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1 Some statistical properties of three-dimensional zonostrophic turbulence

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5 We conduct in-depth analysis of statistical flow properties from direct numerical simulations that reproduce
6 gas giants macroturbulence, namely large-scale zonal winds. Our numerical model has been specifically
7 designed to simulate a recent laboratory device that reports zonal jets in the configuration of deep turbulent
8 planetary layers (Cabanes *et al.* 2017). In this framework, the so-called zonostrophic regime is achieved
9 when large topographical variations of the fluid layer combine with rapid rotation in a well developed
10 three-dimensional (3D) turbulent flow. At steady state, strongly energetic, zonally dominated, large-scale
11 axisymmetric structures emerge scaling with Rhines' theoretical scale. This model differs from the shallow-
12 layer scenario where the flow is confined to a quasi-two-dimensional (2D) fluid shell and the anisotropic β -
13 effect arises from latitudinal variation of the Coriolis force. Thus, we aim to reveal, in the specific framework
14 of the deep-layer scenario, signatures of the zonostrophic regime and of a β -topography in statistical flow
15 properties. To do so, we run two large-scale 3D direct numerical simulations in a cylindrical geometry of a
16 highly turbulent and rapidly rotating flow. These two simulations use similar set of parameters but with and
17 without topographical β -effect. We propose a comparative phenomenological description of the temporal
18 and spatial statistics of the three components of the velocity field. Interestingly, we report that peculiar
19 correlations occur between the vertical and radial flow components when a β -topography is imposed and
20 show a feature possibly due to a zonostrophic dynamics in 3D frequency spectra. These results suggest
21 the development of new tools to remotely investigate gas giants zonal winds by extracting statistical flow
22 properties from direct observations. Ultimately our analysis may support the relevance of the deep models
23 in the study of prevalent features of planetary dynamics.

24 **Keywords:** Zonostrophic turbulence, zonal jets.

25 1. Introduction

26 The strong east-west winds on the gas giants, such as Jupiter (Bagenal *et al.* 2006) and
27 Saturn (Del Genio *et al.* 2009), are unmistakable markers of powerful underlying dynamics,
28 inaccessible to direct measurements. Understanding how these intense zonal winds emerge
29 and interact with deeper planetary envelopes, is a major scientific challenge for ongoing
30 spacecraft missions as well as for numerical and laboratory studies. While classical homoge-
31 neous three-dimensional (3D) turbulence leads to a direct cascade of the energy, turbulence
32 theory predicts that large scale structures naturally emerge from small-scale motions in
33 two-dimensional (2D) systems. This well-known phenomenon is called an inverse energy
34 cascade (Kraichnan 1967). Consequently, macro-turbulence on Jupiter has been interpreted
35 to result from the inverse energy cascade within the quasi-2D shallow troposphere (Cho and
36 Polvani 1996) (i.e., the planetary weather layer). Similar assumptions are used in global
37 circulation models designed to simulate large-scale circulations observed in the Earth's
38 atmosphere and oceans. It is then common to invoke that the vertical dimension (along the
39 planetary radius) is much smaller than any typical horizontal scale, thus reducing simulations
40 to a quasi-2D configuration (e.g. solving shallow-water equations on a planetary spherical
41 surface). However, recent studies have revived the possibility to generate Jupiter like jets
42 from a 3D convective envelope, extending through the molecular layer between the surface

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43 weather layer and the deeper dynamo region (Christensen 2001, Heimpel *et al.* 2005, Kaspi
 44 *et al.* 2009, Heimpel *et al.* 2016, Cabanes *et al.* 2017). These studies invoke that rapid
 45 planetary rotation leads to flow invariance along the planetary spin axis. This dynamical
 46 confinement due to a dominant Coriolis acceleration forces the flow to be invariant along
 47 the rotation axis and sets the required conditions for the inverse cascade to feed large scale
 48 structures. This duality between 2D and 3D dynamics, inverse and direct cascades, is at the
 49 core of fundamental geophysical fluid dynamics (Pouquet and Marino 2013). In both shallow
 50 and deep configurations, the dynamics is quasi-2D, namely depth invariant, and large scales
 51 develop on the horizontal flow components. In the shallow model, horizontal is defined by the
 52 spherical planetary surface and depth is along the planetary radius. In the deep model, the
 53 relevant horizontal and depth components are defined to be respectively normal and along
 54 the planetary spin axis. This defines a complete framework to study the specificity of the
 55 shallow and deep-layer scenarios.

56
 57 In the specific context of planetary systems, the large scale structures formed by quasi-2D
 58 flows display an additional zonal anisotropy in the form of east-west dominant motions.
 59 Such features appear in spherical geometry where the boundary curvature of planetary fluid
 60 layers preferentially channels energy into the large-scale zonal motions via the so-called
 61 β -effect (Vallis and Maltrud 1993). Large-scale drag eventually halts the inverse energy
 62 cascade that propagates up to the Rhines' length scale $L_{Rh} = \pi\sqrt{2U/\beta}$, and sets the
 63 characteristic width of zonal jets (Rhines 1975, Sukoriansky *et al.* 2007). Here U is the
 64 root-mean-square (rms) fluid velocity. In shallow turbulence, where quasi-bidimensionality
 65 results from spatial confinement, zonal anisotropisation arises from latitudinal variation of
 66 the Coriolis force with $\beta = 2\Omega \cos\theta/R$, where Ω is the rotation rate, θ is the latitude and
 67 R is the radius of the planet. In deep turbulence, where quasi-bidimensionality results from
 68 strong anisotropy induced by the Coriolis force, zonal anisotropy emerges from variations
 69 of the axial fluid column height h with cylindrical radius r , with $\beta = (2\Omega/h)dh/dr$.
 70 Both scenarios gather all suitable physical ingredients and zonal jets develop, provided
 71 that the Coriolis force dominates over fluid inertia (i.e, low Rossby number Ro), which
 72 in turn, must dominate over viscous dissipative effects (i.e., large Reynolds number Re
 73 and small Ekman number E). Depending on the details of the system considered, the
 74 large-scale jets can remain weak and meandering like in the Earth's oceans (Maximenko
 75 *et al.* 2005), or ultimately reach the so-called zonostrophic regime where zonal energy
 76 dominates over turbulent fluctuations. These differing regimes of jet flow behaviors can be
 77 characterized in terms of the zonostrophy index R_β (Galperin *et al.* 2010). This index is
 78 the ratio of the Rhines scale and the transitional scale $L_\beta \approx (\epsilon/\beta^3)^{1/5}$ above which the
 79 β -effect affects the energy cascade, with ϵ being the inverse energy transfer rate through
 80 the system. Simplistically, $R_\beta \approx L_{Rh}/L_\beta$ may be regarded as a measure of how strongly
 81 anisotropic the turbulent flow may be. Zonostrophic flows are argued to exist at $R_\beta \geq 2.5$
 82 (Galperin *et al.* 2006, Sukoriansky *et al.* 2007). Jovian mid-latitude jets exceed this threshold
 83 (Galperin *et al.* 2014) with a typical value $R_\beta \approx 5$, while Earth's oceans are limited to
 84 $R_\beta \approx 1.5$ (Galperin *et al.* 2006). Strongly zonostrophic regime is the focus of the present paper.

85
 86 As of now, although these shallow and deep models have been systematically challenged,
 87 there has been no successful attempt to contrast their relevance in the study of the prevalent
 88 features of planetary dynamics. Today, new observations from the ongoing Juno NASA's
 89 mission pave the way to conduct in-depth analyzes of Jupiter's flow properties. The upcoming
 90 challenge will be to identify, using the measurements of the Jovian anisotropic turbulent flow,
 91 typical signatures of the shallow and deep turbulent scenarios. For that purpose, laboratory
 92 and numerical models for planetary jets are now requested to come up with a full description

93 of turbulent flow properties in the framework of zonostrophic systems. However, planetary-like
 94 jets have been reproduced mainly in numerical models with simplifying assumptions and
 95 limitations (Cho and Polvani 1996, Jones and Kuzanyan 2009). Jets dynamics is sometimes
 96 studied in idealized numerical models solving, in the framework of β -plane approximation,
 97 for 2D quasi-geostrophic turbulence in single (Scott and Dritschel 2012, 2013) or multiple
 98 (Berloff *et al.* 2011) planetary layers in vertical. Attractive alternative studies equally retrieve
 99 basic jets properties, in idealized 2D system with β -plane, by modeling kinetic energy transfer
 100 from small turbulent scales to large zonal scales using stochastic methods or closure models
 101 (Bouchet *et al.* 2013, Marston *et al.* 2016). It is then common, when a planetary like spherical
 102 configuration is considered, that the shallow scenario is modeled solving for three-dimensional
 103 general circulation model (GCM) of a thin shell in the outer atmosphere, assuming weak
 104 depth variations along the planetary radius (Schneider and Liu 2009). Numerical studies
 105 designed to simulate the deep model (in which rotation leads to axial flow invariance of
 106 a 3D-turbulent system) are only considered using hyper-diffusion or stress-free boundary
 107 conditions (Christensen 2001, Heimpel *et al.* 2005, 2016). Note that, contrary to shallow-layer
 108 models, where depth invariance is assumed and not produced by the dynamics, vertical
 109 motions including wave motions may play an important role in deep-layer 3D turbulent
 110 models (see e.g. Yarom and Sharon 2014). Previous laboratory studies (see for instance the
 111 recent studies by Di Nitto *et al.* 2013, Smith *et al.* 2014, Zhang and Afanasyev 2014, Read
 112 *et al.* 2015, Galperin *et al.* 2016) have reached weakly zonostrophic regimes (with $R_\beta < 2.5$).

113

114 In the present paper, we extend the analysis initiated by Cabanes *et al.* (2017) of the
 115 first laboratory/numerical model generating planetary-like zonal jets in a deep 3D-layer.
 116 In addition to a large-scale rotating experiment, Cabanes *et al.* (2017) performed a 3D
 117 turbulent direct numerical simulation (DNS) with a cylindrical β -topography designed to
 118 emulate part of a 3D convective planetary layer. Contrary to previous numerical simulations
 119 aforementioned, 3D Navier-Stokes equations have been solved in realistic experimental
 120 conditions using no-slip boundary conditions. Deep-seated jets have been observed with
 121 similar flow properties in the numerical simulation and in its laboratory realization. The
 122 strength and scale of this multiple jets system have been shown to be comparable relative to
 123 those observed on the gas planets. In the direct continuity of these first results, we propose
 124 here a phenomenological description of flow statistics in this high resolution direct numerical
 125 simulation referred to as DNS1. In order to investigate the specific role of the β -topography
 126 and its impact on turbulent flow properties, we run a second DNS using similar dynamical
 127 conditions but without layer depth variation (labeled as DNS2). Spatial and temporal
 128 spectral flow description, comparatively in the β and non- β simulations, allows us to extract
 129 the signatures of the zonostrophic turbulence in a fully 3D system with realistic dissipative
 130 effects. In particular, we evidence the peculiar correlation of the vertical and radial flow
 131 components under topographic β -effect and report, for the first time, a signature of the
 132 zonostrophic regime in frequency spectra. Our findings may open new horizons to remotely
 133 investigate the dynamics of gas giants' zonal winds using phenomenological description of
 134 turbulent flow properties.

135

136 2. Numerical setups

137 We consider the motion of an incompressible fluid with constant kinematic viscosity ν and
 138 constant density ρ . The fluid is contained inside a circular cylinder of radius R rotating with a
 139 constant rate Ω around the vertical axis. In our first model DNS1 (see figure 1(a)), the vertical
 140 extend of the fluid layer $h(r)$ is axisymmetric and corresponds to the analytical paraboloid

141 shape of a free surface that depends on the rotation rate and on the gravitational acceleration
142 g given by

$$h(r) = h_0 - \frac{\Omega^2 R^2}{4g} \left(1 - 2\frac{r^2}{R^2}\right), \quad (1)$$

143 where h_0 is the fluid height in the absence of rotation. This upper paraboloidal surface is
144 assumed to be stress-free whereas both side and bottom boundaries are no-slip. This appro-
145 priately models the experimental setup described in Cabanes *et al.* (2017) and also mimics
146 topographic variations near planetary poles. The center, with minimum fluid height, corre-
147 sponds to the pole, while the edge of the domain corresponds to lower latitudes. The simulation
148 is set so that the nature of the β parameter (i.e., cylindrically-radial topographic variations)
149 and the large dynamical jets' aspect ratio (i.e., vertical jets' depth larger than their horizontal
150 width L_{Rh}) reproduce a deep planetary layer. We have also run a second DNS (labeled as
151 DNS2 in the following), dynamically similar to the first run DNS1 but this time with a top
152 flat stress-free surface. All parameters characterizing the system remain the same, but the
153 uniform layer depth is now h_0 , which corresponds to the limit case $\beta = 0$. This second case
154 is run as a reference, as there is no topographic effect in that case. It is also close to other
155 rotating turbulence experiments, as for instance by Yarom and Sharon (2014). For the two
156 cases, energy is injected through a small-scale steady horizontal volumic force corresponding
157 to Taylor-Green vortices. The forcing is given by the following formula:

$$\mathbf{F} = \begin{cases} \begin{bmatrix} AS(r) \sin(\pi k_f x) \cos(\pi k_f y) \sin(\pi k_f z) \\ AS(r) \cos(\pi k_f x) \sin(\pi k_f y) \sin(\pi k_f z) \\ 0 \end{bmatrix} & \text{if } 0 \leq z \leq 1/k_f, \\ \mathbf{0} & \text{elsewhere.} \end{cases} \quad (2)$$

158 The function $S(r)$ is given by

$$S(r) = \frac{1}{2} [\tanh(60(r - 0.15)) - \tanh(60(r - 0.92))], \quad (3)$$

159 and is used to avoid the generation of intense vortical structures close to the outer boundary
160 and at the center of the cylindrical domain. A is the ratio of the force amplitude to the
161 centrifugal acceleration and can be seen as a forcing Rossby number ($A = 2 \times 10^{-3}$ in the
162 following). We focused on the configuration where the forcing is at relatively small scales
163 (i.e., with a typical horizontal wavenumber $k_f = 12$) to ensure scale separation between the
164 injection of the energy and the jet scale, but not too small so that the local Reynolds number
165 is still large. Note that it is very demanding numerically to allow for a scale separation
166 between the injection and the large-scale flows, while solving at the same time for the inertial
167 range associated with the three dimensional direct cascade. Contrary to previous numerical
168 simulations, designed to model deep-seated jets with negligible role of viscous dissipation,
169 we solve for a viscous flow with realistic no-slip boundary conditions applied on the bottom
170 and side walls. In particular, multiple banded flows are not found in the most recent,
171 high-resolution models that couple the molecular envelope to the deeper dynamo region.
172 In these models, bottom dissipation damps the higher-latitude deep jets out of existence
173 (Heimpel and Gómez Pérez 2011, Gastine *et al.* 2014, Jones 2014). Here, long-lived jets can
174 persist even under conditions including viscous dissipation and no-slip conditions.

175

176 We solve the Navier-Stokes equations in the rotating frame using the spectral element solver
177 Nek5000 (Fischer *et al.* 2007, 2008). Variables are decomposed into Lagrange polynomials
178 based on Gauss-Lobatto-Legendre quadrature points inside hexahedral elements. For both
179 simulations, we use polynomials of order 11 so that each element contains 12^3 grid points.
180 The original mesh corresponds to a full circular cylinder of unit radius and unit height with
181 53760 elements. In order to match the upper paraboloidal boundary in the DNS1 model, we

182 multiply the vertical coordinate of each element by the local required fluid depth. The mesh
 183 is also refined close to all boundaries. We use a third order explicit extrapolation scheme for
 184 the nonlinear convective terms and the linear Coriolis and forcing terms, and a third-order
 185 implicit Backward Difference scheme for the linear diffusive terms. The code is parallelized
 186 using MPI and we typically used 2048 processors for both simulations. In order to gather
 187 the appropriate conditions for generating dominant zonal flows, we consider the following set
 188 of dimensionless parameters. The Ekman number (i.e., ratio of viscous and Coriolis forces,
 189 defined here as $E = \nu/(2\Omega h_0^2)$) is fixed at $E = 5 \times 10^{-7}$, the cylinder radius is equal to
 190 the fluid averaged depth $h_0 = R$ and the ratio of centrifugal to gravitational accelerations,
 191 relevant for case DNS1, is $\Omega^2 R/g = \pi$. These parameters lead to the geometries and flows
 192 shown in figure 1.

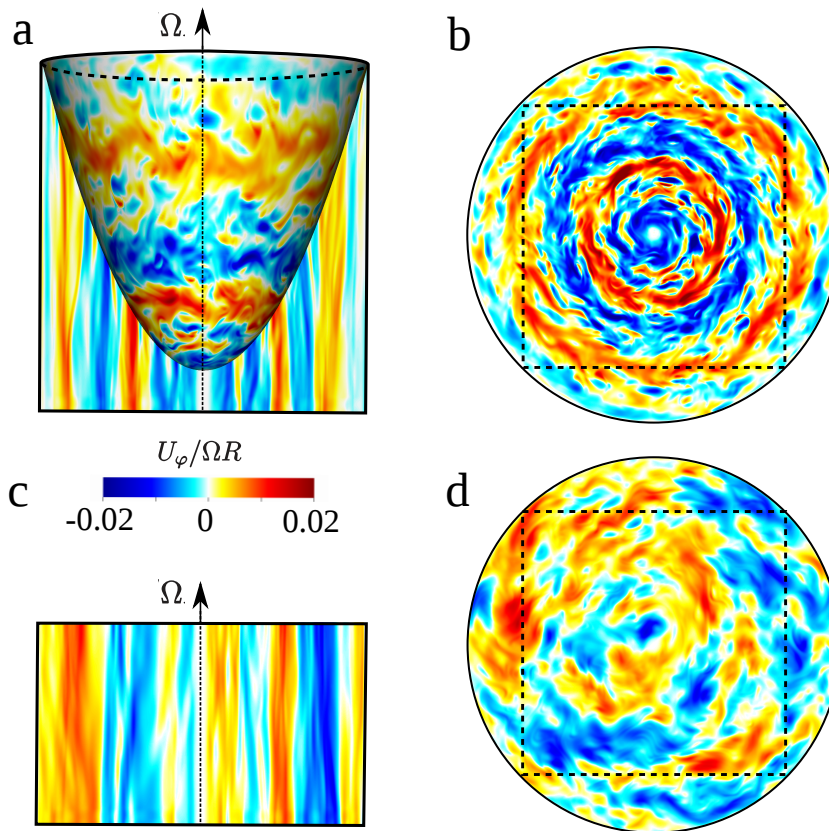


Figure 1. Sidecut and top view of DNS1 (a-b) and DNS2 (c-d) after $t = 1000$ rotations showing the instantaneous non-dimensional azimuthal velocity field at $E = 5 \times 10^{-7}$, $Ro = 2 \times 10^{-3}$ and $Re = 4 \times 10^3$, based on the rms flow velocity and the averaged fluid depth. The dashed squares in (b) and (d) show the location where spectral analysis is performed (see section 4).

193 3. Global properties and visualizations

194 The time evolutions of the rms values of all three velocity cylindrical components (u_r, u_ϕ, u_z)
 195 are shown in figure 2. After a very short transient phase where all components grow rapidly
 196 (few rotation times), a secondary transient growth occurs in both cases on much longer
 197 timescales (typically several hundreds of rotation times) until complete saturation eventu-
 198 ally occurs. In case DNS1, this slow secondary growth only affects the azimuthal compo-
 199 nent whereas radial and vertical components are similar and saturate quickly around a lower

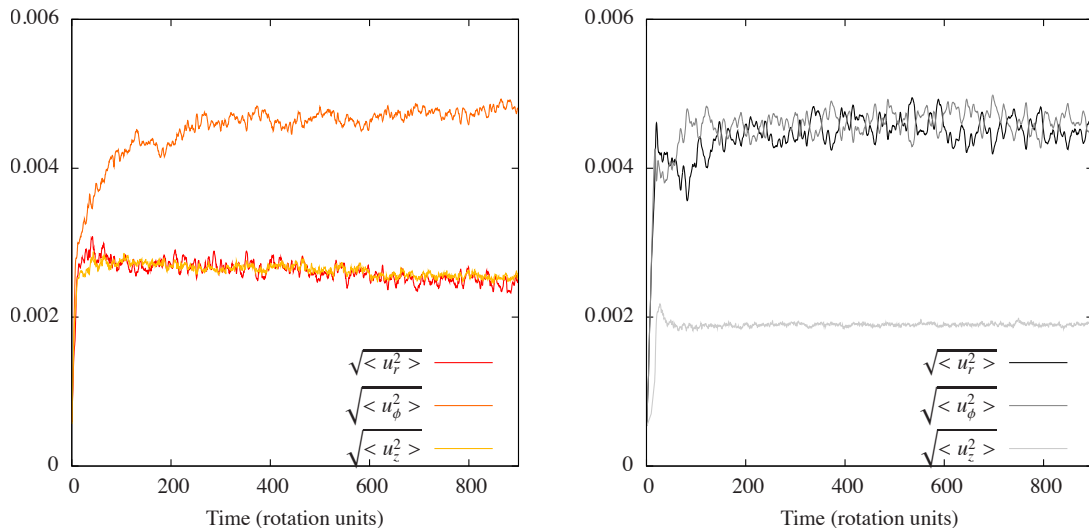


Figure 2. Time evolution of the rms velocity components for the case DNS1 with topographical effects (left) and the case DNS2 without topographical effects (right). (Colour online)

200 value. In case DNS2, both horizontal components are similar and saturate at large amplitude
 201 whereas the vertical component behaves differently and saturates at lower amplitude. This
 202 is the first clear distinction between the so-called zonostrophic regime when compared with
 203 the regular rapidly-rotating regime: the anisotropy induced by the rotation does not affect
 204 all velocity components in the same way. Without topographical effects, the horizontal flow
 205 remains isotropic whereas a clear zonal anisotropy is observed when topographical effects are
 206 important.

207 We show in figure 1 sidecut and top views, after 900 rotations, of the instantaneous zonal
 208 velocity field. For case DNS1, the flow displays five prograde/retrograde jets (red/blue), with
 209 strong, instantaneous signature, and with similar properties to those observed on the gas
 210 planets (see Cabanes *et al.* 2017, for details). More than 50% of the total surface kinetic
 211 energy concentrates in the zonal velocity component. This large scale surface flow is seated
 212 on a deep, nearly invariant geostrophic flow, confirming that the system is quasi-2D under
 213 dynamical conditions ($Ro \ll 1$). Case DNS2 is also nearly invariant along the rotation axis
 214 but does not display such axisymmetry. Large-scale flows are still apparent but irregularly
 215 distributed across the horizontal plane.

216
 217 More quantitatively, the establishment of the zonostrophic regime in DNS1 has been
 218 argued in detail by Cabanes *et al.* (2017) and can be summed-up as follows: using the rms of
 219 the total velocity U_{rms}^T , the estimate of the Reynolds $Re = U_{rms}^T h_o / \nu = 10^3$ and the Rossby
 220 $Ro = U_{rms}^T / 2\Omega h_o = 10^{-3}$ non-dimensional numbers attests that parameters range achieved
 221 in DNS1 and DNS2 is suitable to grow large-scale structures in 3D-turbulent flows. The
 222 additional β -topography in DNS1 leads to the existence of a meaningful zonostrophic inertial
 223 range whose width is estimated by the zonostrophic index $R_\beta \approx L_{Rh} / L_\beta$. The total rms
 224 velocity U_{rms}^T and the mean value $\beta = 73 \text{ m}^{-1} \text{ s}^{-1}$ are used to estimate Rhines wavenumber

225 $k_{Rh} = 1/L_{Rh}$. The transitionnal wavenumber $k_\beta \approx 1/L_\beta$ is defined using a revised expression
 226 $k_\beta = 0.73(\beta^3/\epsilon)^{1/5}$ found in Galperin *et al.* (2014) and confirmed in Cabanes *et al.* (2017).
 227 The upscale energy transfer rate is estimated as $\epsilon \simeq (U_{rms}^T)^2/2\tau_E$ based on the total kinetic
 228 energy, the Ekman spinup-time $\tau_E = h_o/\sqrt{\nu\Omega}$ (Read *et al.* 2015) and it has been shown in
 229 Cabanes *et al.* (2017) that this estimate for ϵ gives good predictions of the inverse energy
 230 transfer rate recovered in kinetic energy spectra. The resulting zonostrophic inertial range,
 231 delimited by typical wavenumbers k_{Rh} and k_β , is reported figure 3(a). A typical estimate
 232 for the zonostrophic index gives $R_\beta \approx 3.4$ (this index goes up to 3.7 in the laboratory, more
 233 calculation details are given in Cabanes *et al.* (2017)). This value significantly exceeds the
 234 2.5 threshold required to set the zonostrophic regime.

235

236

Let us first discuss the zonal spectrum computed from spectral decomposition of the ax-
 237 isymmetric component of the flow, here based on the time and azimuthal average of the zonal
 238 velocity. As shown in figure 3a, we recover the theoretical prediction $\overline{E}_\varphi(k_\perp) = C_\varphi\beta^2k_\perp^{-5}$,
 239 with k_\perp the orthogonal wavenumber, perpendicular to the azimuthal component, and $C_\varphi = 2$
 240 the zonal constant (Sukoriansky *et al.* 2002, 2007, Galperin *et al.* 2014). To distinguish
 241 between the axisymmetric and non-axisymmetric components of the flow, one defines the
 242 origine to be the rotation spin axis for spectral analysis. The axisymmetric flow is then a one
 243 dimensional profile along the radius in our cylindrical geometry. Following this approach, the
 244 orthogonal wavenumber corresponds to the radial wavenumber, $k_\perp = k_r$. We note here that
 245 cylindrical harmonic decomposition of the flow would have been more appropriated than time
 246 and azimuthal average of the velocity field to compute the axisymmetric flow component.
 247 However, to facilitate comparison with spectral analysis in the laboratory reported in Cabanes
 248 *et al.* (2017) (which was performed in Cartesian coordinates due to the lack of a complete
 249 cylindrical field) we decided to adopt the same data processing for statistical description
 250 of the flow. We ascertain however that both approaches lead to comparable results. Thus,
 251 consistently with aforementioned studies (Sukoriansky *et al.* 2002, 2007, Galperin *et al.*
 252 2014), kinetic energy distribution follows a k_r^{-5} slope on both sides of the transitional
 253 wavenumber, it achieves maximum values at large scales and drops at wavenumbers smaller
 254 than the theoretical Rhines wavenumber. This characteristic spectrum of zonostrophic flows
 255 has been commented in details by Cabanes *et al.* (2017) and confirms that the present model
 256 is comparable with Jupiter's jets flow. In the flat geostrophic case (DNS2), we verify that
 257 energy of the time and azimuthal average of the zonal velocity is two orders of magnitude
 258 smaller than in case DNS1 and that Rhines scale is not a relevant scale in the dynamics.
 259 DNS2 is in the rapidly-rotating but non-zonostrophic regime ($Ro \ll 1$, $Re \gg 1$ and $\beta = 0$).
 260 These two simulations set the way to investigate the role of the topographic β -effect in
 261 3D-turbulent rotating flows and to characterize the specificity of the deep-layer scenario.
 262 Based on Cabanes *et al.* (2017), who attested the establishment of a zonostrophic regime in
 263 DNS1, we now extend the study to a full spatial and temporal statistical characterization of
 264 the three components of velocity fluctuations.

265

266 4. Azimuthal fluctuations

267 Because the velocity field is quasi-invariant with depth, we restrict our flow description
 268 to a horizontal plane normal to the rotation axis, specifically $z = 0.15$. Figure 1(b) and
 269 (d) shows horizontal cuts of the zonal velocity. At steady state, we observe large scale
 270 non-axisymmetric structures when the β -topography is absent (see DNS2 figure 1(d)). On the
 271 contrary, a paraboloidal surface preferentially channels energy into the zonal direction leading
 272 to prograde/retrograde jets formation (see DNS1 figure 1(b)). To facilitate the spectral

273 analysis of the velocity fluctuations, we extrapolate them with spectral accuracy on a high
 274 resolution 512×512 Cartesian square grid (dashed lines in figure 1). 2D spectral analyses
 275 are computed along both Cartesian coordinates (x, y) to obtain energetic distribution of the
 276 zonal u_φ , radial u_r and vertical u_z flow components as a function of the total wavenumber
 277 $k = \sqrt{k_x^2 + k_y^2}$, with k_x and k_y the two Cartesian wavenumbers.

278

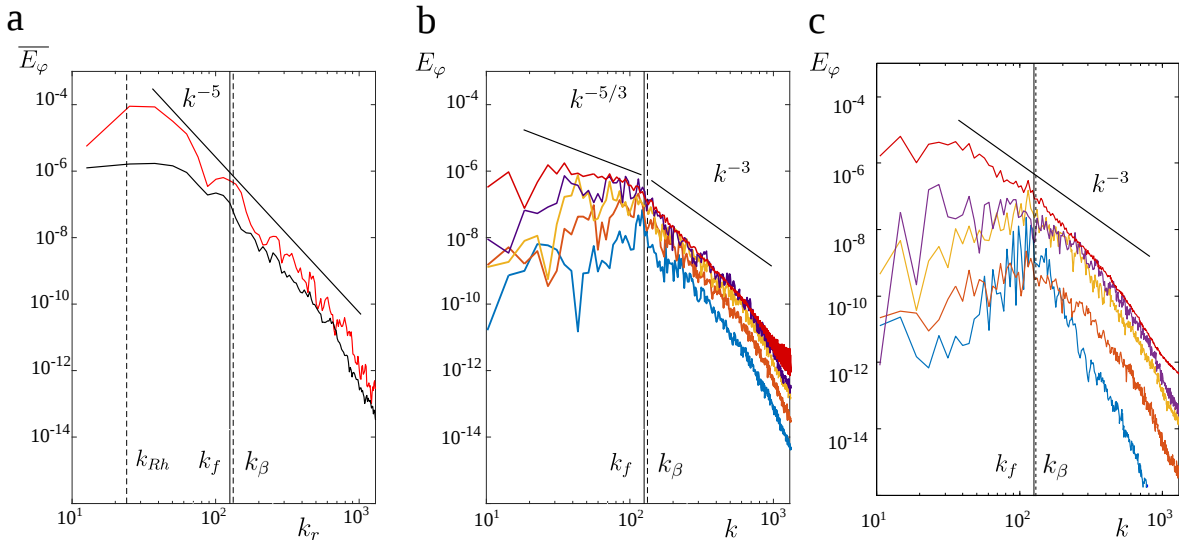


Figure 3. (a) Kinetic energy spectra of the time and azimuthal averaged zonal velocity field for DNS1 (red solid line) and DNS2 (black solid line). The theoretical energy spectrum $\overline{E_\varphi}(k_r)$ for the zonostrophic regime (Sukoriansky *et al.* 2007) is represented by a black straight line as a function of the radial wavenumber k_r . Theoretical values of the Rhines k_{Rh} , the transitional k_β and the injection k_f wavenumbers are respectively shown as dashed grey, vertical dashed black, and solid lines. Typical value of k_β is estimated using $\epsilon \simeq (U_{rms}^T)^2 / 2\tau_E$, with τ_E the Ekman spin-up-time. (b-c) 2D zonal kinetic energy spectra E_φ as a function of the total wavenumber k . Power spectra are given at different times $t = 0, 5, 8, 31$ and 1000 rotations (blue, orange, yellow, purple and red) for DNS1 (b) and DNS2 (c). We also report the injection wavenumber k_f as a vertical line and the transitional wavenumber k_{beta} as a vertical dashed line. (Colour online)

279 Figure 3(b) and (c) show instantaneous 2D-spectra of zonal fluctuations, at different time
 280 steps from initial forcing to steady state. Fluctuations are obtained by subtracting the time
 281 averaged flow to the instantaneous velocity field in our Cartesian grid. We ascertain here
 282 that temporal and azimuthal average are equivalent in our Cartesian grid to extract the
 283 axisymmetric flow component. At initial time, energy is injected from basal forcing at scale
 284 k_f (blue curves). Temporal evolution of the zonal spectra clearly shows how energy transits
 285 in time from initial injection at small-scale to the formation of large-scale structures. With
 286 topographic β -effect, narrow but clear inertial ranges develop on both sides of the forcing scale
 287 following the double cascade scenario predicted by Kolmogorov-Batchelor-Kraichnan (KBK)
 288 (Kraichnan 1967) in 2D-turbulence (figure 3(b)). The existence of two inviscid invariants
 289 (energy and enstrophy) of the 2D Navier-Stokes equations implies an inverse cascade of
 290 kinetic energy to large scales ($k < k_f$) with a power-law of $E(k) \propto k^{-5/3}$, and a direct cascade
 291 of enstrophy to small scales ($k > k_f$) inducing the power-law $E(k) \propto k^{-3}$. These features are
 292 well retrieved in the instantaneous azimuthal velocity of our 3D system, which is dynamically
 293 quasi-2D because of the effect of strong rotation. This double cascade scenario has previously
 294 been observed in 2D turbulence numerical simulations (Boffetta and Musacchio 2010) as well
 295 as in forced rotating flows (Campagne *et al.* 2014). It is now reported in the present work
 296 when topographical effects are dominant, although we are limited by the relatively small scale
 297 separation between the energy injection scale and the container's typical length scale. Indeed,
 298 we note that previous statistical analyses of the zonostrophic regime have been reported in

299 studies solving for 2D vorticity equations (Galperin *et al.* 2010, Sukoriansky *et al.* 2012). In
 300 these studies, the injection wavenumber k_f is much larger than the transitional wavenumber
 301 k_β (namely $k_f \gg k_\beta$). The inverse cascade consequently develops on a much wider inertial
 302 range $k < k_f$, leading to a $k^{-5/3}$ spectrum spreading on both sides of the k_β wavenumber. In
 303 the present work, injection and transitional wavenumbers are approximatively identical, due
 304 to numerical constraints, leading the inverse cascade to develop on wavenumbers $k < k_\beta \approx k_f$
 305 and the direct cascade to develop on wavenumbers $k > k_\beta \approx k_f$. This lack of scale separation
 306 is waiting for extended numerical capacity to increase k_f , while maintaining turbulence at
 307 the injection scale, and finally disclose the dynamics around the transitional wavenumber k_β
 308 in 3D-turbulent zonostrophic flows. Similarly, the k^{-5} slope on wavenumbers $k > k_\beta$ observed
 309 Figure 3(a) is disputable as it occurs beyond the injection wavenumber k_f .

310

311 When no topographic effect is present (figure 3(c)), the inverse cascade still transfers
 312 energy to large scale vortices. There is however no equivalent to the Rhines scale so that
 313 the cascade continuously feeds large-scale flows that finally saturate due to frictional effect
 314 on the lateral and bottom rigid no-slip boundaries. The resulting kinetic energy spectrum
 315 is apparently modified compared to the case with topography, and we observe instead a
 316 slope closer to k^{-3} , although we cannot be fully conclusive since there is less than a decade
 317 available. One possible explanation is the presence of so-called condensates formed by a 2D
 318 turbulence inverse cascade in close or periodic containers (Chertkov *et al.* 2007) that follow
 319 a steeper power-law $E(k) \propto k^{-3}$. Note that in the context of forced rotating turbulence at
 320 Rossby and Reynolds number similar to the ones considered here, both $k^{-5/3}$ and k^{-3} slopes
 321 have been observed in the inverse cascade range depending on the anisotropy of the forcing
 322 (Sen *et al.* 2012, Pouquet *et al.* 2013). In addition, the large-scale friction due to the side
 323 boundary might affect differently the non-axisymmetric flow without topography compared
 324 to the dominantly axisymmetric flow in the presence of topography, thus leading to different
 325 spectral slopes at large-scales. The reason why this finite size effect does not occur on the
 326 azimuthal component of the zonostrophic regime has to be investigated, but may relate
 327 to the dynamical, non-rigid, confinement imposed by the Rhines scale (Vallis and Maltrud
 328 1993). Accurate determination of the slope remains to be validated using larger numerical
 329 simulations (i.e. larger scale separation) and laboratory experiments.

330

331 5. Radial and vertical fluctuations

332 Figure 4(a) reports 2D spectra at steady state of the vertical and radial fluctuations (note that
 333 time averaged radial and vertical flows are both negligible) in the horizontal planes displayed
 334 in figure 1. With topographic β , the amplitude of those two components is much smaller
 335 than the azimuthal one (consistent with figure 2(a)). Also, both radial and vertical velocity
 336 fluctuations are limited to scales that do not exceed Rhines wavenumber. On the contrary, the
 337 large scale vortices formed when the top surface is flat have the same signature on the radial
 338 and zonal component: we retrieve a $E(k) \propto k^{-3}$ power-law behavior (see figure 3(c) and 4(a)).

339

340 Interestingly, with topographic β , vertical velocity component is slightly more energetic
 341 than in the flat case at all scales (dashed lines in figure 4). It traduces that radial motions,
 342 when impacting the paraboloidal topography, convert into vertical motions due to the
 343 non-penetrative condition of the topographic surface. Consequently, radial and vertical
 344 velocities are correlated. To attest this spatial correlation we report in figure 4(b) and
 345 (c) the joint probability distribution functions (PDF) of the vertical and radial velocities,
 346 with and without topography. Without topography, the PDF plotted in figure 4(c) shows

347 that vertical and radial motions are not correlated. Under strong rotation and without
 348 topographical variations in a 3D rapidly-rotating system, the vertical velocity component
 349 actually behaves as a quasi-passive scalar. However, when a paraboloidal topography is
 350 imposed, the PDF shown in figure 4(b) shows that positive/positive and negative/negative
 351 values of the radial/vertical velocities are correlated. In other words, inward (outward)
 352 radial motions, when they impact (move away from) the topography, convert into downward
 353 (upward) vertical motions. We note that these correlations are maximum for radius where β
 354 is maximum (not shown). Joint PDF of the other velocity components are not reported in
 355 this manuscript as they do not display specific correlations.

356

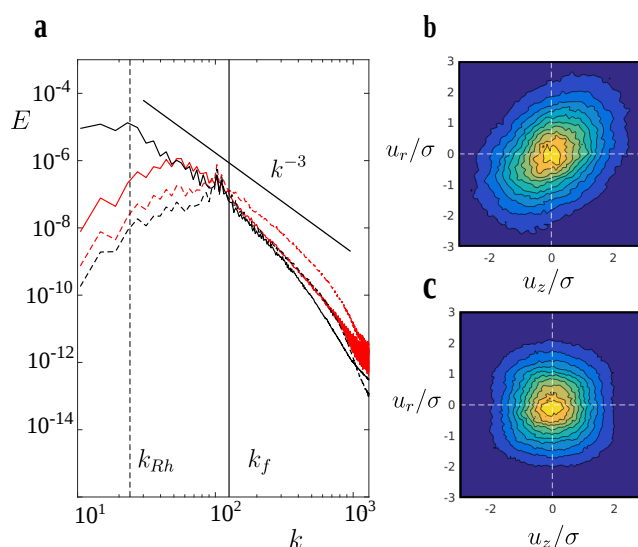


Figure 4. (a) 2D kinetic energy spectra of the radial (solid lines) and vertical (dashed lines) velocity field components, computed from the square grid defined figure 1. Power spectra have been time averaged once steady state is reached. DNS1 and DNS2 are plotted respectively in red and black. k_f and k_{Rh} are respectively the injection and Rhines wavenumbers. Right: joint probability distribution functions normalized by standard deviation σ of the vertical and radial velocity components of the fluctuations for DNS1 (b) and DNS2 (c). (Colour online)

357 These correlations of the radial and vertical flows are a specificity of 3D-rotating turbulence
 358 under topographical β -effect, as it is the case in 3D-convective planetary layers. Similar flow
 359 measurements on gas planets' surface may highlight the dynamics underlying the jets. Note
 360 that velocity correlations are obtained from a numerical simulation where the free surface
 361 is not deformable. In the laboratory, the iso-pressure paraboloidal free-surface is non-rigid
 362 and deformations can occur (typically of a few *mm* for maximum radial velocities). Even
 363 if these deformations appear to be weak, it may attenuate the observed correlation. Direct
 364 measurements of velocity correlations in the laboratory will be further required to confirm
 365 our results.

366 6. Frequency spectra

367 We now focus on the frequency spectra and on the possible signatures associated with the
 368 observed large-scale flows. To complete the description of our system and extract all its
 369 specificities, we report in figure 5 frequency spectra of the instantaneous velocity for the
 370 three components of the flow. Frequency spectra result from time series spanning over 1000

371 rotations and are averaged in the spectral domain over 7500 probes, located in the horizontal
372 plane displayed in figure 1.

373

374 These frequency spectra can be split into two frequency ranges separated by the cutoff
375 frequency $\omega = 2\Omega$ of the inertial waves. In the range $\omega > 2\Omega$, the dynamics evolve on short
376 time scales and the effect of rotation is moderate. Turbulent energy is nearly isotropically
377 distributed over the three components of the flow (see figure 5). We notice that these
378 high frequency velocities are only weakly affected by the β -topography: all spectra of both
379 simulations are almost indistinguishable. We observe a ω^{-3} power law tentatively explained
380 by a Taylor hypothesis: the high-frequency spectrum corresponds to the advection by the
381 large-scale quasi-steady energetic patterns of the direct cascade described in figure 3.

382

383 The low frequency range $\omega < 2\Omega$, where rotation dominates (i.e., low Rossby number) and
384 3D inertial waves develop, shows the most striking signature of the β effect. In simulation
385 DNS2, when no β -topography is present, rotation acts to separate the horizontal flow from the
386 vertical flow (Campagne *et al.* 2015). This horizontal anisotropisation is well observed
387 in the frequency spectra figure 5(b) and was already reported in the spatial spectra. In
388 simulation DNS1, where a β -topography is imposed, the flow undergoes both zonal and
389 horizontal anisotropisation in the range of inertial wave frequencies. Zonal flow dominates
390 over the radial flow component, which in turn, dominates the vertical flow component. These
391 two successive component anisotropies and their frequency spectrum signature are a novelty
392 of our study. Once confirmed from further data analysis, it could establish a new diagnostic
393 of zonostrophic flow tool enabled by our high resolution, turbulent, 3D numerical simulations.

394

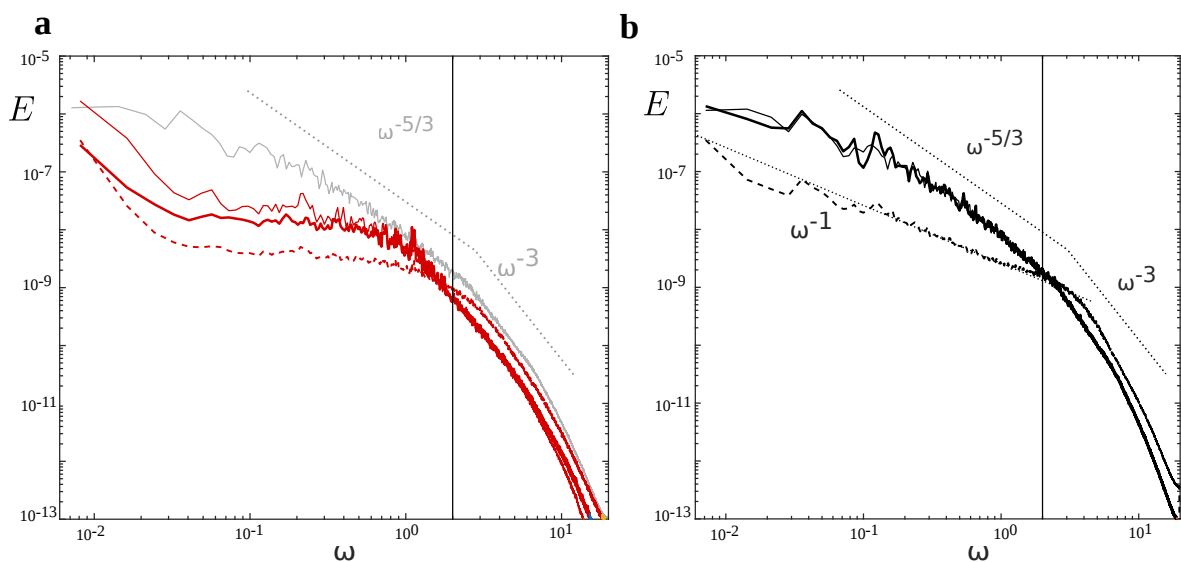


Figure 5. Frequency energy spectra computed from 1000 rotations time series of the zonal (thin solid lines), radial (thick solid lines) and vertical (dashed lines) velocity field components for DNS1 (a) and DNS2 (b). Note that for comparison purpose, we also report on a), the frequency spectra of the zonal component obtained from DNS2 (grey solid line). (Colour online)

395

396 A striking feature of these frequency spectra is the energetic plateau within the range of
397 inertial waves frequency, when a β -topography is imposed (figure 5(a)). The particularity
398 of a β -topography in rotating system is that it injects potential vorticity and generates
399 Rossby waves that propagate in the zonal direction. Our observations indicate that such a
dynamics strongly affects turbulent statistics in a way that energy is equally distributed over

400 a large frequency range. Similar features are observed in frequency spectrum of simplified
 401 2D global circulation models used to simulate planetary atmospheres (James *et al.* 1994).
 402 We note that the lowest frequencies in our spectra, which reflect the stationary zonal jets,
 403 correspond to a maximum of energy. In the absence of β -topography, frequency spectra follow
 404 a ω^{-1} power-law for vertical velocity and for the lowest frequencies of horizontal velocities
 405 (figure 5(b)). The ω^{-1} power-law behavior is a prevalent feature in turbulent flows related to
 406 the statistical properties of coherent structures (Herauld *et al.* 2015). The horizontal velocities
 407 at intermediate frequencies exhibit a nearly $\omega^{-5/3}$ power-law, in agreement with the recent
 408 laboratory studies of Yarom and Sharon (2014) and Campagne *et al.* (2015).

409

410 7. Conclusions

411 Our phenomenological description of turbulent properties in the so-called zonostrophic
 412 regime, and comparatively with the geostrophic (non-zonostrophic) regime, constitutes a
 413 coherent and advanced analysis of the first numerical/laboratory model for deep-seated
 414 planetary jets presented in Cabanes *et al.* (2017). We report spectral evidences that an
 415 inverse energy cascade occurs under rapid rotation in a 3D-turbulent cylindrical system. We
 416 recover several statistics observed in more classical regimes of forced rotating turbulence
 417 but also show some new results specific to the case with topography. The presence of a
 418 paraboloidal surface shape induces a strong zonal anisotropy of the turbulent flow similarly to
 419 deep convective planetary layers. It results that the dominant zonal flow structures into jets
 420 and that the statistical flow properties follow the double cascade scenario reminiscent of the
 421 2D turbulence paradigm (Kraichnan 1967), at least for the azimuthal flow velocity. Our high
 422 resolution numerical simulations allow us to access the flow in 3D and at all times. We observe
 423 that radial and vertical flow components are correlated due to topographical deformation of
 424 the top surface. Power spectrum of the vertical velocity field is consequently more energetic
 425 in the zonostrophic regime where radial motions are converted into vertical motions. This
 426 result constitutes an interesting signature of the deep-layer scenario. Ultimately, we report
 427 frequency spectra where all three components of the flow are strongly affected by the β -effect.
 428 This feature is probably due to the formation of 3D Rossby waves that are non-existent at
 429 constant fluid depth in DNS2. The upcoming data from current and future spacecraft missions
 430 will have to be analyzed in view of these spectral analyses that report statistical properties of
 431 a deep turbulent planetary layer. Additionally, more theoretical work is requested to explain
 432 this 3D zonostrophic regime that is now accessible to both experimental and numerical
 433 investigations.

434

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442 References

- 443 Bagenal, F., Dowling, T.E. and McKinnon, W.B., *Jupiter: the planet, satellites and magnetosphere*, Vol. 1,
444 2006 (Cambridge University Press).
- 445 Berloff, P., Karabasov, S., Farrar, J.T. and Kamenkovich, I., On latency of multiple zonal jets in the oceans.
446 *J. Fluid Mech.*, 2011, **686**, 534–567.
- 447 Boffetta, G. and Musacchio, S., Evidence for the double cascade scenario in two-dimensional turbulence. *Phys.*
448 *Rev. E*, 2010, **82**, 016307.
- 449 Bouchet, F., Nardini, C. and Tangarife, T., Kinetic theory of jet dynamics in the stochastic barotropic and 2D
450 Navier-Stokes equations. *J. Stat. Phys.*, 2013, **153**, 572–625.
- 451 Cabanès, S., Aurnou, J., Favier, B. and Le Bars, M., A laboratory model for deep-seated jets on the gas giants.
452 *Nature Physics*, 2017, **13**, 387–390.
- 453 Campagne, A., Gallet, B., Moisy, F. and Cortet, P.P., Disentangling inertial waves from eddy turbulence in a
454 forced rotating-turbulence experiment. *Phys. Rev. E*, 2015, **91**, 043016.
- 455 Campagne, A., Gallet, B., Moisy, F. and Cortet, P.P., Direct and inverse energy cascades in a forced rotating
456 turbulence experiment. *Phys. Fluids*, 2014, **26**, 125112.
- 457 Chertkov, M., Connaughton, C., Kolokolov, I. and Lebedev, V., Dynamics of energy condensation in two-
458 dimensional turbulence. *Phys. Rev. Lett.*, 2007, **99**, 084501.
- 459 Cho, J.Y. and Polvani, L.M., The morphogenesis of bands and zonal winds in the atmospheres on the giant
460 outer planets. *Science*, 1996, **273**, 335.
- 461 Christensen, U.R., Zonal flow driven by deep convection in the major planets. *Geophys. Res. Lett.*, 2001, **28**,
462 2553–2556.
- 463 Del Genio, A.D., Achterberg, R.K., Baines, K.H., Flasar, F.M., Read, P.L., Sánchez-Lavega, A. and Showman,
464 A.P., *Saturn atmospheric structure and dynamics*, 2009 (Springer).
- 465 Di Nitto, G., Espa, S. and Cenedese, A., Simulating zonation in geophysical flows by laboratory experiments.
466 *Phys. Fluids*, 2013, **25**, 086602.
- 467 Fischer, P.F., Loth, F., Lee, S., Lee, S., Smith, D. and Bassiouny, H., Simulation of high-Reynolds number
468 vascular flows. *Computer Methods in Applied Mechanics and Engineering*, 2007, **196**, 3049–3060.
- 469 Fischer, P.F., Lottes, J.W. and Kerkemeier, S.G., nek5000 Web page, [Http://nek5000.mcs.anl.gov](http://nek5000.mcs.anl.gov) 2008.
- 470 Galperin, B., Sukoriansky, S., Dikovskaya, N., Read, P., Yamazaki, Y. and Wordsworth, R., Anisotropic tur-
471 bulence and zonal jets in rotating flows with a β -effect. *Nonlinear Process. Geophys.*, 2006, **13**, 83–98.
- 472 Galperin, B., Hoemann, J., Espa, S., Di Nitto, G. and Lacorata, G., Anisotropic macroturbulence and diffusion
473 associated with a westward zonal jet: From laboratory to planetary atmospheres and oceans. *Phys. Rev. E*,
474 2016, **94**, 063102.
- 475 Galperin, B., Sukoriansky, S. and Dikovskaya, N., Geophysical flows with anisotropic turbulence and dispersive
476 waves: flows with a β -effect. *Ocean Dynamics*, 2010, **60**, 427–441.
- 477 Galperin, B., Young, R.M., Sukoriansky, S., Dikovskaya, N., Read, P.L., Lancaster, A.J. and Armstrong, D.,
478 Cassini observations reveal a regime of zonostrophic macroturbulence on Jupiter. *Icarus*, 2014, **229**, 295–320.
- 479 Gastine, T., Heimpel, M. and Wicht, J., Zonal flow scaling in rapidly-rotating compressible convection. *Phys.*
480 *Earth Planet. Inter.*, 2014, **232**, 36–50.
- 481 Heimpel, M., Aurnou, J. and Wicht, J., Simulation of equatorial and high-latitude jets on Jupiter in a deep
482 convection model. *Nature*, 2005, **438**, 193–196.
- 483 Heimpel, M., Gastine, T. and Wicht, J., Simulation of deep-seated zonal jets and shallow vortices in gas giant
484 atmospheres. *Nature Geoscience*, 2016, **9**, 19–23.
- 485 Heimpel, M. and Gómez Pérez, N., On the relationship between zonal jets and dynamo action in giant planets.
486 *Geophys. Res. Lett.*, 2011, **38**.
- 487 Hèrault, J., Pétrélis, F. and Fauve, S., Experimental observation of $1/f$ noise in quasi-bidimensional turbulent
488 flows. *Europhysics Letters*, 2015, **111**, 44002.
- 489 James, P., Fraedrich, K. and James, I., Wave-zonal-flow interaction and ultra-low-frequency variability in a
490 simplified global circulation model. *Quarterly Journal of the Royal Meteorological Society*, 1994, **120**, 1045–
491 1067.
- 492 Jones, C., A dynamo model of Jupiter's magnetic field. *Icarus*, 2014, **241**, 148–159.
- 493 Jones, C.A. and Kuzanyan, K.M., Compressible convection in the deep atmospheres of giant planets. *Icarus*,
494 2009, **204**, 227–238.
- 495 Kaspi, Y., Flierl, G.R. and Showman, A.P., The deep wind structure of the giant planets: Results from an
496 anelastic general circulation model. *Icarus*, 2009, **202**, 525–542.
- 497 Kraichnan, R.H., Inertial Ranges in Two Dimensional Turbulence. *Phys. Fluids*, 1967, **10**, 1417–1423.
- 498 Marston, J., Chini, G. and Tobias, S., Generalized quasilinear approximation: application to zonal jets. *Phys.*
499 *rev. Lett.*, 2016, **116**, 214501.
- 500 Maximenko, N.A., Bang, B. and Sasaki, H., Observational evidence of alternating zonal jets in the world ocean.
501 *Geophys. Res. Lett.*, 2005, **32** L12607.
- 502 Pouquet, A. and Marino, R., Geophysical Turbulence and the Duality of the Energy Flow Across Scales. *Phys.*
503 *Rev. Lett.*, 2013, **111**, 234501.
- 504 Pouquet, A., Sen, A., Rosenberg, D., Mininni, P. and Baerenzung, J., Inverse cascades in turbulence and the
505 case of rotating flows. *Physica Scripta*, 2013, **2013**, 014032.
- 506 Read, P., Jacoby, T., Rogberg, P., Wordsworth, R., Yamazaki, Y., Miki-Yamazaki, K., Young, R., Sommeria,

- 507 J., Didelle, H. and Viboud, S., An experimental study of multiple zonal jet formation in rotating, thermally
 508 driven convective flows on a topographic beta-plane. *Phys. Fluids*, 2015, **27**, 085111.
- 509 Rhines, P.B., Waves and turbulence on a beta-plane. *J. Fluid Mech.*, 1975, **69**, 417–443.
- 510 Schneider, T. and Liu, J., Formation of jets and equatorial superrotation on Jupiter. *J. Atmos. Sci.*, 2009, **66**,
 511 579–601.
- 512 Scott, R.K. and Dritschel, D.G., The structure of zonal jets in geostrophic turbulence. *J. Fluid Mech.*, 2012,
 513 **711**, 576–598.
- 514 Scott, R. and Dritschel, D., Halting scale and energy equilibration in two-dimensional quasigeostrophic turbu-
 515 lence. *J. Fluid Mech.*, 2013, **721**, R4.
- 516 Sen, A., Mininni, P.D., Rosenberg, D. and Pouquet, A., Anisotropy and nonuniversality in scaling laws of the
 517 large-scale energy spectrum in rotating turbulence. *Phys. Rev. E*, 2012, **86**, 036319.
- 518 Smith, C.A., Speer, K.G. and Griffiths, R.W., Multiple Zonal Jets in a Differentially Heated Rotating Annulus.
 519 *J. Phys. Oceanogr.*, 2014, **44**, 2273–2291.
- 520 Sukoriansky, S., Dikovskaya, N. and Galperin, B., On the arrest of inverse energy cascade and the Rhines scale.
 521 *J. Atmos. Sci.*, 2007, **64**, 3312–3327.
- 522 Sukoriansky, S., Dikovskaya, N., Grimshaw, R. and Galperin, B., Rossby waves and zonons in zonostrophic
 523 turbulence; in *AIP Conference Proceedings*, Vol. 1439, 2012, pp. 111–122.
- 524 Sukoriansky, S., Galperin, B. and Dikovskaya, N., Universal spectrum of two-dimensional turbulence on a
 525 rotating sphere and some basic features of atmospheric circulation on giant planets. *Phys. Rev. Lett.*, 2002,
 526 **89**, 124501.
- 527 Vallis, G.K. and Maltrud, M.E., Generation of mean flows and jets on a beta plane and over topography. *J.*
 528 *Phys. Oceanogr.*, 1993, **23**, 1346–1362.
- 529 Yarom, E. and Sharon, E., Experimental observation of steady inertial wave turbulence in deep rotating flows.
 530 *Nature Physics*, 2014, **10**, 510–514.
- 531 Zhang, Y. and Afanasyev, Y., Beta-plane turbulence: Experiments with altimetry. *Phys. Fluids*, 2014, **26**,
 532 026602.