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# The Fourier Laplace Approach of Proving The Riemann Hypothesis

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## Abstract

The Author presents a Fourier Laplace transformation of The Analytic Continuation Formula of The Riemann Zeta Function and showed that the zeros of the Analytic Continuation Formula will always be real. A general formula for these zeros is also obtained which when substituted into the ACF it is shown to be the root of the ACF of the Riemann Zeta Function.

**Keywords:** *Riemann zeta function; Fourier transform; Laplace transform; Analytic Continuation Formula (ACF).*

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## Introduction

Over the years, mathematicians have studied the Analytic Continuation Formula (ACF) of the Riemann Zeta Function by representing the exponential and the trigonometric functions in the ACF as Taylors Series. The author in this work chose to represent the exponential and the trigonometric functions as Fourier Cosine transform and Laplace transform respectively since Fourier and Laplace transformations have a meeting.

The Relationship between Fourier and Laplace transform can be explained as follows:

Consider the function

$$f(t) = \begin{cases} e^{-st}g(t) & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Then the Fourier transform of  $f(t)$  is given by

$$F\{f(t)\} = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{(is-x)t} g(t) dt = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-\rho t} g(t) dt ; \rho = (x - is) = \frac{1}{\sqrt{2\pi}} L\{g(t)\}$$

Thus, the Fourier transform of  $f(t) = \frac{1}{\sqrt{2\pi}} \times$  the Laplace transform of  $g(t)$  as defined above.

**(2.0) A closer look at the  $\varepsilon(t)$**

Riemann defined the analytical continuation formula as [1];

$$\varepsilon(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \int_1^{\infty} x^{-3/4} \psi(x) \cos\left(\frac{t}{2} \log x\right) dx \quad (1)$$

Such that

$$\psi(x) = \sum_{n \geq 1}^{\infty} e^{-n^2 \pi x} \quad (2)$$

The Fourier Cosine transform of (2) gives

$$\sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \quad (3)$$

Examining the trigonometric term in (1), it is obvious that one can write  $\cos\left(\frac{t}{2} \log x\right)$  in its

Laplace transform as;

$$\mathcal{L} \left[ \cos \left[ \left( \frac{1}{2} \log x \right) t \right] \right] = \frac{S}{\left( \frac{1}{2} \log x \right)^2 + S^2} \quad (4)$$

By injecting (3) and (4) into (1), one obtains:

$$\varepsilon(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \int_1^{\infty} x^{-3/4} \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \left[ \frac{S}{\left( \frac{1}{2} \log x \right)^2 + S^2} \right] dx \quad (5)$$

This becomes:

$$\frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \int_1^{\infty} x^{-3/4} \left[ \frac{S}{\left(\frac{1}{2} \log x\right)^2 + S^2} \right] dx \quad (6)$$

By using the Classical estimate [15]

$$\sum_{P \leq x} \frac{\log P}{P-1} = \log x + c + \theta(1) \quad (7)$$

$$\log x = \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \quad (8)$$

Substituting (8) into (6), one arrives at

$$\frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \int_1^{\infty} x^{-3/4} dx \quad (9)$$

If we take the limit of (9) as  $b \rightarrow \infty$ , we will have;

$$\lim_{b \rightarrow \infty} \varepsilon(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \lim_{b \rightarrow \infty} \int_1^b x^{-3/4} dx \quad (10)$$

On integrating (10), we obtain;

$$\frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \left[ 4x^{1/4} \right]_1^{b \leq \infty} \quad (11)$$

Looking closely at the bound of the Integrand in (11),  $b$  must be equal to  $x$  in the RHS. Thus, we have;

$$\frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \left[ 4b^{1/4} - 4 \right] \quad (12)$$

Since  $b = x$ , equation (12) gives;

$$\varepsilon(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right] \left[ 4x^{1/4} - 4 \right] \quad (13)$$

For  $\varepsilon(t) = 0$ , it implies that;

$$\begin{aligned} \frac{1}{2} &= \left(t^2 + \frac{1}{4}\right) \left[ 4x^{1/4} \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right. \\ &\quad \left. - 4 \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right] \quad (14) \end{aligned}$$

$$\begin{aligned} \frac{1}{2} &= \left(\frac{4t^2 + 1}{4}\right) \left[ 4x^{1/4} \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right. \\ &\quad \left. - 4 \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right] \quad (15) \end{aligned}$$

This results to:

$$\begin{aligned} 2 &= (4t^2 + 1) \left[ 4x^{1/4} \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right. \\ &\quad \left. - 4 \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right] \quad (16) \end{aligned}$$

On further simplification

$$\frac{2}{(4t^2 + 1)} = \left[ 4x^{1/4} \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left(\frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1)\right)^2 + S^2} \right] \right]$$

$$-4 \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \quad (17)$$

$$\frac{2}{\left[ 4x^{1/4} - 4 \right] \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right)} = (4t^2 + 1) \quad (18)$$

$$\frac{1}{\left[ 4x^{1/4} - 4 \right] \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right)} = \frac{1}{2} (4t^2 + 1) \quad (19)$$

This gives:

$$\frac{1}{\left[ 4x^{1/4} - 4 \right] \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right)} - \frac{1}{2} = 2t^2 \quad (20)$$

We can see that (20) is the same as:

$$\frac{1}{2} \left[ \left[ \left( 4x^{1/4} - 4 \right) \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right) \right]^{-1} - \frac{1}{2} \right] = t^2 \quad (21)$$

Making  $t^2$  the subject of (21), we obtain:

$$\frac{\left[ \left( 4x^{1/4} - 4 \right) \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right) \right]^{-1}}{2} - \frac{1}{4} = t^2 \quad (22)$$

and

$$2^{-1} \left[ \left( 4x^{1/4} - 4 \right) \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right) \right]^{-1} - \frac{1}{4} = t^2 \quad (23)$$

Thus

$$2^{-1} \left[ (4x^{1/4} - 4) \left( \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \left[ \frac{S}{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2} \right] \right) \right]^{-1} - 2^{-2} = t^2 \quad (24)$$

This means that

$$t^2 = \frac{2 \left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2}{2 \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] [4x^{1/4} - 4] S} - \frac{1}{4} \quad (25)$$

Such that:

$$t^2 = \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2}{4 \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] [(x^{1/4} - 1)] S} - \frac{1}{4} \quad (26)$$

One can now determine the general expression for  $t$ , the root of  $\varepsilon(t)$  from (26) such that:

$$t = \pm \left[ \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2}{4 \left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] [(x^{1/4} - 1)] S} - \frac{1}{4} \right]^{1/2} \quad (27)$$

We can write equation (27) as

$$t = \pm \left[ \frac{1}{4} A - \frac{1}{4} \right]^{1/2} \quad (28)$$

Such that

$$A = \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2}{\left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi}} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right] \right] [(x^{1/4} - 1)] S} \quad (29)$$

The author in his work [15] obtained a general formula for the zeros of the analytic continuation formula of the Riemann Zeta Function as:



$$t = \pm \left[ \frac{(z-1)\gamma(z)\pi^{-z/2} \prod_p (P^{-z} - 1)^{-1}}{\varepsilon_e k \sum_{d \leq n} 2n} - z(z-1) - \frac{1}{k} \right]^{1/2}; \text{ such } k = 4 \quad (30)$$

Where

$$\varepsilon_e = \gamma(z) \left[ \prod_p \left( \frac{1}{p^z} - 1 \right) \sum_{d \leq n}^{\infty} 2n \left( kz + \frac{G(t)}{(z-1)} \right) \pi^{z/2} \right]^{-1} \quad (31)$$

and

$$\gamma(z) = -\frac{z}{2}(z-1)\pi^{-z/2}\Gamma(z/2) \left[ \left( kz + \frac{G(t)}{(z-1)} \right) \pi^{z/2} \right] \quad (32)$$

If we equate equations (29) and (30), we will get;

$$t = \pm \left[ \frac{1}{4}A - \frac{1}{4} \right]^{1/2} = \pm \left[ \frac{(z-1)\gamma(z)\pi^{-z/2} \prod_p (P^{-z} - 1)^{-1}}{4\varepsilon_e \sum_{d \leq n} 2n} - z(z-1) - \frac{1}{4} \right]^{1/2} \quad (33)$$

We can show that:

$$A = \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2 + S^2}{\left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right]} \right] \left[ (x^{1/4} - 1) \right] S} = (z-1) \left[ \frac{\gamma(z)\pi^{-z/2} \prod_p (P^{-z} - 1)^{-1}}{\varepsilon_e \sum_{d \leq n} 2n} - z \right] \quad (34)$$

On further Simplification, we obtain:

$$A = \frac{1}{\left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right]} \right] \left[ (x^{1/4} - 1) \right]} \left[ \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2}{S} + S \right] \\ = (z-1) \left[ \frac{\gamma(z)\pi^{-z/2} \prod_p (P^{-z} - 1)^{-1}}{\varepsilon_e \sum_{d \leq n} 2n} - z \right] \quad (35)$$

Equation (35) can be elegantly rewritten as;

$$A = \frac{1}{\left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right]} \right] \left[ (x^{1/4} - 1) \right]} \left[ \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2}{S} + S \right]$$

$$= (z - 1) \left[ \frac{\gamma(z)}{\varepsilon_e \pi^{z/2} \prod_p (P^{-z} - 1) \sum_{d \leq n} 2n} - z \right] \quad (36)$$

Since

$$\frac{\gamma(z)}{\varepsilon_e} = \prod_p \left( \frac{1}{p^z} - 1 \right) \sum_{d \leq n} 2n \left( kz + \frac{G(t)}{(z-1)} \right) \pi^{z/2} \quad (37)$$

Then RHS of equation (36) equals to:

$$(z - 1) \left[ (k - 1)z + \frac{G(t)}{(z - 1)} \right]_{k=4} \quad (38)$$

Thus by this equation (35) gives:

$$A = \frac{1}{\left[ \sum_{n \geq 1}^{\infty} \sqrt{\frac{2}{\pi} \left[ \frac{n^2 \pi}{n^2 \pi + S^2} \right]} \right] \left[ (x^{1/4} - 1) \right] \left[ \frac{\left( \frac{1}{2} \sum_{P \leq x} \frac{\log P}{P-1} - c - \theta(1) \right)^2}{S} + S \right]} = (z - 1) \left[ (k - 1)z + \frac{G(t)}{(z - 1)} \right]_{k=4} \quad (39)$$

The followings are the Numerical values of A for the first fifty zeros of the Riemann Analytic

Continuation Formula.

S/N	A	t	S/N	A	t
1	800.1618191	14.134725142	26	34220.00572	92.491899271
2	1768.704601	21.022039639	27	35836.50771	94.651344041
3	2503.171988	25.010857580	28	36765.71402	95.870634228
4	3703.692348	30.424876126	29	39071.41980	98.831194218
5	4339.873125	32.935061588	30	41062.22773	101.317851006
6	5651.883152	37.586178159	31	43036.94893	103.725538040
7	6698.366262	40.918719012	32	44476.96121	105.446623052
8	7509.941116	43.327073281	33	45941.44482	107.168611184
9	9218.978044	48.005150881	34	49311.23101	111.029535543
10	9910.737595	49.773832478	35	50064.75739	111.874659177
11	11224.41983	52.970321478	36	52277.45163	114.320220915
12	12745.71551	56.446247697	37	54035.56485	116.226680321
13	14089.28653	59.347044003	38	56446.00031	118.790782866
14	14803.02111	60.831778525	39	58923.82897	121.370125002
15	16959.57357	65.112544048	40	60464.69124	122.946829294

16	17999.80392	67.079810529	41	61760.02777	124.256818554
17	19347.80796	69.546401711	42	65043.01859	127.516683880
18	20775.70086	72.067157674	43	67163.56233	129.578704200
19	22925.80077	75.704690699	44	68736.92831	131.087688531
20	23806.30539	77.144840069	45	71287.58335	133.497737203
21	25178.67630	79.337375020	46	72638.26763	134.756509753
22	27497.52501	82.910380854	47	76305.16423	138.116042055
23	28721.41508	84.735492981	48	78105.83231	139.736208952
24	30573.71456	87.425274613	49	79664.60316	141.123707404
25	31549.23293	88.809111208	50	81925.00163	143.111845808

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