S layer if the inverse proximity effect is not small. In the absence of the external magnetic field $B_z(d_f) = 0$ and we finally obtain that the magnetic field induced in the superconductor is

$$B_z = -4\pi M_0 Q \exp(x/\lambda_0), \quad (4)$$

where $Q = \int_0^{d_f} \lambda^{-2}(x') x' dx'$. Note that in the con-

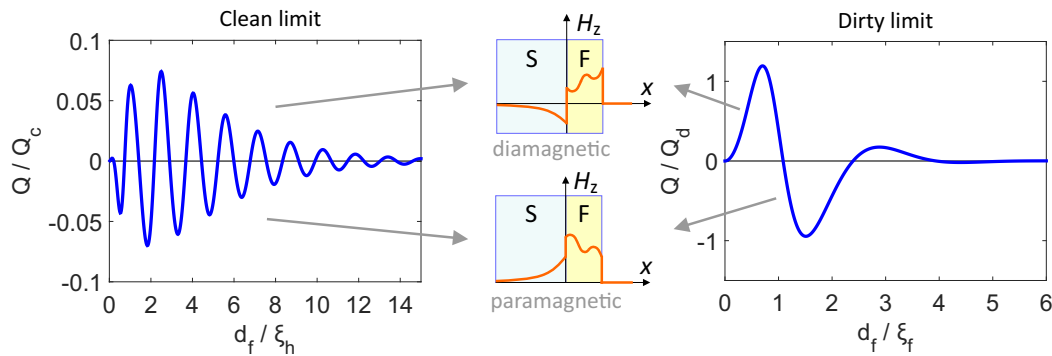


FIG. 2. The dependence of the magnetic kernel Q on the ferromagnet thickness for the clean and dirty limits. For the clean limit we take $h = 10\pi T$ and choose the temperature T in a way that $\Delta = 2\pi T$. Also we define the values $Q_d = \pi^2 \sigma_f \xi_f^2 \tanh(\Delta/2T)/(2\hbar c^2)$ and $Q_c = e^2 \nu_0 \xi_h^2 (v_F/c)^2$ where σ_f is the conductivity of the ferromagnet and ν_0 is the density of states at the Fermi level per unit spin projection and per unit volume.

ventional regime when the superconducting condensate penetrating the F layer reveals a diamagnetic response with $Q > 0$ the magnetic field induced in the S layer is anti-parallel to the magnetization \mathbf{M}_0 . In the case when d_f is of the order of the superconducting coherence length in the ferromagnet ξ_f the basic estimate gives $Q \sim (\xi_f/\lambda)^2 \sim 10^{-2}$ for the S/F structures based, e.g., on thin Nb films. Considering the experiments probing the change in the magnetic moment in SF structures when cooling down through the superconducting critical temperature and taking the typical magnetization corresponding to Fe or Co films $4\pi M_0 \sim 10^4$ Oe we find $B_z \sim 10^2$ Oe which is an easily measurable value.

More accurate estimate for the value Q can be obtained within the Usadel formalism for dirty S/F bilayers²⁰. In Fig. 1(c) we plot the typical profile of the screening parameter inside the F layer while the behavior of the kernel Q is shown in the right panel of Fig. 2. Remarkably, the value Q oscillates as a function of d_f being either positive or negative which corresponds to anti-parallel or parallel directions of \mathbf{B} and \mathbf{M}_0 for different values of d_f . Somewhat similar results for the magnetic response have been previously found for the S/F structures with a low transparent barrier at the interface in²¹, but the magnetic field generated inside the superconductor was neglected.

Interestingly, the value Q and, thus, the magnetic field in the S layer can be substantially increased provided the magnetization in the F layer has a non-collinear structure - this is relevant to the recent experiments¹⁴. To illustrate the origin of this effect let us assume that the ferromagnet consists of two layers: one layer F_1 occupying the region $0 < x < d_1$ has the magnetization M_0 along the z -axis while another layer F_2 of the thickness d_2 has the same magnetization M_0 but directed along the y -axis. We choose d_2 to be much larger than the normal metal coherence length $\xi_n = \sqrt{D_f/T}$ but still much less than λ . The vector potential in the F bilayer has two components: $\mathbf{A} = (A_{0y} + 4\pi M_0 x) \mathbf{e}_y + A_{0z} \mathbf{e}_z$ for $0 < x < d_1$ and $\mathbf{A} = (A_{0y} + 4\pi M_0 d_1) \mathbf{e}_y + [A_{0z} - 4\pi M_0 (x - d_1)] \mathbf{e}_z$ for $d_1 < x < d_1 + d_2$. The functions $A_{y0}(x)$ and $A_{z0}(x)$ vary over

the distances $\sim \lambda$ and are almost constants in the F film. Integrating Eq. (1) analogous to the case of the S/F bilayer we find the magnetic field induced in the S layer:

$$B_z = -4\pi M_0 Q_z e^{x/\lambda_0}, \quad B_y = -4\pi M_0 Q_y e^{x/\lambda_0}, \quad (5)$$

where

$$Q_z = \int_0^{d_1} \frac{x' dx'}{\lambda^2(x')}, \quad Q_y = \int_{d_1}^{d_1+d_2} \frac{(x' - d_1) dx'}{\lambda^2(x')}. \quad (6)$$

In Fig. 3 we schematically show the spatial profiles of the spontaneous magnetic field for parallel and perpendicular orientations of the magnetic moments in the fer-

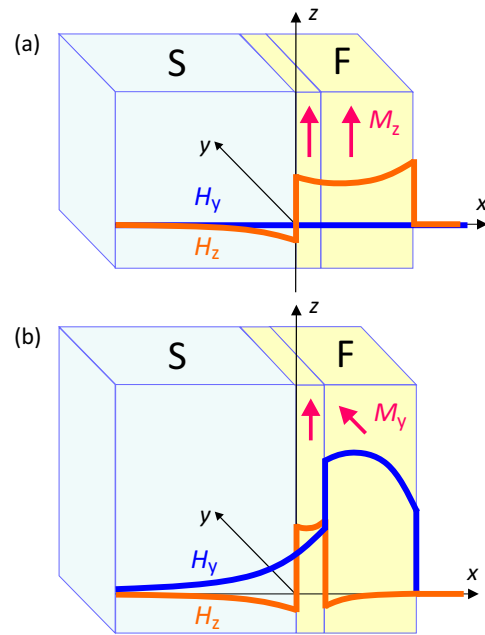


FIG. 3. The profiles of the spontaneous magnetic field in the superconductor-ferromagnet-ferromagnet trilayers when the magnetic moments in the two F layer are (a) parallel to each other and (b) perpendicular to each other.

romagnetic layers. Remarkably, for perpendicular orientation the estimate for the value Q_y based on the Usadel formalism gives $Q_y \sim (\xi_n/\lambda)Q_z \gg Q_z$ where $\xi_n = \sqrt{D_f/T} \gg \xi_f$. As a result, in the experiments where the angle θ between the magnetic moments in two F layers can be tuned one should observe a strong increase in the magnetic moment of the S layer for $\theta = \pi/2$ as compared to the case $\theta = 0$ (see Fig. 3). Namely this behavior has been recently discovered in experiments with the Au/Nb/ferromagnet structures¹⁴.

Note that all described phenomena should become more pronounced provided one deals with the clean S/F and S/F₁/F₂ sandwiches. In this case the Cooper pairs penetrate over much larger distances into the ferromagnet which strengthens the proximity effect and, thus, the amplitude of the induced magnetic field in the superconductor. In the clean limit the relation $\mathbf{j}_s(\mathbf{A})$ becomes non-local and we may write it in a very generic form

$$\mathbf{j}_s(x) = -\frac{c}{4\pi} \int \mathbf{A}(x')K(x, x')dx'. \quad (7)$$

Deep inside the superconductor for $|x| \gg \xi_N = \hbar v_F/T$ the kernel K in this relation has the standard London form characterized by the penetration depth λ_0 . Then the profile of the magnetic field generated in the superconductor is again determined by Eq. (4) but the value Q now can be expressed through the nonlocal kernel K (see supplementary material):

$$Q = \int_{-x_0}^{d_f} dx \int_0^{d_f} dx' x' [K(x, x') - \lambda_0^{-2} \delta(x - x') \theta(-x)], \quad (8)$$

where $\theta(x)$ is the Heaviside step function and the integration limit x_0 satisfies the condition $\xi_N \ll x_0 \ll \lambda_0$.

The calculations of the screening parameter Q are presented in the supplementary material. In Fig. 2 we show the typical dependence of the value Q on the ferromagnet thickness. The interference between different quasiparticle trajectories results in the oscillations of $Q(d_f)$ with the period of the order of the length $\xi_h = \hbar v_F/h$ which is known to characterize clean ferromagnets¹. The envelope of the oscillating Q increases as $\propto (d_f/\xi_N)^2$ at small d_f/ξ_N values [here $\xi_N = \hbar v_F/(4\pi T)$] and decays exponentially at $d_f \gg \xi_N$.

Up to now several experimental papers reported the evidences of the long ranged (at distances larger than ξ) magnetic field generation in S/F systems which could be naturally explained by the above theory. In particular, in Ref.¹⁶ the polarized neutron reflectometry was used to study the magnetic field profile in a single V(40 nm)/Fe(1 nm) bilayer. Below T_c these measurements indicate the appearance of the spontaneous magnetic field penetrating the S layer at the distance ~ 20 nm from the S/F interface. This distance definitely exceeds the coherence length $\xi_0 \sim 5 \div 10$ nm for vanadium. Another manifestation of the long ranged field generation was recently reported in¹⁴, where the muon

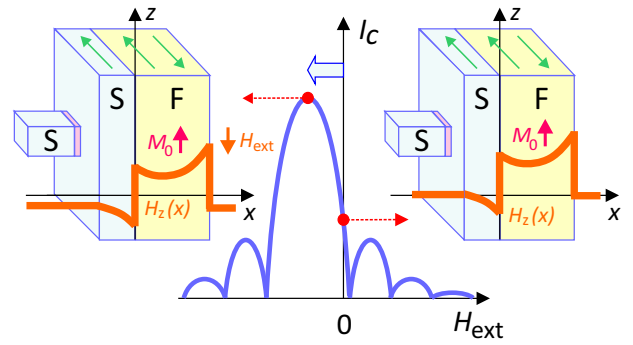


FIG. 4. Shift in the Fraunhofer critical current oscillations for the Josephson junction with one electrode covered by the ferromagnetic layer.

spin-rotation experiments in Au/Nb/ferromagnet structure revealed a remote magnetic field in Au at the large distance (more than 50 nm which strongly exceeds the ξ value in Nb films) from the ferromagnet. In¹⁴ the composite F layer was used allowing a non-collinear magnetic configuration. The remote field was much more pronounced in a perpendicular configuration in accordance with the above arguments regarding the generation of the long-range magnetic moment in SFF' systems. All these observations are pretty hard to explain by the standard theory of the inverse proximity effect characterized by a rather short length scale $\sim \xi_0$. In contrast, our results clearly demonstrate the current and magnetic field generation at much larger distances $\sim \lambda \gg \xi$ from the S/F interface. Note that repeating the experiments¹⁴ without the applied external field one may expect the generation of the magnetic field in the Au layer solely by the spontaneous current predicted in our work.

Sure, the muon spin-rotation experiments permit the direct measurement of the spontaneous magnetic field in S/F structures. An alternative way to detect the currents associated with these spontaneous magnetic fields can be based on the use of different types of local transport probes positioned at the outer boundary of the superconductor. Scanning, e.g., the outer surface by the normal metal tip of the scanning tunneling microscope one can measure the changes in the local density of states caused by the Doppler shift of the quasiparticles energy in the presence of the superflow²²⁻²⁴. For the case of the superconducting probe one can also propose a simple and elegant experimental setup revealing this effect. It consists of the Josephson junction where one of the electrodes has the thickness of the order of λ and is covered by the ferromagnetic layer (see Fig. 4). Certainly, the superconducting probe should be small enough not to perturb the measured magnetic field distribution. The electromagnetic proximity effect should result in the shift of the Fraunhofer dependence of the critical current on the external magnetic field. Note that the proper choice of the magnetic field can compensate this shift and restore the bare critical current (see Fig. 4).

To sum up, we have revealed a very general mecha-

nism of the long-range electromagnetic proximity effect in the S/F structures which results in the strong spread of the stray magnetic field into the superconductor from the ferromagnet. The screening currents accompanying this magnetic field spread appear in the region where in the normal state all the stray fields are completely absent. The only nonzero electromagnetic characteristic in this region is the vector potential which is usually an unobservable quantity. In this sense the current generation which accompanies the superconducting transition illustrates the crucial role of the vector potential in quantum physics of the SF structures.

SUPPLEMENTARY MATERIAL

See supplementary material for a microscopical calculation of the electromagnetic response in the dirty and clean limits.

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SUPPLEMENTARY MATERIAL FOR “ELECTROMAGNETIC PROXIMITY EFFECT IN PLANAR SUPERCONDUCTOR-FERROMAGNET STRUCTURES”

A. Magnetic response: dirty limit

To estimate the value Q in the dirty limit we use the Usadel formalism. The spatial profile $\lambda^{-2}(x)$ is determined by the anomalous Green function $\hat{f}(x) = (f_s + \mathbf{f}_t \hat{\sigma}) i \sigma_y$ which is a 2×2 matrix in the spin space ($\hat{\sigma}$ is the vector of the Pauli matrices):

$$\lambda^{-2}(x) = \frac{16\pi^2 T \sigma}{\hbar c^2} \sum_{\omega > 0} (|f_s|^2 - |\mathbf{f}_t|^2). \quad (\text{S1})$$

In Eq. (S3) the summation is performed over the positive Matsubara frequencies ω , σ is the normal state conductivity and inside the F layer the singlet and triplet components f_s and \mathbf{f}_t satisfy the Usadel equations

$$D \partial_x^2 f_s = 2\omega f_s + 2i\mathbf{h}\mathbf{f}_t, \quad D \partial_x^2 \mathbf{f}_t = 2\omega \mathbf{f}_t + 2i\mathbf{h}f_s, \quad (\text{S2})$$

where D is the diffusion coefficient and \mathbf{h} is the exchange field. For simplicity let us assume the conductivity of the F layer σ_f to be much smaller than the one of the superconductor (σ_s) which allows to impose the rigid boundary conditions at the S/F interface for the \hat{f} function: $f_s = f_{s0}$ and $\mathbf{f}_t = 0$ where Δ is the gap function and $f_{s0} = \Delta / \sqrt{\omega^2 + \Delta^2}$. Assuming Δ to be real we can immediately write the solution of Eqs. (S2): $f_s = \text{Re}(F)$, $f_{tz} = i \text{Im}(F)$ where $F = f_{s0} \cosh[q(x - d_f)] / \cosh(qd_f)$ and $q^2 = 2(\omega + ih)/D_f$. The functions $f_s(x)$ and $f_{tz}(x)$ are oscillating and, therefore, there are regions where $|f_{tz}| > |f_s|$ and the local Meissner response is paramagnetic.

To calculate the value Q for the S/F bilayer in the dirty limit it is convenient to rewrite the expression for the screening parameter λ^{-2} through the function $F = f_s + f_{tz}$:

$$\lambda^{-2}(x) = \frac{16\pi^2 T \sigma}{\hbar c^2} \sum_{\omega > 0} \text{Re}(F^2). \quad (\text{S3})$$

Note that the frequencies ω enter only the amplitude f_{s0} which allows to calculate the sum in Eq. (S3) analytically:

$$\sum_{\omega > 0} \frac{\Delta^2}{\omega^2 + \Delta^2} = \frac{\Delta}{4T} \tanh\left(\frac{\Delta}{2T}\right). \quad (\text{S4})$$

Also the value Q is defined by the following integral:

$$\int_0^{d_f} \frac{x \cosh^2[q(x - d_f)] dx}{\cosh^2(qd_f)} = \frac{q^2 d_f^2 - 1}{4q^2 \cosh^2(qd_f)} + \frac{1}{4q^2}. \quad (\text{S5})$$

The last term in Eq. (S5) is purely imaginary and it drops out from the value Q . Finally, after algebraic transformations we obtain:

$$Q = \frac{\pi^2 \sigma_f \xi_f^2 \Delta}{2\hbar c^2} \tanh\left(\frac{\Delta}{2T}\right) \text{Im} \left[\frac{q^2 d_f^2 - 1}{\cosh^2(qd_f)} \right], \quad (\text{S6})$$

where $\xi_f = \sqrt{D_f/h}$.

B. Magnetic response: clean limit

To calculate the magnetic response kernel in the clean limit we use the approach analogous to the one in [1, 2] which is based on the solution of the Eilenberger equation [3]. We consider a clean S/F bilayer with $d_f \ll \lambda$ and $d_s \gg \lambda$. Inside the S layer the Eilenberger equations for the quasiclassical Green functions read

$$\hbar \mathbf{v}_F \hat{\mathbf{D}} f = -2\omega f + 2\Delta g, \quad \hbar \mathbf{v}_F \hat{\mathbf{D}}^* f^\dagger = 2\omega f^\dagger - 2\Delta^* g, \quad (\text{S7})$$

where g is the normal Green function, f and f^\dagger are the anomalous Green functions satisfying the normalization

condition $f^\dagger f + g^2 = 1$, $\hat{\mathbf{D}} = (\nabla + 2ie\mathbf{A}/\hbar c)$ is the gauge-invariant momentum operator, ω is the Matsubara frequency, and $\mathbf{v}_F = v_x \hat{\mathbf{x}}_0 + v_y \hat{\mathbf{y}}_0 + v_z \hat{\mathbf{z}}_0$ is the vector of the quasiparticle velocity. For simplicity we neglect the spatial variations of the gap potential Δ inside the S layer which is accurately justified in the vicinity of T_c or in the case of strong mismatch between Fermi velocities in the S and F layers which damps the proximity effect. Note, however, that such variations of Δ should not qualitatively change the final results, so our simple model makes sense even at low temperatures. Inside the F layer we get:

$$\hbar \mathbf{v}_F \hat{\mathbf{D}} f = -2(\omega + ih)f, \quad \hbar \mathbf{v}_F \hat{\mathbf{D}}^* f^\dagger = 2(\omega + ih)f^\dagger. \quad (\text{S8})$$

We focus only on the solutions of Eq. (S8) near the S/F interface in the region of the thickness $\sim (\hbar v_F/T) \ll \lambda$, where the deviations of the electromagnetic response from the bulk London relation $\mathbf{j}(\mathbf{A})$ are most pronounced. So, again, we put $A_y(x) = A_0 + 4\pi M_0 x$ neglecting the variations of $A_y(x)$ close to the interface.

The analytical solution of the Eilenberger equation for the function g allows us to calculate the superconducting current flowing along S/F interface:

$$j_y(x) = -4\pi e \nu_0 T \sum_{\omega > 0} \langle \text{Im}[g(x)] v_y \rangle, \quad (\text{S9})$$

where ν_0 is the density of states at the Fermi level per unit spin projection and per unit volume, and the brackets denote averaging over the Fermi surface: $\langle \dots \rangle = (4\pi)^{-1} \int_0^\pi d\theta \int_0^{2\pi} d\varphi (\dots) \sin\theta$. Here θ and φ are the spherical angles at the Fermi surface so that $v_x = v_F \cos\theta$ and $v_y = v_F \sin\theta \cos\varphi$. Further, it is convenient to represent this current as the sum $j_y(x) = j_M + j_{surf}(x)$, where $j_M = -(4/3)(e^2 T v_F^2 \nu_0 A_0(0)/\hbar c) \sum_{\omega > 0} (\Delta^2/\Omega^3)$ coincides with the standard Meissner current flowing in the bulk superconductor and $j_{surf}(x)$ is the surface current induced by the magnetization. Finally, integrating $j_{surf}(x)$ over x we find the desired value of the magnetic response kernel Q .

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