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AN APPLICATION OF THE REDUCED-BASIS METHOD FOR DARCY FLOWS

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Context and motivations

- Subsurface flow modeling (history-matching, optimization of field performance, ...).
- Constraints :
 - Long simulation times (grid's size, time iterations, ...)
 - Simulations should be rerun for different input parameters.

Two-phase flow model

- Unknowns : water saturation S , pressure P .
- Parameter : water viscosity $\mu \in \mathcal{P} \subset \mathbb{R}^+$.
- Equations in $\Omega \subset \mathbb{R}^2$:

$$\operatorname{div} \mathbf{v} = 0, \quad (1)$$

$$\mathbf{v} = -\lambda_T(S, \mu) \mathcal{K} \nabla P, \quad (2)$$

$$\phi \frac{\partial S}{\partial t} + \operatorname{div} (f_w(S, \mu) \mathbf{v}) = 0, \quad (3)$$

subject to inflow-outflow boundary conditions.

Objective and difficulties

Build a reduced model for the pressure problem (1) - (2) at the final time step of the simulation.

Difficulties :

- Nonlinear coupling between P and S .
- No affine parameter dependance of λ_T with respect to μ .

Finite volume discretization

Discret finite volume approximation of the Darcy fluxes:

$$\text{if } \sigma = K|L : F_{K,\sigma}^{\text{int}} = \lambda_\sigma(\mu) \mathcal{K}_\sigma D_\sigma P_N^{n+1}, \quad (4)$$

$$\text{if } \sigma \in \mathcal{E}_K^{\text{Dir}} : F_{K,\sigma}^{\text{Dir}} = \lambda_T(S_K^n, \mu) \mathcal{K}_\sigma D_\sigma P_N^{n+1}, \quad (5)$$

where $D_\sigma P_N^{n+1} = P_{N,K}^{n+1} - P_{N,L}^{n+1}$ if $\sigma = K|L$ and $D_\sigma P_N^{n+1} = P_{N,K}^{n+1} - P_\sigma^{\text{Dir}}$ if $\sigma \in \mathcal{E}_K^{\text{Dir}}$.

Discret conservation of the volume:

$$\sum_{\sigma \in \mathcal{E}_K^{\text{int}}} F_{K,\sigma}^{\text{int}} + \sum_{\sigma \in \mathcal{E}_K^{\text{Dir}}} F_{K,\sigma}^{\text{Dir}} = 0. \quad (6)$$

Reduced model for the pressure problem

- $\mathcal{P}_{\text{Greedy}} := \{\mu_n\}_{n=1}^N \subset \mathcal{P}$.
- $Q_N = \operatorname{span} \{P_N(\cdot; \mu_n) \mid \mu_n \in \mathcal{P}_{\text{Greedy}}\}$ an N -dimensional subspace of Q_N .
- $Z_N := \{\zeta_n\}_{n=1}^N$ be an orthonormalized basis of Q_N .
- Algebraic form of RB-approximation

$$\mathbb{A}_N \mathbf{p}_N = \mathbf{f}_N, \quad (7)$$

where

$$(\mathbb{A}_N)_{n,m} = \sum_{\sigma \in \mathcal{E}^{\text{int}}} \lambda_\sigma(\mu) \mathcal{K}_\sigma D_\sigma \zeta_n D_\sigma \zeta_m + \sum_{\sigma \in \mathcal{E}^{\text{Dir}}} \lambda_T(S_K, \mu) \mathcal{K}_\sigma \zeta_{n,K} \zeta_{m,K}, \quad (\mathbf{f}_N)_n = \sum_{\sigma \in \mathcal{E}^{\text{Dir}}} \lambda_T(S_K, \mu) \mathcal{K}_\sigma P_\sigma^{\text{Dir}} \zeta_{m,K}.$$

Visual comparison

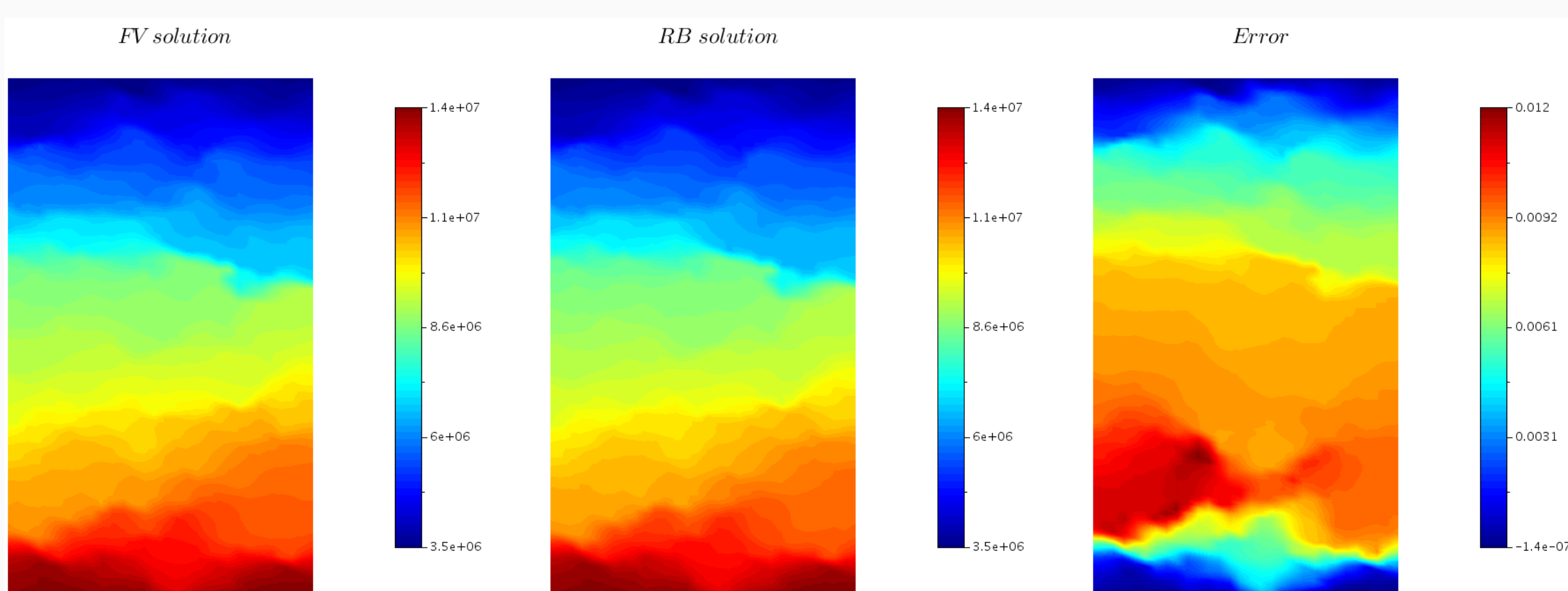


Figure 1: Comparison between truth solution (left), reduced basis approximations for $N = 20$ (middle) and relative error (right) for $\mu = 27$ cP

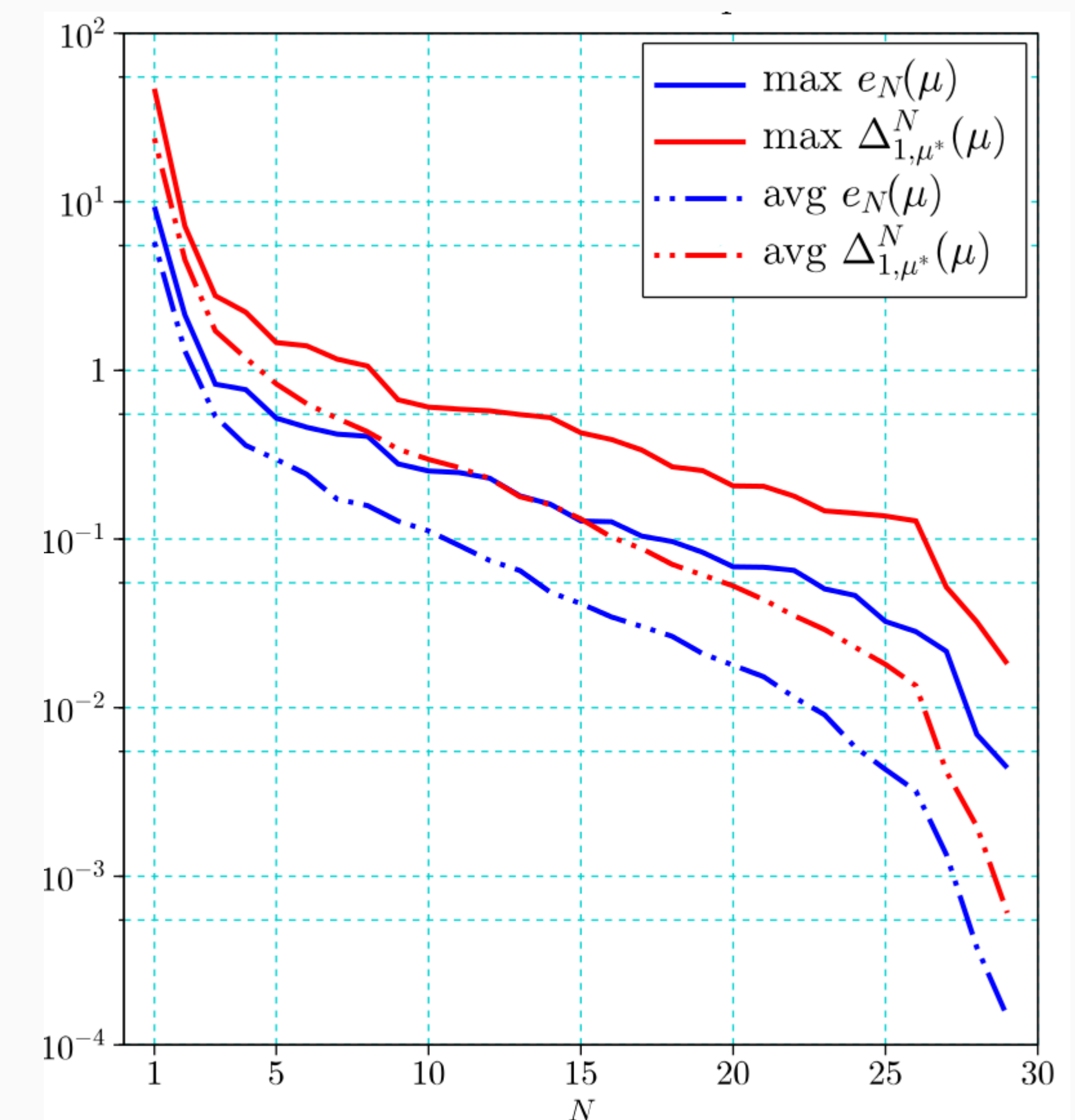
A posteriori error estimation

Define

$$\langle \mathcal{R}_N(P_N(\mu)), q \rangle = \sum_{\sigma \in \mathcal{E}} \lambda_\sigma(\mu) \mathcal{K}_\sigma D_\sigma P_N(\mu) D_\sigma q. \quad (8)$$

Then

$$\|P_N(\mu) - P_N(\mu)\|_{1,\mathcal{N},\mu^*} \leq \Delta_{1,\mu^*}^N(\mu) := \frac{1}{\alpha_N(\mu)} \|\mathcal{R}_N(P_N(\mu))\|_{-1,\mathcal{N},\mu^*},$$



Efficient RB approximation

Use the Empirical Interpolation Method (EIM) to approximate $\lambda_T(S, \mu)$ by

$$\lambda_T^J(\mu) = \sum_{j=1}^J \Theta_j^J(\mu) \xi_j. \quad (9)$$

One obtains

$$\left(\sum_{j=1}^J \Theta_j^J(\mu) \mathbb{A}_{N,j}^J \right) \mathbf{p}_N^J(\mu) = \sum_{j=1}^J \Theta_j^J(\mu) \mathbf{f}_{N,j}^J, \quad (10)$$

where

$$(\mathbb{A}_{N,j}^J)_{n,m} = \sum_{\sigma \in \mathcal{E}^{\text{int}}} \xi_{j,\sigma} \mathcal{K}_\sigma D_\sigma \zeta_n D_\sigma \zeta_m + \sum_{\sigma \in \mathcal{E}^{\text{Dir}}} \xi_{j,K} \mathcal{K}_\sigma \zeta_{n,K} \zeta_{m,K}, \quad (\mathbf{f}_{N,j}^J)_m = \sum_{\sigma \in \mathcal{E}^{\text{Dir}}} \xi_{j,K} \mathcal{K}_\sigma P_\sigma^{\text{Dir}} \zeta_{m,K}.$$

Convergence of the greedy algorithm

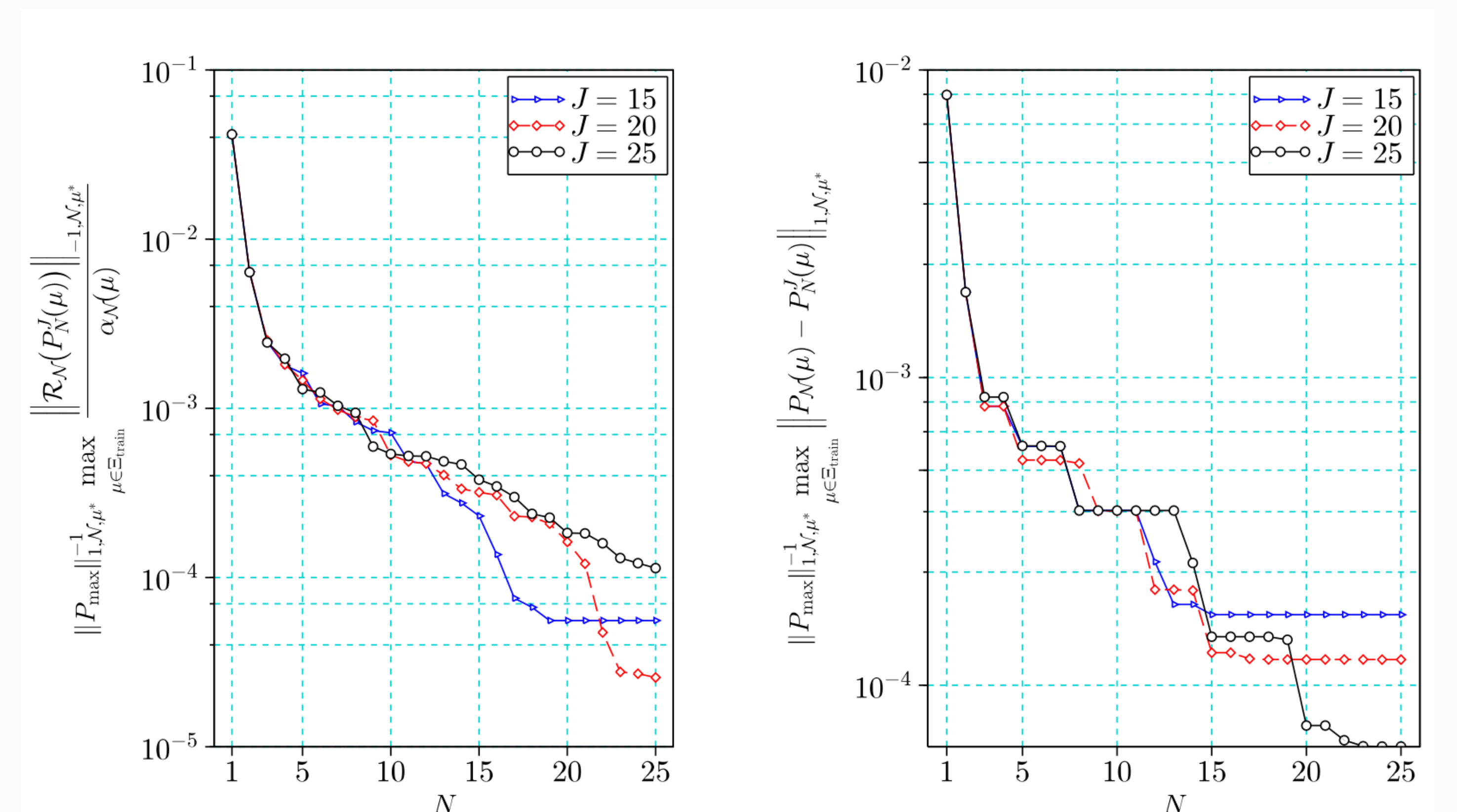


Figure 2: Convergence of the reduced-basis approximation for different values of J (layer 85 of the SPE10 test-case)

Conclusions

- Reduction of the pressure problem with a non-affine dependence in the parameter at the final time step.
- Certification of the RB model (a posteriori error estimate in energy norm).
- Efficient implementation of the RB method, i.e. the reduced problems can be constructed in complexity independent of \mathcal{N} .
- (Theoretical) Certification of the EIM-RB model.