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# Design and Synthesis of Arbitrary Group Delay Filters for Integrated Analog Signal Processing

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*Abstract*—This article proposes procedures for the design and synthesis of wideband filter networks for integrated analog signal processing (ASP) applications. In this context, filters with engineered group delay are required. In the state-of-theart of ASP, this is solved using discrete devices or system in package solutions like SAW filters. The proposal intends to generate network topologies compatible with an implementation in integrated circuit technologies. An example of the design and synthesis of doubly terminated ladder networks are presented to illustrate the proposal. The distortion on the group delay characteristic due to filter transformation is highlighted. It is also shown the trade-off between bandwidth and group delay swing. Finally, a synthesized network is sized for implementation in a 130 nm BiCMOS technology, and the results of its electrical simulation are compared with that of an ideal network.

#### I. INTRODUCTION

At high-frequency ranges, like microwave, millimeter-wave and terahertz frequencies, digital signal processing (DSP) circuits face significant difficulties, such as high power consumption, decreasing signal levels, and limited performances of interfacing A/D converters. Contrasting with digital signal processing, analog signal processing (ASP) is regaining attention in recent years as a way to attain higher frequencies with lower power consumption [1]–[3]. Therefore, ASP is more suitable for real-time signal processing in wideband communications, radar, and spectrum sniffing [1]–[3].

The design of wideband filters with engineered group delay (also known as phasers, chirp filters, dispersive delay lines or reflective array compressor devices) is the core problem in analog signal processing [1], [2]. Such filter devices enable operations as stretching, compression and reversion of electronic signals in the time domain [4].

Several methods for the design of filters with arbitrary group delay for different microwave technologies have been mentioned in [1]. However, only B. Xiang et al. succeeded to implement and demonstrate three versions of an integrated distributed amplifier-based transversal filter in [3], [5], [6]. Such filters present roughly linear group delays obtained through complicated design procedures. The solutions still suffer from poorly behaved responses in magnitude and group delay.

B. Xiang's implementations illustrate the complexity of current engineered group delay integrated filter architectures and design techniques. This work focuses on the design and the synthesis of wideband filter networks with an arbitrary group delay to promote newer integrated solutions. An example of the design and synthesis of doubly terminated ladder networks is presented using passive devices. An active implementation is also feasible for a real-time ASP system.

#### **II. DESIGN AND SYNTHESIS PROCEDURES**

#### A. Generation of Filter Polynomials

In filter theory, it is common to represent the forward transmission coefficient  $(S_{21})$  and the input reflection coefficient  $(S_{11})$  of a lossless passive network as the ratio of polynomials [7, Sec. 3.2.1]

$$S_{21} = \frac{P\left(s\right)}{E\left(s\right)} \tag{1a}$$

$$S_{11} = \frac{F(s)}{E(s)},$$
 (1b)

where E(s) must be a Hurwitz polynomial.

The first step in the design of filters consists in finding the polynomials in (1) for the lossless lowpass prototype filter. For this, the present work uses the method described in [2]. For convenience, the flowgraph of the procedure used to synthesize the prototype filter polynomials is shown in Fig.1 and is briefly described below.

The procedure starts with the phase specification of the prototype filter,  $\phi(\Omega)$ , where  $\Omega$  is a normalized frequency, with  $\Omega = 1$  being the cutoff frequency of the lowpass prototype filter. Since the designer is generally more concerned with the filter group delay,  $\phi(\Omega)$  can be obtained by integrating the group delay specification of the prototype filter,  $\tau_a(\Omega)$ .

Once the filter order N is defined, a set of N+1 frequency values in the interval [0, 1] is created, preferably, with frequencies equally spaced in the interval. The extreme values of the interval are required to be in the set. Subsequently, a set of phase values are calculated from this set of frequencies and the phase specification.

The denominator polynomial, E(s), is obtained through the recursive procedure given by

$$\begin{pmatrix} 1, \\ n = 0 \end{pmatrix}$$

$$E_n(s) = \begin{cases} s + \alpha_0, & n = 1 \\ \alpha - E_n(s) + (s^2 + \Omega^2) & F_n(s) \\ \alpha - E_n(s) + (s^2 + \Omega^2) & F_n(s) \end{cases}$$

$$\left(\alpha_{n-1}E_{n-1}(s) + (s^{2} + \Omega_{n-1}^{2})E_{n-2}(s), \quad n \ge 2$$
(2)



Fig. 1. Flowgraph of the procedure to synthesize the lossless lowpass prototype filter polynomials E(s), P(s) and F(s), starting from a specified phase  $\phi$  or group delay  $\tau_g$  and filter order N.

where

$$\alpha_{i} = \begin{cases} \frac{\Omega_{1}}{\tan \phi_{1}}, & i = 0\\ \frac{\Omega_{i+1}^{2} - \Omega_{i}^{2}}{\alpha_{i-1} - \frac{\Omega_{i+1}^{2} - \Omega_{i-1}^{2}}{\alpha_{i+1} - \Omega_{i-2}^{2}}}, & i \ge 1\\ \frac{\Omega_{i+1}^{2} - \Omega_{i-1}^{2}}{\frac{\Omega_{i+1}^{2} - \Omega_{i-2}^{2}}{\alpha_{i-2} - \frac{\Omega_{i+1}^{2}}{\tan \phi_{i+1}}}} & (3) \end{cases}$$

As a magnitude of one (i.e., 0 dB) is generally desired in the filter passband, the transfer function numerator polynomial, P(s), is determined by the following procedure: first, the polynomial  $E(s)|_{s=j\Omega}$ , where  $s = j\Omega$  is the complex frequency, is approximated by a truncated Chebyshev expansion through [8]

$$P(j\Omega) \approx \sum_{i=0}^{M} c_i T_i(\Omega), \qquad (4)$$

where M is the desired order of the P(s) polynomial and should be less than or equal to the filter order N (i.e., the order of the E(s) polynomial),  $T_i$  is the Chebyshev polynomial of the first kind of order i, and  $c_i$  is given by

$$c_{i} = \begin{cases} \frac{1}{\pi} \int_{-1}^{1} \frac{\sqrt{E(j\Omega)E(-j\Omega)}T_{i}(\Omega)}{\sqrt{1-\Omega^{2}}} d\Omega, & i = 0\\ \frac{2}{\pi} \int_{-1}^{1} \frac{\sqrt{E(j\Omega)E(-j\Omega)}T_{i}(\Omega)}{\sqrt{1-\Omega^{2}}} d\Omega, & i > 0 \end{cases}$$
(5)

Then, the resulting polynomial is normalized according to

$$P_{norm.}(j\Omega) = \frac{P(j\Omega)}{\max_{-1 < \Omega < 1} \left\{ \frac{P(j\Omega)}{|E(j\Omega)|} \right\}},\tag{6}$$

where  $\max_{1 \le x \le 1} \{f(x)\}$  returns the maximum value of f(x)in the interval  $-1 \le x \le 1$ , so that  $|S_{21}| \le 1$ . The P(s)polynomial is obtained by evaluating (6) for  $\Omega = \frac{s}{i}$ , i.e.,

$$P(s) = P_{norm.}(j\Omega)\Big|_{\Omega = \frac{s}{j}}.$$
(7)

The last polynomial, F(s), the numerator of the input impedance function, is determined through the energy conservation law, that can be rewritten as,

$$F(s) F(-s) = E(s) E(-s) - [P(s)]^{2}.$$
 (8)

Since F(s) F(-s) has 2N roots, which are symmetrically distributed about the imaginary axis or lie in pairs in the imaginary axis, the F(s) polynomial is formed by taking all the roots in the left half plane and one of the roots of each pair laying in the imaginary axis.

#### B. Synthesis of Lumped Element Filter Networks

The second step in the design of filters consists in synthesizing the filter network from the generated polynomials. The authors in [2] use the coupling matrix technique to synthesize the filter in a waveguide cross-coupled microwave structure.

This technique is targeted to the synthesis of microwave filters and is only capable of generating bandpass coupling filter topologies of, generally, narrow bandwidth, since coupling elements are commonly assumed to be frequency invariant admittance or impedance inverters.

Another option is to use the classical circuit synthesis approach. In this approach, an electrical network can be synthesized from the [ABCD] (or "chain") matrix representation of the filter [7, Ch. 7].

In the specific case of lumped ladder networks, the element values of the network can be found by expanding  $S_{11}$  in a continued fraction [9, Sec. 2.5].

After the synthesis of the lowpass prototype filter, the resulting network can be transformed to another class of filter (e.g., highpass, bandpass, bandstop) and/or scaled in frequency [9, pp. 32-41], without being subjected to any bandwidth restriction.

#### III. DESIGN AND SYNTHESIS EXAMPLE

A fourth order filter presenting a positive slope group delay is synthesized through a doubly terminated ladder network to illustrate the design and synthesis procedures. The synthesis of such structure is appealing as it is well known that it presents low sensitivity to components values variation [10, Sec. 3.3.8]. Another particular advantage of ladder networks is that they are generally used as a starting point for the synthesis of active networks.

A group delay swing of 1s over the passband of the lowpass prototype filter (i.e.,  $0 \le \Omega \le 1$ ) was specified. The expressions for the transfer function and the input impedance of the prototype filter, obtained using the algorithm shown in



Fig. 2. Synthesized networks: (a) lowpass prototype starting with a series inductor, (b) resulting network from the bandpass transformation and scaling of Fig.2a to a bandwidth of 500 MHz, centered around 1 GHz (components values in Table I), (c) lowpass prototype starting with a shunt capacitor, (d) the resulting network from the bandpass transformation and scaling of Fig.2c to a bandwidth of 500 MHz, centered around 1 GHz (components values in Table I), (c) lowpass prototype starting with a shunt capacitor, (d) the resulting network from the bandpass transformation and scaling of Fig.2c to a bandwidth of 500 MHz, centered around 1 GHz (components values in Table I).



Fig. 3. Frequency response and group delay for the lowpass prototype networks, with the band of interest highlighted in yellow.

Fig.1, are presented in (9), while Fig.2a and Fig.2c show two tantamount networks that were synthesized by expanding (9b) into a continued fraction [9, Sec. 2.5]. The frequency response and group delay of the equivalent lowpass prototype networks of Fig.2a and Fig.2c are shown in Fig.3, with the band of the filter presenting the desired group delay highlighted in yellow.

$$S_{21}(s) = \frac{0.89430}{s^4 + 2.1408s^3 + 3.1524s^2 + 2.3190s + 0.90249}$$
(9a)  
$$S_{11}(s) = \frac{s^4 + 0.96301s^3 + 1.3245s^2 + 0.09532s + 0.12131}{s^4 + 2.1408s^3 + 3.1524s^2 + 2.3190s + 0.90249}$$
(9b)

Since a ladder structure can only realize transfer functions with transmission zeros in the  $j\omega$ -axis [10, p. 89], to be able to synthesize a ladder network, the order of the P(s) polynomial was specified equal to zero in the algorithm. This resulted in a minimum phase transfer function. Due to the relation existing between the magnitude and phase of a causal system (i.e., Bode relations), the lack of finite frequency transmission zeros prevents obtaining a flatter magnitude characteristic in the filter passband, unchanged the specified group delay.

An order of four has been chosen for the filter so that an almost linear group delay characteristic is obtained, with

 TABLE I

 Components values for the networks of Fig.2b and Fig.2d.

	Network of	Network of		Network of	Network of
	Fig.2b	Fig.2d		Fig.2b	Fig.2d
C1	$999.8\mathrm{fF}$	$10.81\mathrm{pF}$	L1	$27.03\mathrm{nH}$	$2.499\mathrm{nH}$
C2	$10.70\mathrm{pF}$	$1.010\mathrm{pF}$	L2	$2.526\mathrm{nH}$	$26.74\mathrm{nH}$
C3	$1.219\mathrm{pF}$	$8.865\mathrm{pF}$	L3	$22.16\mathrm{nH}$	$3.048\mathrm{nH}$
C4	$3.130\mathrm{pF}$	$3.453\mathrm{pF}$	L4	$8.632\mathrm{nH}$	$7.825\mathrm{nH}$

 TABLE II

 Technology parameters values for the components of the network of Fig.2d.

	Canacit	Width		Internal	No. of	Line
	Capacit.			diameter	turns	width
C1	$10.689\mathrm{pF}$	$73\mu\mathrm{m}$	L1	$143.335\mu{ m m}$	3	$11.99\mu m$
C2	$1.010\mathrm{pF}$	$22\mu m$	L2	$137.310\mu\mathrm{m}$	9	$5.055\mu{ m m}$
C3	$8.788\mathrm{pF}$	66 µm	L3	$162.965\mu{ m m}$	3	$8.065\mu\mathrm{m}$
C4	$3.448\mathrm{pF}$	$41\mu\mathrm{m}$	L4	$163.000\mu\mathrm{m}$	5	$8.235\mu{ m m}$

a maximum attenuation of -3 dB in the passband (at the cutoff frequency). Increasing the filter order would increase the passband attenuation, with negligible improvement in the group delay characteristic.

Beyond the shown networks, several others can be obtained. For example, it is common to replace the series inductors by shunt capacitors, by surrounding it by admittance inverters [9, Sec. 2.8], [7, Sec. 7.2]. Likewise, a shunt capacitor can be transformed into a series inductor through the use of impedance inverters. The diversity of networks that can be obtained allows finding a network that is better suited for implementation in a particular technology.

Applying a bandpass transformation to the lowpass prototype networks of Fig.2a and Fig.2c and scaling them to a center frequency of 1 GHz and a bandwidth of 500 MHz centered around this center frequency, the networks of Fig.2b and Fig.2d are obtained, respectively. The components values for both networks are listed in Table I.

Considering the ideal components values in Table I, the frequency response and the group delay for the bandpass networks of Fig.2b and Fig.2d are displayed in Fig.4a with a



Fig. 4. Frequency response and group delay for: (a) the bandpass networks of Fig.2b and Fig.2d considering: ideal components (continuous trace) and technology models (dotted trace), (b) an equivalent bandpass network, but with a bandwidth of 200 MHz centered around 1 GHz (network not shown).

continuous trace. The band of interest is highlighted in yellow. The same metrics are illustrated in Fig.4b for a bandpass network with the same center frequency but a bandwidth of 200 MHz.

Considering the overall bandwidths of the filters in Fig.4, the graphs illustrate the group delay distortion due to the bandpass transformation [11] and the trade-off that exists between group delay swing and bandwidth [7, Sec. 3.8.2], showing that the group delay swing increases as the bandwidth decreases, and vice-versa. For analog signal processing, these effects imply a tradeoff, as a wide group delay swing is generally desired.

The components of the circuit of Fig.2d were sized for implementation in the ST BiCMOS9MW technology. The models *cmim3pM2* and *indsym\_lamwM2* were used for, respectively, capacitors and inductors. Table II lists the dimensioned parameters values for each component. The results of the nominal simulation of the scheme of Fig.2d, considering the accurate models above, are superimposed to the ideal curves of Fig.4a.

The attenuation in the passband of the simulated response if Fig.4a is mainly due to the high series resistance associated with the inductor  $L_2$ . The rounding of the transfer function affects the group delay characteristic. Parasitics also affect matching and consequently, the reflection coefficient response.

#### IV. CONCLUSION

This work proposed procedures for the design and synthesis of wideband filters of arbitrary group delay or phase. The described procedures allow the implementation of filters in integrated technologies by finding the better-suited network for a particular technology. An example is given, focusing on the synthesis of the popular doubly terminated ladder network. Such architecture was chosen due to its low sensitivity characteristic to element values variation and its simplicity. Passive or active integrated implementations are possible. The trade-off between group delay swing and bandwidth has been illustrated. The effect of group delay distortion caused by the lowpass to bandpass transformation is highlighted. The electrical simulation of the synthesized passive network in a 130 nm BiCMOS technology has been performed, and the obtained results are in agreement with the target transfer function.

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