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## Decentralized Controller Design for Large Scale Switched Takagi-Sugeno Systems with H∞ Performance Specifications

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#### **Abstract:**

This paper investigates the design of decentralized controllers for a class of large scale switched nonlinear systems under arbitrary switching laws. A global large scale switched system can be split into a set of smaller interconnected switched Takagi-Sugeno fuzzy subsystems. In this context, to stabilize the overall closed-loop system, a set of switched non-PDC output-feedback controllers is considered. The latter is designed based on Linear Matrix Inequalities (LMI) conditions obtained from a multiple switched non-quadratic Lyapunov-like candidate function. The controllers proposed herein are synthesized to satisfy  $H_{\infty}$  performances for disturbance attenuation. Finally, a numerical example is proposed to illustrate the effectiveness of the suggested decentralized switched controller design approach.

Key Words: Large Scale Switched Fuzzy System, Decentralized non-PDC Controllers, Arbitrary Switching Laws.

#### I. INTRODUCTION

During the last few decades, several complex systems are appeared to meet the specific needs of the world population. In this context, we can quote as examples the networked power systems, water transportation networks, traffic systems, as well as other systems in various fields. Generally speaking, establish a mathematical model for large scale systems is a complex task, especially when the system is considered as a whole. Hence, to overcome these difficulties, an alternative to the global modelling approach has been explored. It consists in decomposing the overall large-scale system in a finite set of interconnected low-order subsystems [1].

Among these complex systems, switched large-scale interconnected system have attracted considerable attention since they provide a convenient modelling approach for many physical systems that can exhibit both continuous and discrete dynamic behavior. In this context, several studies dealing with the stability analysis and stabilization issues for both linear and nonlinear switched interconnected large-scale systems have been explored [1]-[8]. Hence, the main challenge to deal with such problems consists in determining the conditions ensuring the stability of the whole systems with consideration to the interconnections effects between its subsystems. Nevertheless, few works based on the approximation property of Takagi-Sugeno (TS) fuzzy models for nonlinear problems have been achieved to deal with the stabilization of continuous-time large-scale switched nonlinear systems [3], [8]-[12]. For example, by

using the PDC design method, an output-feedback decentralized controller has been developed in [9] for a class of T-S fuzzy switched large-scale systems. In the same way, the authors of [10] have studied the design of an adaptive fuzzy output-feedback control for a class of switched uncertain nonlinear large-scale systems with unknown dead zones and immeasurable states. Recently, an observer-based decentralized control scheme was developed in [11] for a class switched non-linear largescale systems. In the same context, an adaptive fuzzy decentralized output-feedback tracking control has been explored in [12] for a class of switched nonlinear largescale systems under the assumption that the large-scale system was composed of subsystems interconnected by their outputs. The stability of the whole closed-loop system and the tracking performance were achieved by using the Lyapunov function and under constrained switching signals with dwell time.

This paper presents the design of decentralized robust controllers for a class of switched TS interconnected large-scale systems with external bounded disturbances. More specifically, the primary contribution of this paper consists in proposing a LMI based methodology, in the non-quadratic framework, for the design of robust output-feedback decentralized switched non-PDC controllers for a class of large scale switched nonlinear systems under arbitrary switching laws.

The remainder of the paper is organized as follows. Section 2 presents the considered class of switched TS interconnected large-scale system, followed by the problem statement. The design of the decentralized

switched non-PDC controllers is presented in section 3. A numerical example is proposed to illustrate the efficiency of the proposed approach in section 4. The paper ends with conclusions and references.

## II. PROBLEM STATEMENT AND PRELIMIARIES

Let us consider the class of nonlinear hybrid systems S composed of n continuous time switched nonlinear subsystem  $S_i$  represented by switched TS models. The n state equations of the whole interconnected switched fuzzy system S are given as follows; for i = 1,...,n:

$$\begin{cases} \dot{x}_{i}(t) = \sum_{j_{i}=1}^{m_{i}} \sum_{s_{j}=1}^{r_{j_{i}}} \xi_{j_{i}}(t) h_{s_{j_{i}}}(z_{j_{i}}(t)) \begin{bmatrix} A_{hj}x_{i}(t) + B_{hj}u_{i}(t) + B_{hj}^{w}w_{i}(t) \\ + \sum_{\alpha=1,\alpha\neq i}^{n} (F_{i,\alpha,hj}x_{\alpha}(t) + B_{hj}^{w}w_{\alpha}(t)) \end{bmatrix} \\ y_{i}(t) = \sum_{i=1}^{m_{i}} \xi_{j_{i}}(t) C_{hj}x_{i}(t) \end{cases}$$

where  $x_i(t) \in \mathbb{R}^{\eta_i}$ ,  $y_i(t) \in \mathbb{R}^{\rho_i}$ ,  $u_i(t) \in \mathbb{R}^{\nu_i}$  represent respectively the state, the measurement (output) and the input vectors associated to the  $i^{th}$  subsystem.  $w_i(t) \in \mathbb{R}^{\nu_i}$  is an  $L_2$ -norm bounded external disturbance associated to the  $i^{th}$  subsystem.  $m_i$  is the number of switching modes of the  $i^{th}$  subsystem.  $r_{j_i}$  is the number of fuzzy rules associated to the  $i^{th}$  subsystem in the  $j_i^{th}$  mode; for i=1,...,n,  $j_i=1,...,m_i$  and  $s_{j_i}=1,...,r_{j_i}$ ,  $A_{s_{j_i}} \in \mathbb{R}^{\eta_i \times \eta_i}$ ,  $B_{s_{j_i}} \in \mathbb{R}^{\eta_i \times \nu_i}$ ,  $B_{s_{j_i}}^w \in \mathbb{R}^{\eta_i \times \nu_i}$  and  $C_{l_{j_i}} \in \mathbb{R}^{\rho_i \times \eta_i}$  are constant matrices describing the local dynamics of each polytops;  $B_{s_{j_i}}^w \in \mathbb{R}^{\eta_i \times \nu_a}$  and  $F_{i,\alpha,s_{j_i}} \in \mathbb{R}^{\eta_i \times \eta_a}$  express the interconnections between subsystems.  $z_{j_i}(t)$  are the premises variables and  $h_{s_{j_i}}(z_{j_i}(t))$  are positive membership functions satisfying

the convex sum proprieties 
$$\sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}}(z_{j_i}(t)) = 1; \quad \xi_{j_i}(t)$$
 is

the switching rules of the  $i^{th}$  subsystem, considered arbitrary but assumed to be real time available. These are defined such that the active system in the  $l_i^{th}$  mode lead to:

$$\begin{cases} \xi_{j_i}(t) = 1 & \text{if } j_i = l_i \\ \xi_{j_i}(t) = 0 & \text{if } j_i \neq l_i \end{cases}$$
 (2)

**Notations:** In order to lighten the mathematical expression, one assumes the scalar  $\underline{N} = \frac{1}{n-1}$ , the index i associated to the  $i^{th}$  subsystem to denote the mode  $j_i$ . The premise entries  $z_{i}$ , will be omitted when there is no ambiguities

and the following notation is employed for fuzzy matrices:

$$G_{hj} = \sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} G_{s_{j_i}}$$
 and  $Y_{h_j,h_j} = \sum_{s_{j_i}=1}^{r_{j_i}} \sum_{k_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} h_{k_{j_i}} Y_{s_{j_i},k_{j_i}}$ 

Moreover, for matrices of appropriate dimensions we will

denote: 
$$\dot{X}_{hj} = \frac{dX_{h_{ji}}}{dt}$$
 and  $(\dot{X}_{hj})^{-1} = \frac{d(X_{h_{ji}})^{-1}}{dt}$ . As usual, a star (\*) indicates a transpose quantity in a symmetric matrix and  $sym(G) = G + G^T$ . The time  $t$  will be omitted when there is no ambiguity. However, one denotes  $t_{j \to j^+}$  the switching instants of the  $i^{th}$  subsystem between the current mode  $j$  (at time  $t$ ) and the upcoming mode  $j^+$  (at time  $t^+$ ), therefore we have:

$$\begin{cases} \xi_{j} \ t = 1 \\ \xi_{j^{+}} \ t = 0 \end{cases} \text{ and } \begin{cases} \xi_{j} \ t^{+} = 0 \\ \xi_{j^{+}} \ t^{+} = 1 \end{cases}$$
 (3)

In the sequel, we will deal with the robust output-feedback disturbance attenuation for the considered class of large-scale system S. For that purpose, a set of decentralized output-feedback switched non-PDC control laws is proposed as; for i=1,...,n:

$$u_{i}(t) = \sum_{i=1}^{m_{i}} \xi_{j_{i}}(t) K_{hj} \left(X_{hj}^{9}\right)^{-1} y_{i}(t)$$
(4)

where the matrices  $K_{hj} = \sum_{k_i=1}^{r_{j_i}} h_{s_{j_i}} z_{j_i} t K_{k_{j_i}}$  and

$$X_{hj}^9 = \sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} \left( z_{j_i} \left( t \right) \right) X_{s_{j_i}}^9$$
 are the fuzzy gains to be

synthesized with 
$$X_{s_h}^9 = \left(X_{s_h}^9\right)^T > 0$$
.

Substituting (4) into (1), one expresses the overall closed-loop dynamics  $S_{cl}$  as, for i = 1,...,n:

$$\dot{x}_{i} = \sum_{j=1}^{m_{i}} \xi_{j} \left\{ \left[ A_{hj} + B_{hj} K_{hj} \left( X_{hj}^{9} \right)^{-1} C_{hj} \right] x_{i} + \sum_{\alpha=1, \alpha \neq i}^{n} F_{i,\alpha,hj} x_{\alpha} \right\}$$
(5)

Thus, the problem considered in this study can be resumed as follows:

**Problem 1:** The objective is to design the controllers (4) such that the closed-loop interconnected large-scale switched TS system (6) satisfies a robust  $H_{\infty}$  performance.

**Definition 1:** The switched interconnected large-scale system (1) is said to have a robust  $H_{\infty}$  output-feedback performance if the following conditions are satisfied:

• Condition 1 (Stability condition): With zero disturbances input condition  $w_i \equiv 0$ , for i = 1,...,n, the closed-loop dynamics (5) is stable.

• Condition 2 (Robustness condition): For all non-zero  $w_i \in L_2[0 \quad \infty)$ , under zero initial condition  $x_i(t_0) \equiv 0$ , it holds that for i = 1, ..., n,

$$J_{i} = \int_{0}^{+\infty} x_{i}^{T} x_{i} dt \le \varsigma_{i}^{2} \int_{0}^{+\infty} \left( w_{i}^{T} w_{i} + \sum_{\alpha=1, \alpha \neq i}^{n} w_{\alpha}^{T} w_{\alpha} \right) dt \qquad (6)$$

where  $\varphi_i^2$  is a positive scalars which represents the disturbance attenuation level associated to the  $i^{th}$  subsystem.

From the closed-loop dynamics (5), it can be seen that several crossing terms among the gain controllers  $K_{hj}$  and

the system's matrices 
$$\left(B_{hj}K_{hj}\left(X_{hj}\right)^{-1}C_{hj}\right)$$
 are present.

Hence, in view of the wealth of interconnections characterizing our system, these crossing terms lead surely to very conservative conditions for the design of the proposed controller. In order to decouple the crossing terms  $\left(B_{hj}K_{hj}\left(X_{hj}\right)^{-1}C_{hj}\right)$  appearing in the equation (5),

and to provide LMI conditions, we use an interesting property called the descriptor redundancy [13]-[16]. In this context, the closed-loop dynamics (6) can be alternatively expressed as follows. First, from (4), we introduce null terms such that, for i = 1, ..., n:

$$0\dot{y}_i = -y_i + C_{hi}x_i \tag{7}$$

and:

$$0 = u_i - K_{hj} \left( X_{hj}^9 \right)^{-1} y_i \tag{8}$$

Then, by considering the augmented state vectors  $\tilde{x}_i^T = \begin{bmatrix} x_i^T & y_i^T & u_i^T \end{bmatrix}$ ,  $\tilde{x}_{\alpha}^T = \begin{bmatrix} x_{\alpha}^T & y_{\alpha}^T & u_{\alpha}^T \end{bmatrix}$  and disturbances  $\tilde{w}_{i,\alpha}^T = \begin{bmatrix} \tilde{w}_i^T & \tilde{w}_{\alpha}^T \end{bmatrix}$ , the closed-loop dynamics of the large-scale system (1) under the non-PDC controller (4) can be reformulated as follows, for  $i = 1, \ldots, n$ :

$$E\dot{\tilde{x}}_{i} = \tilde{A}_{hj,hj}\tilde{x}_{i} + \sum_{\alpha=1,\alpha\neq i}^{n} \left( \tilde{F}_{i,\alpha,hj}\tilde{x}_{\alpha} + \tilde{B}_{hj}^{w\alpha}\tilde{w}_{i,\alpha} \right)$$
(9)

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{A}_{hj,hj} = \begin{bmatrix} A_{hj} & 0 & B_{hj} \\ 0 & -I & K_{hj} \left( X_{hj}^{\gamma} \right)^{-1} \\ C_{hj} & 0 & -I \end{bmatrix},$$

$$\tilde{F}_{i,\alpha,hj} = \begin{bmatrix} F_{i,\alpha,hj} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{B}_{hj}^{w\alpha} = \begin{bmatrix} \underline{N}B_{hj}^{w} & 0 \\ 0 & B_{hj}^{w\alpha} \\ 0 & 0 \end{bmatrix}$$

Note that the system (9) is a large scale switched descriptor. Hence, it is worth pointing out that the output-feedback stabilization problem of the system (1) can be

converted into the stabilization problem of the augmented system (9).

**Remark:** It may be hard to work with the first formulation of the closed-loop dynamics (6), due to the large number of crossing terms. However, the goal of our study can now be achieved by considering the augmented closed-loop dynamics (9) expressed in the descriptor form. In this context, the second condition of the definition 1, given by equation (6), can be reformulated as follows:

$$\int_{0}^{+\infty} \tilde{y}_{i}^{T} Q \tilde{y}_{i} dt \leq \varsigma_{i}^{2} \int_{0}^{+\infty} \sum_{\alpha=1,\alpha\neq i}^{n} \tilde{w}_{i,\alpha}^{T} \Xi \tilde{w}_{i,\alpha} dt$$
 (10)

with 
$$\Xi = \begin{bmatrix} \underline{N}I & 0 \\ 0 & I \end{bmatrix}$$
  $Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 

To conclude the preliminaries, let us introduce the following lemma, which will be used in the sequel.

*Lemma* [17]: Let us consider two matrices A and B with appropriate dimensions and a positive scalar  $\tau$ , the following inequality is always satisfied:

$$A^{T}B + B^{T}A \le \tau A^{T}A + \tau^{-1}B^{T}B \tag{11}$$

## III. LMI Based Decentralized Controller Design

In this section, the main result for the design of a robust  $H_{\infty}$  decentralized switched non-PDC controller (4) ensuring the closed-loop stability of (5) and the  $H_{\infty}$  disturbance rejection performance (10) is presented. It is summarized by the following theorem.

**Theorem:** Assume that for each subsystem i of (1), the active mode is denoted by  $j_i$  and, for  $j_i=1,...,m_i$  and  $s_{j_i}=1,...,r_{j_i}$ ,  $\dot{h}_{s_{j_i}}\left(z(t)\right)\geq\lambda_{s_{j_i}}$ . The overall interconnected switched TS system (1) is stabilized by a set of n decentralized switched non-PDC control laws (4) according to the definition 1, if there exists, for all combinations of i=1,...,n,  $j_i=1,...,m_i$ ,  $j_i^+=1,...,m_i$ ,  $s_{j_i}=1,...,r_{j_i}$ ,  $k_{j_i}=1,...,r_{j_i}$ ,  $k_{j_i}=1,...,r_{j_i}$  and  $l_{j_i}=1,...,r_{j_i}$ , the matrices  $X_{k_{j_i}}^1=\left(X_{k_{j_i}}^1\right)^T>0$ ,  $X_{k_{j_i}}^5=\left(X_{k_{j_i}}^5\right)^T>0$ ;  $X_{k_{j_i}}^9=\left(X_{k_{j_i}}^9\right)^T>0$   $W_{s_{j_i}s_{j_i}k_{j_i}}^1$ ,  $K_{k_{j_i}}$  and the scalars,  $\tau_{1,i}$ , ...  $\tau_{i-1,i}$ ,  $\tau_{i+1,i}$ ,...,  $\tau_{n,i}$  (excepted  $\tau_{i,i}$  which don't exist since there is no interaction between a subsystem and himself), such that the LMIs described by (12), (12), (14) and (15) are satisfied.

$$X_{k'_{i}}^{1} + W_{s_{i}k_{i}l_{i}} > 0 (12)$$

$$\begin{bmatrix}
-\mu_{j_i \to j_i^+} X_{k_{j_i}}^1 & X_{k_{j_i}}^1 \\
X_{k_{j_i}}^1 & -X_{k_{j_i}^+}^1
\end{bmatrix} \le 0$$
(13)

$$\left( \frac{\Gamma_{s_{\hat{h}}l_{\hat{h}}k_{\hat{h}}}}{\overline{X}_{k_{\hat{h}}}} \right| (*) \\
= \overline{I} < 0$$
(14)

$$\begin{pmatrix}
\Lambda_{s_{h}k_{h}} & * & \\
\hline
X_{k_{h}} & -\tau_{\alpha,i}I & \\
\begin{pmatrix}
\tilde{B}_{s_{h}}^{w,\alpha}
\end{pmatrix}^{T} & 0 & -\underline{N}\varsigma_{i}^{2}I & 0 \\
0 & -I
\end{pmatrix} < 0$$
(15)

$$\text{with} \quad \Lambda_{s_{\hat{h}}l_{\hat{h}}k_{\hat{h}}} = \begin{bmatrix} \Gamma_{s_{\hat{h}}l_{\hat{h}}k_{\hat{h}}} & * \\ \vdots & * \\ X_{k_{\hat{h}}}^2 & 0 & 0 & -I \end{bmatrix} , \quad \tilde{B}_{kj}^{wcz} = \begin{bmatrix} \underline{N}B_{kj}^{w} & 0 \\ 0 & B_{kj}^{wcz} \\ 0 & 0 \end{bmatrix} ,$$

$$\Phi_{s_{j_{i}}l_{j_{i}}k_{j_{k}}k_{j_{k}}} = \sum_{l_{j_{i}}=1}^{r_{j_{i}}} \lambda_{l_{j_{i}}} \left( X_{l_{j_{i}}}^{1} + W_{s_{j_{i}}k_{j_{k}}k_{j_{k}}} \right), X_{k_{j_{i}}} = \begin{bmatrix} X_{k_{j_{i}}}^{1} & 0 & 0 \\ 0 & X_{k_{j_{i}}}^{5} & 0 \\ 0 & 0 & X_{k_{j_{i}}}^{9} \end{bmatrix}, \qquad \text{That is to say:}$$

$$X_{hj+}^{1} - \mu_{j \to j^{+}} X_{hj}^{1} = 0$$

$$1 \text{ of the orderight multiplying}$$

$$\begin{split} \overline{\overline{I}} &= diag \left[ -\tau_{1,i} I \quad \cdots \quad -\tau_{i-1,i} I \quad -\tau_{i+1,i} I \quad \cdots \quad -\tau_{n,i} I \right], \\ \overline{\overline{X}}_{k_{-}} &= \left[ X_{k_{k_{-}}} \quad \cdots \quad X_{k_{k_{-}}} \quad X_{k_{k_{-}}} \quad \cdots \quad X_{k_{k_{-}}} \right], \end{split}$$

$$\Gamma_{s_{\lambda}l_{\lambda}k_{\lambda}} = \begin{bmatrix} sym\left(X_{k_{\lambda}}^{1}A_{s_{\lambda}}^{T}\right) & & & \\ +\tau_{i,\alpha}F_{i,\alpha,s_{\lambda}}F_{i,\alpha,s_{\lambda}}^{T} & (*) & (*) \\ -\Phi_{s_{\lambda}l_{\lambda}k_{\lambda}}F_{i,\alpha}^{T} & & (*) & (*) \\ \hline 0 & -sym\left(X_{k_{\lambda}}^{5}\right) & (*) \\ \hline X_{k_{\lambda}}^{9}\left(B_{s_{\lambda}}\right)^{T} + C_{s_{\lambda}}X_{k_{\lambda}}^{1} & \left(K_{l_{\lambda}}\right)^{T} & -sym\left(X_{k_{\lambda}}^{9}\right) \end{bmatrix} \cdot & \text{Substituting (9) into (22), we can write, } \forall t \neq t_{j \to j^{*}} :$$

**Proof:** The present proof is divided in two parts corresponding to the condition 1 and 2 given in definition 1.

Part 1 (Stability condition 1): With zero disturbances input condition  $\tilde{w}_{i,\alpha} \equiv 0$ , for i = 1,...,n. Let us define the following multiple switched non-quadratic Lyapunov-like candidate functional:

$$V(x_1, x_2, ..., x_n) = \sum_{i=1}^{n} \sum_{j_i=1}^{m_i} \xi_{j_i} v_{j_i}(x_i) > 0$$
 (16)

$$\begin{aligned} &\text{where} \quad v_{j_i} = \tilde{x}_i^T E \ X_{hj}^{-1} \, \tilde{x}_i = \tilde{x}_i^T E \left( \sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} X_{s_{j_i}} \right)^{-1} \tilde{x}_i \quad \text{and} \\ &\text{with} \quad E X_{hj} = X_{hj} E > 0 \quad , \quad X_{hj} = diag \left[ X_{hj}^1 \ X_{hj}^5 \ X_{hj}^9 \, \right] \quad , \\ &X_{hi}^1 = X_{hi}^{1T} \, . \end{aligned}$$

The augmented system (9), and implicitly the closed-loop interconnected switched system (5), is asymptotically stable if:

$$\forall t \neq t_{i \to i^+}, \ \dot{V}(x_1, x_2, ..., x_n) < 0$$
 (17)

and:

$$v_{j_{i}^{+}} t_{j \to j^{+}} \leq \mu_{j \to j^{+}} v_{j_{i}} t_{j \to j^{+}}$$

$$\tag{18}$$

where  $\mu_{i \rightarrow j^+}$  are positive scalars.

First, let us focus on the inequalities (18). Their aim is to ensure the global decreasing behavior of the Lyapunov-like function (16) at the switching time  $t_{i \to j^+}$ . These inequalities are verified if, for i = 1,...,n,  $j_i = 1,...,m_i$  $j_i^+ = 1,...,m_i$  and  $s_{j_i} = 1,...,r_{j_i}$ :

$$E X_{hj^{+}}^{-1} - \mu_{j \to j^{+}} E X_{hj}^{-1} \le 0$$
 (19)

$$X_{hj+}^{1}^{-1} - \mu_{j \to j^{+}} X_{hj}^{1}^{-1} \le 0$$
 (20)

Left and right multiplying by  $X_{hi}^1$ , then using Schur complement, (20) is equivalent to:

$$\begin{bmatrix} -\mu_{j \to j^{+}} X_{hj}^{1} & X_{hj}^{1} \\ X_{hj}^{1} & -X_{hj^{+}}^{1} \end{bmatrix} \leq 0$$
 (21)

Now, let us deal with (17), with the above defined

$$\sum_{i=1}^{n} \left[ sym \left( \dot{\tilde{x}}_{i}^{T} E \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right) + \tilde{x}_{i}^{T} E \left( \dot{X}_{hj} \right)^{-1} \tilde{x}_{i} \right] < 0$$
 (22)

Substituting (9) into (22), we can write,  $\forall t \neq t_{i \rightarrow i^+}$ :

$$\sum_{i=1}^{n} \left[ \tilde{x}_{i}^{T} \left[ sym\left( \left( X_{hj} \right)^{-1} \tilde{A}_{hj,hj} \right) + E\left( \dot{X}_{hj} \right)^{-1} \right] \tilde{x}_{i} \right] < 0$$

$$+ \sum_{\alpha=1,\alpha \neq i}^{n} sym\left( \tilde{x}_{\alpha}^{T} F_{i,\alpha,hj}^{T} \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right)$$

$$> 0$$

$$(23)$$

From (11), the inequality (23) can be bounded by,

$$\sum_{i=1}^{n} \left[ \tilde{x}_{i}^{T} \left[ sym\left( (X_{hj})^{-1} \tilde{A}_{hj,hj} \right) + E\left( \dot{X}_{hj} \right)^{-1} + \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{i,\alpha} \left( X_{hj} \right)^{-1} \tilde{F}_{i,\alpha,hj} \tilde{F}_{i,\alpha,hj}^{T} \left( X_{hj} \right)^{-1} \right] \tilde{x}_{i} + \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{i,\alpha}^{-1} \tilde{x}_{\alpha}^{T} \tilde{x}_{\alpha} \right] < 0$$
(24)

Moreover, since 
$$\sum_{i=1}^{n} \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{i,\alpha}^{-1} x_{\alpha}^{T} x_{\alpha} = \sum_{i=1}^{n} \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{\alpha,i}^{-1} x_{i}^{T} x_{i}$$

$$\forall x_{i} , (24) \text{ is satisfied if, for } i = 1,...,n \text{ and } \forall t \neq t_{i,\alpha,t}^{+} :$$

$$sym\left(\left(X_{hj}\right)^{-1}\tilde{A}_{hj,hj}\right) + E\left(\dot{X}_{hj}\right)^{-1} + \sum_{\alpha=1,\alpha\neq i}^{n} \left[\tau_{i,\alpha}\left(X_{hj}\right)^{-1}\tilde{F}_{i,\alpha,hj}\tilde{F}_{i,\alpha,hj}^{T}\left(X_{hj}\right)^{-1} + \tau_{\alpha,i}^{-1}I\right] < 0$$
(25)

Note that  $EX_{hj} = X_{hj}E > 0$ , left and right multiplying the inequalities (25) respectively by  $X_{hj}$ , the inequality (25) can be rewritten as:

$$sym(\tilde{A}_{hj,hj}X_{hj}) + EX_{hj}(\dot{X}_{hj})^{-1}X_{hj} + \sum_{\alpha=1,\alpha\neq i}^{n} \left[\tau_{i,\alpha}\tilde{F}_{i,\alpha,hj}\tilde{F}_{i,\alpha,hj}^{T} + \tau_{\alpha,i}^{-1}X_{hj}X_{hj}\right] < 0$$
(26)

Now, the aim is to obtain the inequality (14) from (26). This can be achieved with usual mathematical developments. First, note that  $-E\left(\dot{X}_{hj}\right)^{-1}=E\left(X_{hj}\right)^{-1}\dot{X}_{hj}\left(X_{hj}\right)^{-1}\leq -\Phi_{s_h^l h_h^k h_h^{k^*} h} \text{ (see [13] for more details on similar developments). Then, to deals with the term <math>X_{hj}X_{hj}$ , one applies the Schur complement. This ends that part of the proof.

Part 2 (Robustness condition 2): For all non-zero  $\tilde{w}_{i,\alpha} \in L_2[0 \infty)$ , under zero initial condition  $\tilde{x}_i(t_0) \equiv 0$ , it holds for i = 1, ..., N:

$$\sum_{i=1}^{n} \left[ \dot{v}_{i} + \tilde{x}_{i}^{T} Q \tilde{x}_{i}^{T} - \varsigma_{i}^{2} \sum_{\alpha=1, \alpha \neq i}^{N} \tilde{w}_{i,\alpha} \Xi \tilde{w}_{i,\alpha}^{T} \right] < 0$$
 (27)

wich is equivalent to:

$$\sum_{i=1}^{n} \left[ sym \left( \dot{\tilde{x}}_{i}^{T} E \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right) + \tilde{x}_{i}^{T} \left( E \left( \dot{X}_{hj} \right)^{-1} + Q \right) \tilde{x}_{i} \right] < 0 \qquad (28)$$

Substituting (9) into (28), we can write,  $\forall t \neq t_{j \to j^+}$ :

$$\sum_{i=1}^{n} \left[ \tilde{x}_{i}^{T} \left[ sym \left( \tilde{A}_{hj,hj}^{T} \left( X_{hj} \right)^{-1} \right) + Q + E \left( \dot{X}_{hj} \right)^{-1} \right] \tilde{x}_{i} \right]$$

$$+ \sum_{i=1}^{n} \left[ sym \left( \tilde{x}_{\alpha}^{T} F_{i,\alpha,hj}^{T} \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right) - \varsigma_{i}^{2} \tilde{w}_{i,\alpha}^{T} \Xi \tilde{w}_{i,\alpha} \right]$$

$$+ sym \left( \tilde{w}_{i,\alpha}^{T} \left( \tilde{B}_{hj}^{w\alpha} \right)^{T} \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right)$$

$$+ sym \left( \tilde{w}_{i,\alpha}^{T} \left( \tilde{B}_{hj}^{w\alpha} \right)^{T} \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right)$$

$$(29)$$

From (11), the inequality (23) can be bounded by,  $\forall t \neq t_{j \rightarrow j^+}$ :

$$\begin{bmatrix} \sum_{\tilde{x}_{i}^{T}}^{T} \left[ sym\left( \left( X_{hj} \right)^{-1} \tilde{A}_{hjhj} \right) + Q + E\left( \dot{X}_{hj} \right)^{-1} \\ + \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{i,\alpha} \left( X_{hj} \right)^{-1} \tilde{F}_{i,\alpha,hj} \tilde{F}_{i,\alpha,hj}^{T} \left( X_{hj} \right)^{-1} \right]^{\tilde{x}_{i}} \\ + \sum_{\alpha=1,\alpha\neq i}^{n} sym\left( \tilde{w}_{i,\alpha}^{T} \left( \tilde{B}_{hj}^{w\alpha} \right)^{T} \left( X_{hj} \right)^{-1} \tilde{x}_{i} \right) \\ + \sum_{\alpha=1}^{n} \left( \tau_{i,\alpha}^{-1} \tilde{x}_{\alpha}^{T} \tilde{x}_{\alpha} - \varsigma_{i}^{2} \tilde{w}_{i,\alpha}^{T} \Xi \tilde{w}_{i,\alpha} \right) \\ = \frac{1}{\alpha\neq i} \begin{bmatrix} sym\left( \left( X_{hj} \right)^{-1} \tilde{A}_{hjhj} \right)^{-1} \tilde{x}_{i} \\ \tilde{x}_{i} \end{bmatrix} \end{bmatrix}$$

$$(30)$$

Since 
$$\sum_{i=1}^{n} \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{i,\alpha}^{-1} x_{\alpha}^{T} x_{\alpha} = \sum_{i=1}^{n} \sum_{\alpha=1,\alpha\neq i}^{n} \tau_{\alpha,i}^{-1} x_{i}^{T} x_{i} , \quad \forall x_{i} \quad \text{and} \quad \forall t \neq t_{i \to i^{+}}, \quad (24) \text{ is satisfied if:}$$

$$\sum_{i=1}^{n} \sum_{\alpha=1,\alpha\neq i}^{n} \left( \tilde{X}_{i}^{T} \left( \underbrace{N \left( sym \left( \left( X_{hj} \right)^{-1} \tilde{A}_{hjhj} \right) + Q + E \left( \dot{X}_{hj} \right)^{-1} \right)}_{+\tau_{i,\alpha} \left( X_{h_{j_{i}}} \right)^{-1} \tilde{F}_{i,\alpha,h_{j_{i}}} \tilde{F}_{i,\alpha,h_{j_{i}}}^{T} \left( X_{h_{j_{i}}} \right)^{-1} + \tau_{\alpha,i}^{-1} I \right) \tilde{x}_{i} \right) < 0$$

$$+ sym \left( \tilde{w}_{i,\alpha}^{T} \tilde{B}_{w,\alpha,h_{j_{i}}}^{T} \left( X_{h_{j_{i}}} \right)^{-1} \tilde{x}_{i} \right) - \varsigma_{i}^{2} \tilde{w}_{i,\alpha}^{T} \Xi \tilde{w}_{i,\alpha}$$

$$(31)$$

The previous equation can be rewritten as follow:

$$\begin{bmatrix} \tilde{x}_i \\ \tilde{w}_{i,\alpha} \end{bmatrix}^T \begin{bmatrix} \Upsilon_{hj,hj,hj} & * \\ \left(\tilde{B}_{hj}^{w\alpha}\right)^T \left(X_{hj}\right)^{-1} & -\varsigma_i^2 \Xi \end{bmatrix} \begin{bmatrix} \tilde{x}_i \\ \tilde{w}_{i,\alpha} \end{bmatrix} < 0$$
(32)

With

$$\begin{split} \boldsymbol{\Upsilon}_{hj,hj,hj} &= \underline{N} \bigg( sym \Big( \big( \boldsymbol{X}_{hj} \big)^{-1} \, \tilde{\boldsymbol{A}}_{hj,hj} \Big) + \boldsymbol{Q} + E \big( \, \dot{\boldsymbol{X}}_{hj} \, \Big)^{-1} \bigg) \\ &+ \tau_{i,\alpha} \, \Big( \boldsymbol{X}_{hj} \, \Big)^{-1} \, \tilde{\boldsymbol{F}}_{i,\alpha,hj} \, \tilde{\boldsymbol{F}}_{i,\alpha,hj}^T \, \Big( \boldsymbol{X}_{hj} \, \Big)^{-1} + \tau_{\alpha,i}^{-1} \boldsymbol{I} \end{split}$$

Left and right multiplying the inequalities (25) respectively by  $\begin{bmatrix} X_{hj} & 0 \\ 0 & I \end{bmatrix}$ , it yields for i=1,...,n and  $\alpha=1,...,n$  with  $\alpha\neq i$ :

$$\begin{bmatrix}
\underline{N}\left(sym\left(\tilde{A}_{hjhj}X_{hj}\right) + X_{hj}QX_{hj} + EX_{hj}\left(\dot{X}_{hj}\right)^{-1}X_{hj}\right) \\
+\left(\tau_{i,\alpha}\tilde{F}_{i,\alpha,hj}\tilde{F}_{i,\alpha,hj}^{T} + \tau_{\alpha,i}^{-1}X_{hj}X_{hj}\right) \\
X_{h_{ji}}\left(\tilde{B}_{hj}^{w\alpha}\right)^{T} - \varsigma_{i}^{2}\Xi
\end{bmatrix} < 0$$
(33)

Finally, to obtain the LMI condition (15), similarly to the first part of this proof, from the property  $-E\left(\dot{X}_{hj}\right)^{-1}=E\left(X_{hj}\right)^{-1}\dot{X}_{hj}\left(X_{hj}\right)^{-1}\ , \quad \text{we can major the derivative } -E\dot{X}_{hj} \quad \text{by } -\Phi_{s_h l_h k_h k_h^*} \quad \text{and then apply the Schur complement.}$ 

#### IV. Numerical example

This section is dedicated to illustrate the effectiveness of the proposed LMI conditions. We consider the following system composed of two interconnected switched TS subsystems given by:

#### Subsystem 1

$$\begin{cases}
\dot{x}_{1} = \sum_{j_{1}=1}^{2} \sum_{s_{j_{1}}=1}^{2} \xi_{j_{1}} h_{s_{j_{1}}} \left[ A_{s_{j_{1}}} x_{1} + B_{s_{j_{1}}} u_{1} + B_{s_{j_{1}}}^{w} w_{1} + F_{1,2,s_{j_{1}}} x_{2} + B_{s_{j_{1}}}^{w_{2}} w_{2} \right] \\
y_{1} = \sum_{j_{1}=1}^{2} \sum_{s_{j_{1}}=1}^{2} \xi_{j_{1}} h_{s_{j_{1}}} C_{s_{j_{1}}} x_{1}
\end{cases}$$
(34)

with 
$$x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}$$
,  $A_{sj_1} = \begin{bmatrix} -2 & Ab_j \\ 0.1 & Aa_{sj} \end{bmatrix}$ ,  $B_{sj_1} = \begin{bmatrix} Bb_j & Ba_{sj} \\ 0 & 1 \end{bmatrix}$ ,

$$\begin{split} C_{sj_1} &= \begin{bmatrix} Ca_{sj} & 0.1 \\ -1 & 1 \end{bmatrix}, Bw_{sj_1} = \begin{bmatrix} wa_{sj} & wb_{j} \\ -.01 & .01 \end{bmatrix}, Bw_{2sj_1} = \begin{bmatrix} .01 & \alpha b_{j} \\ \alpha a_{sj} & .01 \end{bmatrix}, \\ F_{sj_1} &= \begin{bmatrix} .01 & .01 & Fa_{sj} \\ Fb_{j} & .01 & .1 \end{bmatrix}. \end{split}$$

In the **mode 1**, the variables values is given by  $Ab_1 = 1$ ,  $Aa_{11} = -2.1$  ,  $Aa_{21} = -1.1$  ,  $Bb_1 = -1.2$  ,  $Ba_{11} = 0$  ,  $Ba_{21} = 1.2$  ,  $Ca_{11} = -.1$  ,  $Ca_{12} = 1$  ,  $Fb_1 = 0.01$  ,  $Fa_{11} = .01$  ,  $Fa_{21} = .1$  ,  $wb_1 = 0.01$  ,  $wa_{11} = -.01$  ,  $wa_{21} = -.02$ ,  $\alpha b_1 = 0.01$ ,  $\alpha a_{11} = .02$ ,  $\alpha a_{12} = .01$ .

In the mode 2, the variables values is given by:  $Ab_2 = 0.2$  ,  $Aa_{12} = -2$  ,  $Aa_{22} = -3$  ,  $Bb_2 = -1.5$  ,  $Ba_{12} = 1$ ,  $Ba_{22} = 3$ ,  $Ca_{21} = 1$ ,  $Ca_{22} = .1$ ,  $Fb_2 = -0.01$ ,  $Fa_{21} = .2$  ,  $Fa_{22} = .02$  ,  $wb_2 = -0.05$  ,  $wa_{12} = -.05$  ,  $wa_{22} = .01$ ,  $\alpha b_2 = -0.05$ ,  $\alpha a_{21} = .04$ ,  $\alpha a_{22} = .03$ .

The membership functions:  $h_1(x_1(t)) = \sin^2(x_{11}(t))$ ,  $h_{2_{l_1}}(x_1(t)) = \sin^2(x_{12}(t)), \quad h_{i_{21}}(x_1(t)) = 1 - h_{i_{11}}(x_1(t)).$ 

$$\begin{cases} \dot{x}_{2} = \sum_{j_{2}=1}^{2} \sum_{s_{j_{2}}=1}^{2} \xi_{j_{2}} h_{s_{j_{2}}} \left[ A_{s_{j_{2}}} x_{2} + B_{s_{j_{2}}} u_{2} + B_{s_{j_{2}}}^{w} w_{2} + F_{2,1,s_{j_{2}}} x_{1} + B_{s_{j_{2}}}^{w_{1}} w_{1} \right] \\ y_{2} = \sum_{j_{2}=1}^{2} \sum_{s_{j_{2}}=1}^{2} \xi_{j_{2}} h_{s_{j_{2}}} C_{s_{j_{2}}} x_{2} \end{cases}$$
(35)

with 
$$x_2 = \begin{bmatrix} x_{21} & x_{22} & x_{23} \end{bmatrix}^T$$
,  $A_{sj_2} = \begin{bmatrix} -2 & Ab_j & 0 \\ 0 & Aa_{sj} & 0 \\ 0 & .1 & -1.1 \end{bmatrix}$ ,

$$B_{sj_2} = \begin{bmatrix} -.1 & .5 & .1 \\ -.01 & .5 & .01 \\ -.01 & Ba_{sj} & .1 \end{bmatrix} , C_{sj_2} = \begin{bmatrix} .01 & Ca_{sj} & .1 \\ -1 & .1 & 1 \\ .1 & .1 & .1 \end{bmatrix} , K_{l_{1_2}} = K_{2_{1_2}} = \begin{bmatrix} -0.7586 & 0.3474 & 0.1388 \\ 0.3475 & -0.6394 & 0.0852 \\ 0.1389 & 0.0853 & -1.0686 \end{bmatrix}$$

$$F_{sj_2} = \begin{bmatrix} .01 & .001 & Fa_{sj} \\ .01 & .01 & Fb_j \end{bmatrix}, \quad Bw_{1sj_2} = \begin{bmatrix} \alpha b_j & .05 & \alpha a_{sj} \\ .001 & .001 & .001 \end{bmatrix}, \qquad K_{1_{2_2}} = K_{2_{2_2}} = \begin{bmatrix} -0.8513 & 0.1899 & 0.0906 \\ 0.1899 & -0.8347 & 0.0661 \\ 0.0906 & 0.0661 & -0.9930 \end{bmatrix}$$

$$Bw_{sj_2} = \begin{bmatrix} wb_j & .05 & wa_{sj} \\ .001 & .001 & .001 \\ .001 & .001 & .001 \end{bmatrix}.$$

In the **mode 1**, the variables values is given by  $Ab_1 = 2$ ,  $Aa_{11} = -1$ ,  $Aa_{21} = -1.1$ ,  $Ba_{11} = .01$ ,  $Ba_{21} = .02$ ,  $Ca_{11} = -.1$  ,  $Ca_{12} = -.2$  ,  $Fb_1 = 0.1$  ,  $Fa_{11} = .2$  ,  $Fa_{21} = .02$  ,  $wb_1 = -0.01$  ,  $wa_{11} = .01$  ,  $wa_{21} = .001$  ,  $\alpha b_1 = -.01$ ,  $\alpha a_{11} = .01$ ,  $\alpha a_{12} = .001$ .

In the **mode 2**, the variables values is given by:  $Ab_2 = 1$ ,  $Aa_{12} = -2$  ,  $Aa_{22} = -3$  ,  $Ba_{12} = 0.03$  ,  $Ba_{22} = 0.04$  ,  $Ca_{21} = -.4$ ,  $Ca_{22} = -.3$ ,  $Fb_2 = 0.2$ ,  $Fa_{21} = Fa_{22} = .4$ ,  $wb_2 = 0.01$  ,  $wa_{12} = .02$  ,  $wa_{22} = .05$  ,  $\alpha b_2 = .01$  ,  $\alpha a_{21} = .02$ ,  $\alpha a_{22} = .05$ 

and the membership functions  $h_{1}(x_2) = \sin^2(x_{21})$ ,  $h_{2_{12}}(x_2) = \sin^2(x_{22}), h_{i_{22}}(x_2) = 1 - h_{i_{12}}(x_2).$ 

Let us assume that each subsystem switches under within frontier defined by  $H_{11} = 0.9x_{11} + x_{12}$  $H_{12} = -0.2x_{11} + 9x_{12}$  $\mathbf{H}_{21} = -x_{21} + x_{22}$ ,  $H_{22} = x_{21} - 2x_{22}$ . The external disturbances  $w_1$  and  $w_2$ are considered as white noise sequences.

A set of decentralized switched controllers (4) is synthesized based on theorem 1 via the Matlab LMI toolbox. To do so, the lower bounds of the derivatives of membership functions are prefixed  $\lambda_{l_{1_1}} = \lambda_{l_{2_1}} = \lambda_{l_{1_2}} = \lambda_{l_{2_2}} = -6$ , and the disturbance attenuation level by  $\varsigma_1^2 = 1.7$  and  $\varsigma_2^2 = 1.5$ . The solution of the proposed theorem leads to the synthesis of two decentralized non-PDC switched TS controllers (4) with the following gain matrices:

#### 1rst TS switched sub-controller:

$$\begin{split} K_{\mathbf{l}_{\mathbf{l}_{\mathbf{l}}}} &= K_{\mathbf{2}_{\mathbf{l}_{\mathbf{l}}}} = 10^{-2} * \begin{bmatrix} -9.04 & -0.72 \\ -0.72 & -4.21 \end{bmatrix}, \\ K_{\mathbf{l}_{\mathbf{2}_{\mathbf{l}}}} &= K_{\mathbf{2}_{\mathbf{2}_{\mathbf{l}}}} = 10^{-2} * \begin{bmatrix} -15.37 & 5.90 \\ 5.90 & -14.14 \end{bmatrix}, \\ X_{\mathbf{l}_{\mathbf{l}_{\mathbf{l}}}}^{9} &= \begin{bmatrix} 0.2427 & -0.1589 \\ -0.1589 & 0.1892 \end{bmatrix}, \quad X_{\mathbf{2}_{\mathbf{l}_{\mathbf{l}}}}^{9} &= \begin{bmatrix} 0.2494 & -0.1589 \\ -0.1589 & 0.1936 \end{bmatrix}, \\ X_{\mathbf{l}_{\mathbf{2}_{\mathbf{l}}}}^{9} &= \begin{bmatrix} 0.2449 & -0.1056 \\ -0.1056 & 0.3855 \end{bmatrix}, X_{\mathbf{2}_{\mathbf{2}_{\mathbf{l}}}}^{9} &= \begin{bmatrix} 0.2826 & -0.125 \\ -0.125 & 0.42 \end{bmatrix}. \end{split}$$

### $2^{sd}\ TS$ switched sub-controller:

$$K_{1_{1_2}} = K_{2_{1_2}} = \begin{bmatrix} -0.7586 & 0.3474 & 0.1388 \\ 0.3475 & -0.6394 & 0.0852 \\ 0.1389 & 0.0853 & -1.0686 \end{bmatrix},$$

$$K_{1_{2_2}} = K_{2_{2_2}} = \begin{bmatrix} -0.8513 & 0.1899 & 0.0906 \\ 0.1899 & -0.8347 & 0.0661 \\ 0.0906 & 0.0661 & -0.9930 \end{bmatrix},$$

$$X_{\mathsf{l}_{\mathsf{l}_2}}^9 = \begin{bmatrix} 2.0615 & -1.4032 & -0.9036 \\ -1.4032 & 1.4636 & -0.0925 \\ -0.9036 & -0.0925 & 4.035 \end{bmatrix},$$

$$X_{2_{1_{2}}}^{9} = \begin{bmatrix} 2.0064 & -1.361 & -0.7552 \\ -1.361 & 1.5038 & -0.0975 \\ -0.7552 & -0.0975 & 3.7587 \end{bmatrix},$$

$$X_{1_{2_2}}^9 = \begin{bmatrix} 2.1295 & -0.8043 & -0.2885 \\ -0.8043 & 2.1742 & -0.1705 \\ -0.2885 & -0.1705 & 2.9487 \end{bmatrix},$$

$$X_{1_{2_2}}^9 = \begin{bmatrix} 2.1104 & -0.8093 & -0.2917 \\ -0.8093 & 2.1822 & -0.1628 \\ -0.2917 & -0.1628 & 2.9275 \end{bmatrix}.$$

The close-loop subsystems' dynamics are shown in Fig. 1 and Fig. 2, for the initial states  $x_1(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}^T$  and VOL. 0, NO. 0, 0-0, DEC. 0000

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 $x_2(0) = \begin{bmatrix} -1 & 1.5 & -1 \end{bmatrix}^T$ . Moreover, Fig. 3 and Fig.4 shows the control signals as well as the switching modes' evolution. As expected, the synthesized decentralized switched controller stabilizes the overall large scale switched system composed of (33) and (34).

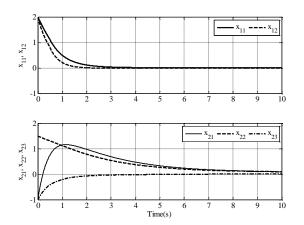


Fig. 1. Closed-loop state responses of the interconnected switched Takagi-Sugeno systems.

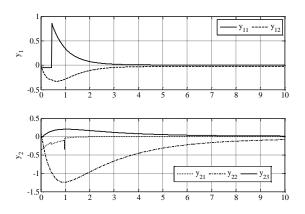


Fig. 2. Outputs trajectories of the overall closed-loop interconnected switched Takagi-Sugeno system.

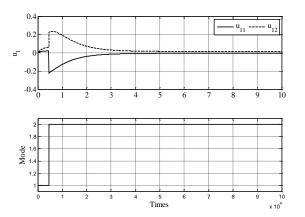


Fig. 3. Control signal and switched laws' evolutions of the first subsystem.

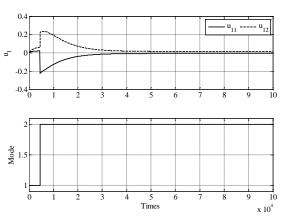


Fig. 4. Control signal and switched laws' evolutions of the second subsystem.

#### v. CONCLUSIONS

This study has focused on large scale switched nonlinear systems where each nonlinear mode has been represented by a fuzzy TS system. To ensure the stability of the whole system in closed-loop, a set of decentralized switched non-PDC controllers has been considered. Therefore, LMI based conditions for the design of decentralized controllers have been proposed through the consideration of a multiple switched non-quadratic Lyapunov-like function candidate and by using the descriptor redundancy formulation. Finally, a numerical example has been proposed to show the effectiveness of the proposed approach. An extension of the proposed approach to general switched systems under asynchronous switches will be the focus of our future works.

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