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AN ALL-SPEED RELAXATION SCHEME FOR THE SIMULATION OF MULTI-MATERIAL FLOWS

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ABSTRACT

We propose an implicit all-speed scheme for the simulation of compressible flows inside gases, liquids and solids in different regimes. The scheme accurately simulates the propagation of material waves. A numerical model for the description of physical interfaces will also be introduced.

INTRODUCTION

We introduce an all-speed relaxation scheme for the simulation of compressible material flows at all speeds. A monolithic Eulerian model describing gases, fluids and elastic solids with the same system of conservation laws is adopted [1].

The numerical scheme is based on the relaxation technique introduced by Jin and Xin [2]. We adopt a fully implicit time integration thanks to the linearity of the transport operator in the relaxation system. The spatial discretization is obtained by a combination of upwind and centered schemes, in order to recover the correct numerical viscosity at all Mach numbers [1].

EULERIAN MODEL FOR COMPRESSIBLE MATERIALS

The conservative form of the equations of a general medium in the deformed configuration is given by

$$\begin{cases} \partial_t \rho + \operatorname{div}_x(\rho u) = 0 \\ \partial_t(\rho u) + \operatorname{div}_x(\rho u \otimes u - \sigma) = 0 \\ \partial_t([\nabla_x Y]) + \nabla_x(u \cdot [\nabla_x Y]) = 0 \\ \partial_t(\rho e) + \operatorname{div}_x(\rho e u - \sigma^T u) = 0. \end{cases} \quad (1)$$

Here ρ is the density, u is the Eulerian velocity field, σ is the Cauchy stress tensor and $[\nabla_x Y]$ is the gradient of the backward characteristics, describing the solid deformation. e is the total energy per unit mass and it is given by the sum of the kinetic energy and the internal energy per unit mass ε . We adopt the following general constitutive law, which is able to describe gases, fluids and elastic solids at the same time:

$$\varepsilon(\rho, s, [\nabla_x Y]) = \underbrace{\frac{\kappa(s)}{\gamma-1} \left(\frac{1}{\rho} - b\right)^{1-\gamma}}_{\text{general gas}} - a\rho + \frac{p_\infty}{\rho} + \underbrace{\frac{\chi}{\rho} (\operatorname{tr} \bar{B} - 2)}_{\text{neohookean solid}}. \quad (2)$$

Here $\kappa(s) = \exp(s/c_v)$ (s being the entropy), \bar{B} is the 2D right Cauchy-Green tensor and $\chi, p_\infty, \gamma, a, b$ are positive constants characterizing a given material.

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ALL-SPEED RELAXATION SCHEME

Letting $\psi = [\rho, \rho u_1, \rho u_2, Y_1^1, Y_1^2, Y_2^1, Y_2^2, \rho e]$ be the conservative variables and $\mathbf{F}(\psi)$ and $\mathbf{G}(\psi)$ the fluxes in the x_1 and x_2 directions, we adopt the relaxation method introduced by Jin and Xin [2]. The relaxation rate η and the relaxation matrices \mathbf{A}_1 and \mathbf{A}_2 are introduced. The fluxes $\mathbf{F}(\psi)$ and $\mathbf{G}(\psi)$ are “relaxed” by the vectors of relaxation variables \mathbf{v} and \mathbf{w} respectively: thanks to this procedure, the advective operator becomes linear. In this framework, a fully implicit time discretization is easily implemented. By employing finite volumes on a Cartesian mesh, the full scheme at first order reads:

$$\begin{cases} \frac{\psi_{ij}^{n+1} - \psi_{ij}^n}{\Delta t} + \frac{\mathbf{v}_{i+1/2,j}^{n+1} - \mathbf{v}_{i-1/2,j}^{n+1}}{\Delta x_1} + \frac{\mathbf{w}_{i,j+1/2}^{n+1} - \mathbf{w}_{i,j-1/2}^{n+1}}{\Delta x_2} = 0 \\ \frac{\mathbf{v}_{ij}^{n+1} - \mathbf{v}_{ij}^n}{\Delta t} + \mathbf{A}_1 \frac{\psi_{i+1/2,j}^{n+1} - \psi_{i-1/2,j}^{n+1}}{\Delta x_1} = \frac{1}{\eta} \left(\mathbf{F}(\psi_{ij}^{n+1}) - \mathbf{v}_{ij}^{n+1} \right) \\ \frac{\mathbf{w}_{ij}^{n+1} - \mathbf{w}_{ij}^n}{\Delta t} + \mathbf{A}_2 \frac{\psi_{i,j+1/2}^{n+1} - \psi_{i,j-1/2}^{n+1}}{\Delta x_2} = \frac{1}{\eta} \left(\mathbf{G}(\psi_{ij}^{n+1}) - \mathbf{w}_{ij}^{n+1} \right). \end{cases} \quad (3)$$

The variables at the interfaces $x_{i+1/2,j}$ inside (3) are computed as follows

$$\mathbf{v}_{i+1/2,j} = f(M_{loc}) (\mathbf{v}_{i+1/2,j})_{upw} + (1 - f(M_{loc})) (\mathbf{v}_{i+1/2,j})_{cent}. \quad (4)$$

This is a convex combination of upwind and centered fluxes, which provides the correct numerical viscosity for each regime, according to the local Mach number M_{loc} . We choose $f(M_{loc}) = \min\{1, M_{loc}\}$.

NUMERICAL VALIDATIONS

As a sample of the numerical results, here we show the results on a 2D Riemann problem (2DRP) involving two contact waves, a shock and a rarefaction in a gas. This is solved with an adaptive mesh refinement technique based on the numerical entropy production criterion [3]. The grid and the density profile obtained at time $t = 0.3$ are shown in Figs. 1(a) and 1(b). The all-speed scheme keeps the contact waves sharp, thanks to (4).

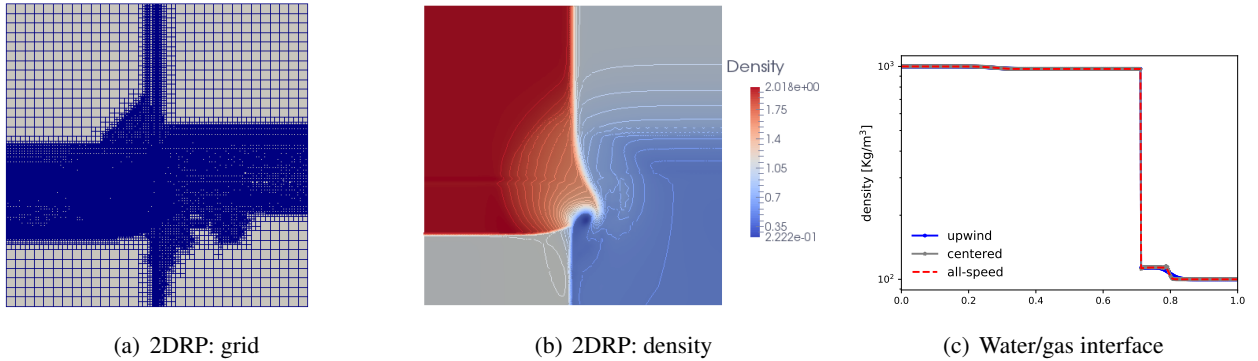


FIGURE 1: (a)-(b): 2D Riemann problem in a gas. (c): multi-material flow (water/gas interface).

In Fig. 1(c) we present a preliminary 1D multi-material test, where a water/gas interface is solved. The Mach number in water (left) is around $3 \cdot 10^{-2}$, whereas in the gas (right) is around 0.15, hence we are dealing with a multi-regime test. The transmission conditions are imposed via immersed boundary extrapolations, thus the interface is sharp by construction.

REFERENCES

- [1] Abbate, Iollo, Puppo. *An all-speed relaxation scheme for gases and compressible materials*. Journal of Computational Physics 351, pp. 1-24, 2017
- [2] Jin, Xin. *The relaxation schemes for systems of conservation laws in arbitrary space dimensions*. Communications on pure and applied mathematics 48(3), pp. 235-276, 1995.
- [3] Puppo, Semplice. *Numerical entropy and adaptivity for finite volume schemes*. Communications in Computational Physics 10(5), pp. 1132-1160, 2011.