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# The complete proof of Goldbach's Conjecture

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# The proof of Goldbach's Conjecture

MATAN COHEN

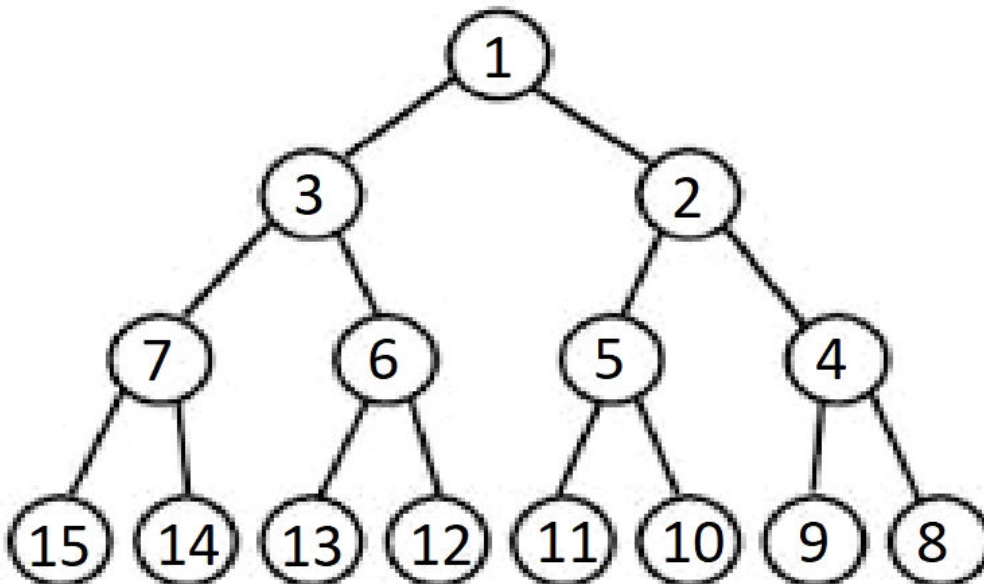
Ariel University of Samaria

## Introduction

Goldbach's Conjecture states that every even number greater than 2 is the sum of two primes. That is:  $\forall 2m \exists p_1, p_2 : 2m = p_1 + p_2, m \in \mathbb{N}$ . This paper uses a binary tree to provide a complete proof to Goldbach's Conjecture.

## Definitions and Formulas

**Definition 1** The following structure is called a Binary Tree. In some parts of the proof, it is simply called **tree**.



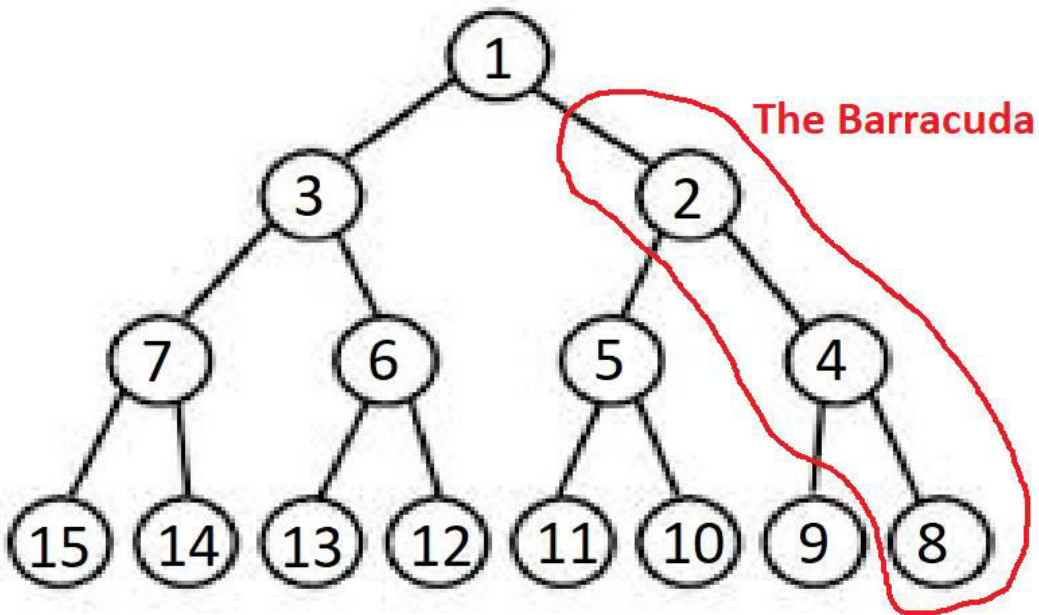
The above tree consists of 4 levels, and contains natural numbers up to 15. Our goal is to proof

Goldbach's Cojecture for an infinite amount of even numbers, For this reason, let us assume that the above tree is **unlimited** and contains **all natural numbers**.

**Definition 2** The symbol  $V$  represents an arbitrary vertex in the tree.

**Definition 3**  $\forall V \in \text{tree}$ , the right child of  $V$  is an even number and the left child of  $V$  is an odd number.

**Definition 3** The **Barracuda** of the tree is the group  $\{V \in \text{tree} : V = 2^k, k \in \mathbb{N}\}$ .

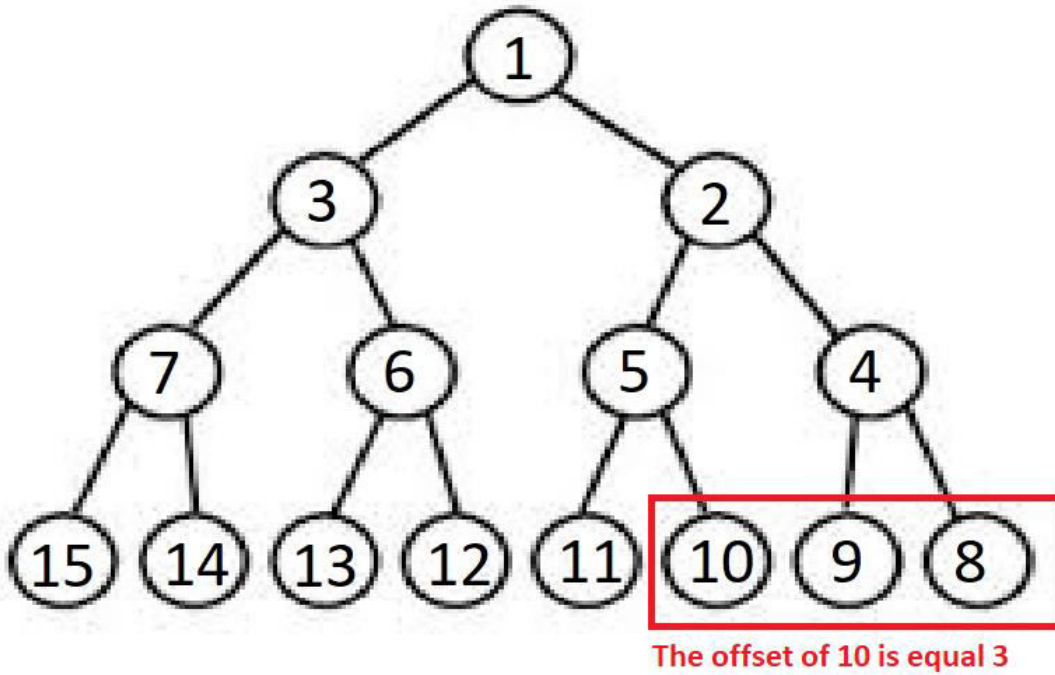


**Definition 4** The level  $k$  of the tree contains  $2^k$  vertices

**Formula 1**  $\forall k \in \mathbb{N} 2^0+2^1+2^2+\dots+2^k = 2^{k+1}-1$

**Definition 5** The sub-tree which consists of the levels  $0,1,2,\dots,k$  contains  $2^0+2^1+2^2+\dots+2^k$  vertices. According to Formula 1, that is  $2^{k+1}-1$  vertices

**Definition 6** the **Offset** of  $V$  is the number of vertices that exist in the level of  $V$  and are located to the right of  $V$ . The offset of  $V$  always includes  $V$  as well.



**Definition 7** In every level of the tree, the offsets of even numbers are odd.

**Definition 8** In every level of the tree, the offsets of odd numbers are even.

## The Proof

**Theory** Goldbach's Cojecture is correct

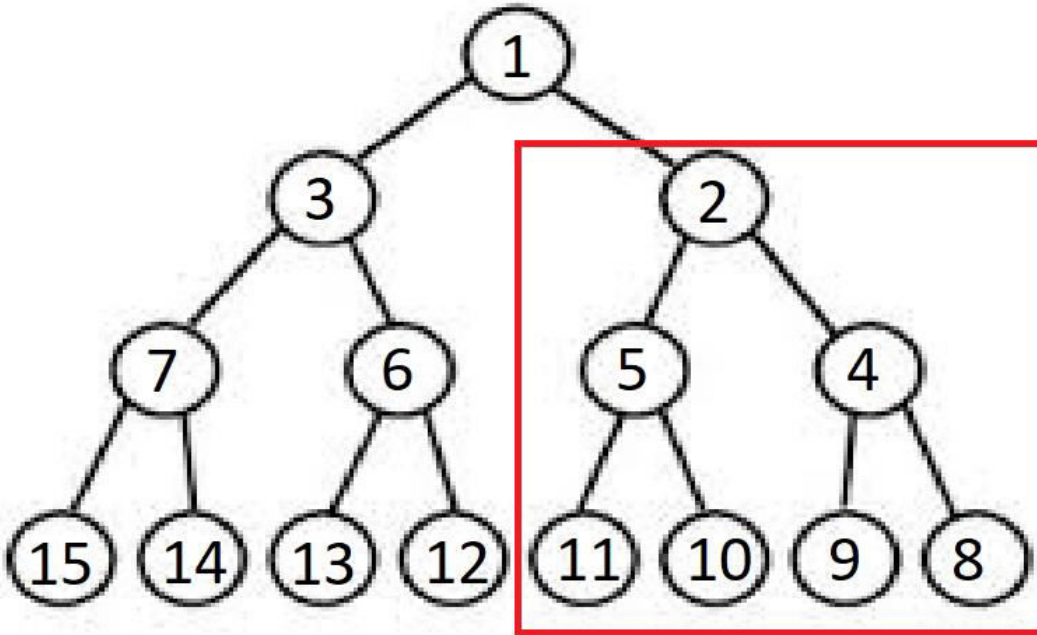
### Proof of the Theory

**Lemma 1** Goldbach's Conjecture is correct for every vertex outside the barracuda, which is even and greater than the number 2.

#### Proof

1. Since the tree is unlimited, Every even number greater than 2 can be found in the tree.
2. Let us choose an arbitrary vertex  $X$  in the tree. so that  $X = 2m$ ,  $m \in \mathbb{N}$ ,  $m > 1$

3. Let Y be the level of X. Let R be the intersection between Y and the Barracuda.
4. Let T be a sub-tree, with a root that connects X and R together. For example, if X = 10 then R = 8 and T will be the following tree:



5. Let A be the offset of vertex X in level Y of the tree T.
6. Let B be the difference between the number of vertices in Y and A, that is:  $B = 2^Y - A$

**Theorem 1.1.** Either A is prime or B is prime.

**Proof of Theorem 1.1:**

1.  $2^Y$  is even. According to *definition 7*, A must be odd. Therefore, B must be odd as well.
2. Given that A and B are odd, Let us assume that both A and B are composite numbers. That is, A and B can be divided only by odd numbers. It means that  $\{\forall^* : A \neq 2^*\}$  and  $\{\forall^\& : B \neq 2^\&\}$ . Therefore  $\{\forall^* \forall^\& : A + B \neq 2^* + 2^\&\}$ , and it follows that  $\{\forall^* \forall^\& \forall y : 2^* + 2^\& \neq 2^y\}$ . Contradiction!
3. Because of the contradiction in *step 2*, at least one of the two numbers (A and B) must be prime.
4. Q.E.D

7. Let  $p_1$  be a prime number which is equal to either A or B. Let us evaluate the term  $X - p_1$ .

**Theorem 1.2:**  $X - p_1$  is prime.

**Proof of Theorem 1.2:**

1. Since  $X$  is even and  $p_1$  is prime, the term  $X - p_1$  is odd. According to *Definition 8*, the offset of  $X - p_1$  is even. Let  $2m$  be the offset of  $X - p_1$ .

2. Because of *Definition 5*,  $X - p_1 = 2^k - 1 + 2m$

3. Let us assume that  $X - p_1$  is a composite number. If so, it can be divided by an odd number greater than 2. Let's call that number  $-1 + 2m$

4. It follows that:

$$\frac{(x - p_1)}{-1 + 2m} = \frac{2^k - 1 + 2m}{-1 + 2m} = 1 + \frac{2^k}{-1 + 2m}$$

5. The number  $2^k$  only has 2 as a prime factor, while odd numbers never have 2 as a prime factor. So  $2^k$  and an odd number will have no prime factors in common. For this reason, the number  $2^k$  cannot be divided by an odd number.

6. From *step 4* and *step 5*, it follows that the term:

$$\frac{2^k}{-1 + 2m}$$

is not a natural number, that is, the odd number  $X - p_1$  cannot be divided by an odd number. So  $X - p_1$  is prime.

7. Q.E.D

8. From Theorem 1.1. and Theorem 1.2. it follows that Lemma 1 is proven.

9. Q.E.D

**Lemma 2** Goldbach's Conjecture is correct for every vertex in the Barracuda (except of the vertex 2).

**Proof of Lemma 2**

Let  $2m$  be an arbitrary even number, so that  $\{2^x \neq 2m > 2^k : m \in \mathbb{N}, k \in \mathbb{N}, x \in \mathbb{N}, x > k\}$ . It is easy to see that:

$$\{\exists n \in \mathbb{N} : 2m = 2^k + 2n, 2n \neq 2^x \}$$

Because  $2n \neq 2^x$ , the number  $2n$  is outside the Barracuda. Hence, according to Lemma 1, Goldbach's Conjecture is correct for the number  $2n$ , so  $2n = p_1 + p_2$ . It follows that:

$$2m = 2^k + p_1 + p_2$$

For the same reason, Goldbach's Conjecture is correct for the number  $2m$ . It means that the weak version of Goldbach's Conjecture must be correct for the number  $2m$ . According to the weak version of Goldbach's Conjecture, every odd number is the sum of 3 primes. Therefore, the number  $2m$  is the sum of 4 primes.

It follows that  $2^k + p_1 + p_2$  is the sum of 4 primes, so  $2^k$  is the sum of 2 primes, thus Goldbach's Conjecture is correct for  $2^k$ .

In other words, Goldbach's conjecture is correct for every vertex *inside the Barracuda*.

Q.E.D

**Proposition 1** If Goldbach's Conjecture is correct for every vertex outside the Barracuda (which is even and greater than 2) and Goldbach's Conjecture is also correct for every vertex inside the Barracuda (except of the vertex 2), then Goldbach's Conjecture is correct for every even number greater than 2.

### **Proof of Proposition 1**

Let **OUT** be the group of all vertices that are outside the Barracuda (and are even and greater than 2).

Let **IN** be the group of all vertices that are inside the Barracuda (except of the vertex 2).

According to Lemma 1, Goldbach's Conjecture is correct for **OUT**

According to Lemma 2, Goldbach's Conjecture is correct for **IN**

It is easy to see that the union **OUT U IN** is the group of all even numbers greater than 2.

Q.E.D.

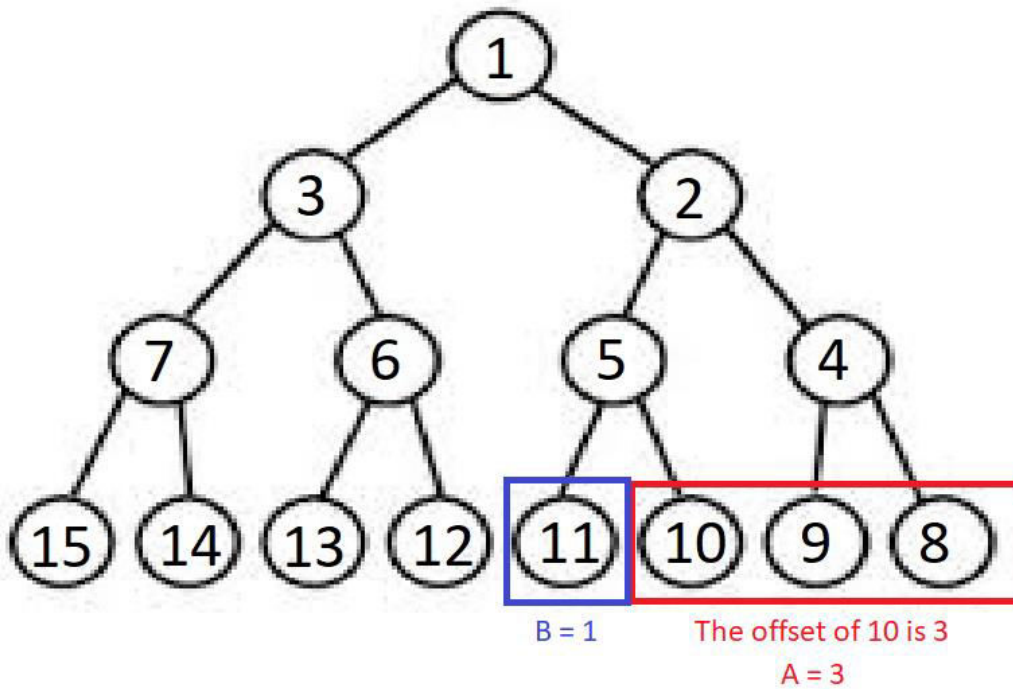
The proof of Proposition 1 results in the proof of the Theory.

**Q.E.D**

Examples:

Does the number 10 satisfy Goldbach's Conjecture?

**X = 10**

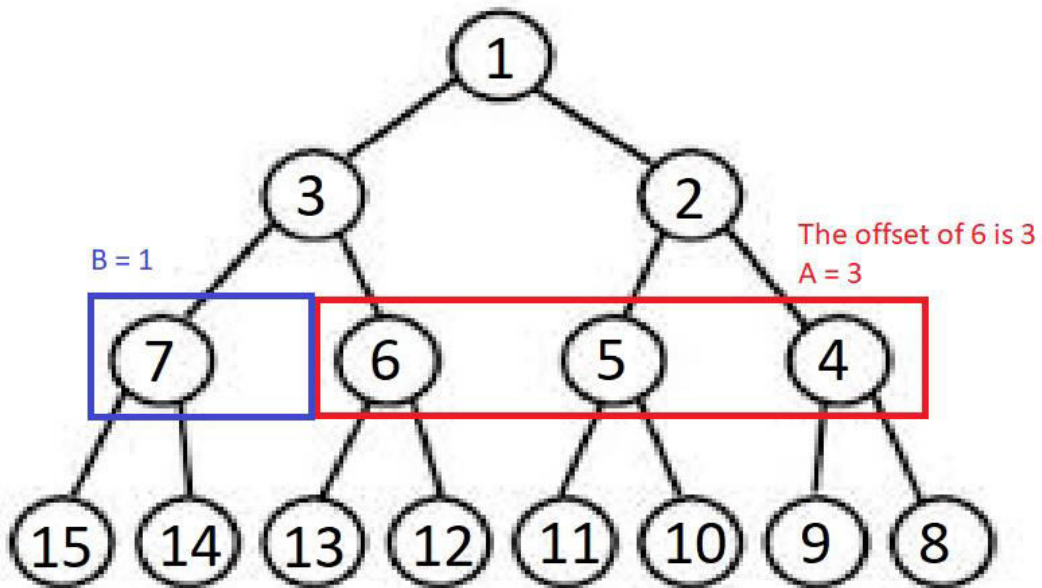


**A = 3 is prime**

**X - A = 10 - 3 = 7 is prime**

Does the number 6 satisfy Goldbach's Conjecture?

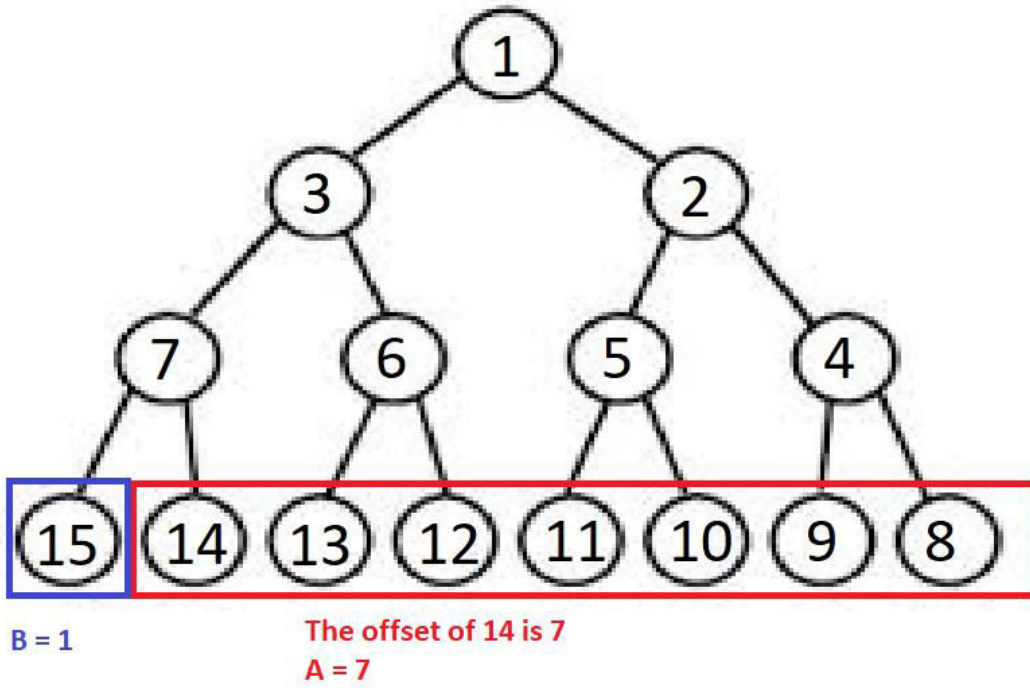
**X = 6**



**A = 3 is prime**  
**X - A = 6 - 3 = 3 is prime**

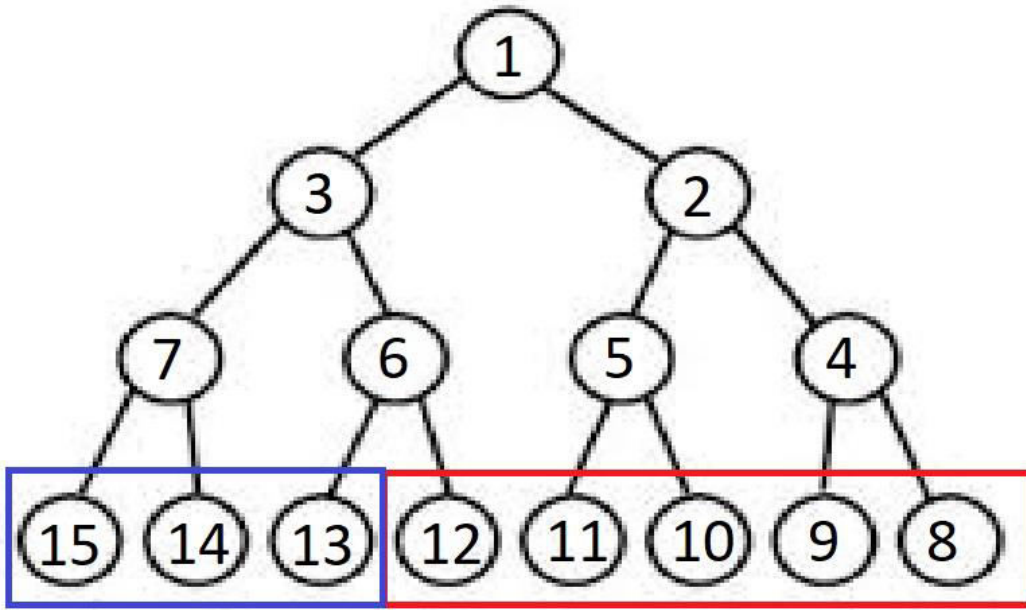
Does the number 14 satisfy Golbach's Conjecture?

$X = 14$



Does the number 12 satisfy Golbach's Conjecture?

A = 12



B = 3

The offset of 12 is 5

A = 5