

Supplementary material for “Inferential procedures for partially observed functional data”*

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Abstract: This supplementary document contains additional simulation results and further results of the data analysis.

Key words and phrases: Bootstrap; covariance operator; functional data; K -sample test; partial observation; principal components.

S1 Extended simulation results

Table S1 is an extended version of Table 1 presented in the main body of the paper. It includes additional simulation results for tests of equal means for non-Gaussian distributed curves and for groups with unequal covariance operators. The same model as in the paper is used except that for the non-Gaussian case independent t_5 distributed coefficients are generated and for the case of unequal covariance operators we set $\lambda_{3,0} = 0.2$. Since the empirical size deviates from the nominal level in some cases, Table S2 additionally reports size-adjusted powers for the same settings using the method described by Lloyd (2005, Subsection 3.2).

Table S3 reports results for tests of equal covariance operators. In addition to the results presented in Table 2 in the main body of the paper it contains results for t_5 distributed coefficients in the model for random curves. Table S4 reports size-adjusted powers for the same settings.

S2 Additional results for the data analysis

Fig. S1 contains additional plots of the covariance function estimates of the heart rate data shown in the main body of the paper. Fig. S2 shows the null estimates of the covariance functions and their leading eigenfunctions that the projection covariance test uses. Components of the score vector standardized by their estimated standard deviation are plotted in Fig. S3

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Table S1

Empirical rejection probability (in %) of the L^2 test, T_{L^2} , and projection test, T_d , of equal means. A dash indicates the same value as on the preceding row. The observation patterns (1)–(9) and mean configurations A–D are described in Section 5 of the paper.

Distrib.	Covar. oper.	Observ. pattern	Mean configuration							
			A		B		C		D	
			T_{L^2}	T_d	T_{L^2}	T_d	T_{L^2}	T_d	T_{L^2}	T_d
Gaussian	Equal	(1)	5.6	6.2	69	60	49	56	52	63
		(2)	5.4	6.7	59	52	28	29	38	50
		(3)	—	—	—	—	50	56	44	62
		(4)	4.4	6.5	66	58	51	57	51	62
		(5)	—	—	—	—	44	49	50	58
		(6)	5.4	7.1	58	51	50	55	42	49
		(7)	—	—	—	—	28	34	37	42
		(8)	5.4	5.8	55	47	34	37	42	48
		(9)	5.4	7.8	37	40	20	23	26	34
Gaussian	Unequal	(1)	4.2	5.2	79	75	58	63	57	67
		(2)	4.0	5.6	66	62	28	32	37	52
		(3)	—	—	—	—	56	62	47	66
		(4)	4.0	5.7	77	72	58	62	55	64
		(5)	—	—	—	—	50	55	53	63
		(6)	3.9	4.9	64	60	55	57	43	52
		(7)	—	—	—	—	29	36	38	46
		(8)	4.5	7.0	64	62	39	42	47	54
		(9)	4.0	6.5	42	48	23	25	27	38
t_5	Equal	(1)	5.4	7.3	72	61	51	58	54	63
		(2)	4.7	7.6	58	53	27	30	38	52
		(3)	—	—	—	—	50	60	44	63
		(4)	5.1	6.4	70	60	52	57	51	60
		(5)	—	—	—	—	46	52	50	60
		(6)	3.7	6.1	56	50	50	54	41	50
		(7)	—	—	—	—	27	32	37	43
		(8)	5.1	7.1	58	52	33	36	44	51
		(9)	5.4	6.6	38	42	21	24	26	34
t_5	Unequal	(1)	5.8	7.4	82	77	59	65	60	68
		(2)	4.7	6.9	68	64	32	35	44	57
		(3)	—	—	—	—	60	66	50	68
		(4)	5.2	6.7	80	76	62	65	59	66
		(5)	—	—	—	—	53	60	56	65
		(6)	3.9	6.1	65	63	57	61	47	57
		(7)	—	—	—	—	32	37	42	50
		(8)	4.8	7.5	65	64	39	42	50	56
		(9)	5.5	6.2	44	50	24	28	30	40

Table S2

Size-adjusted empirical power (in %) for the same settings as in Table S1.

Distrib.	Covar. oper.	Observ. pattern	Mean configuration					
			B		C		D	
			T_{L^2}	T_d	T_{L^2}	T_d	T_{L^2}	T_d
Gaussian	Equal	(1)	66	56	47	52	49	59
		(2)	56	43	25	23	34	41
		(3)	—	—	47	48	40	54
		(4)	68	52	52	48	52	54
		(5)	—	—	45	43	51	51
		(6)	58	46	50	49	42	45
		(7)	—	—	28	29	37	37
		(8)	54	45	34	34	41	45
		(9)	36	33	20	17	26	27
Gaussian	Unequal	(1)	83	73	63	62	62	66
		(2)	72	59	35	29	44	49
		(3)	—	—	62	59	56	64
		(4)	81	72	63	62	61	63
		(5)	—	—	56	55	59	62
		(6)	68	60	60	57	47	53
		(7)	—	—	34	36	43	46
		(8)	67	54	42	36	49	48
		(9)	45	45	25	23	31	35
t_5	Equal	(1)	71	55	50	51	52	57
		(2)	60	44	28	23	39	42
		(3)	—	—	51	47	46	53
		(4)	69	53	51	53	50	56
		(5)	—	—	44	45	49	55
		(6)	60	48	53	52	45	48
		(7)	—	—	31	30	40	40
		(8)	57	44	32	30	43	44
		(9)	38	38	21	20	26	31
t_5	Unequal	(1)	80	71	58	59	58	62
		(2)	68	56	32	27	44	48
		(3)	—	—	61	56	50	60
		(4)	80	71	62	60	59	62
		(5)	—	—	53	54	56	61
		(6)	70	61	61	57	51	54
		(7)	—	—	37	35	46	47
		(8)	66	56	40	35	51	49
		(9)	43	45	23	24	28	36

Table S3

Empirical rejection probability (in %) of the Hilbert–Schmidt norm test, S_{HS} , projection test, S_d , and square root covariance test, S_{sqr} , of equal covariance operators. A dash indicates the same value as on the preceding row. The observation patterns (1)–(5) and covariance configurations A–D are described in Section 5 of the paper.

Distrib.	Observ. pattern	Covariance configuration											
		A			B			C			D		
		S_{HS}	S_d	S_{sqr}	S_{HS}	S_d	S_{sqr}	S_{HS}	S_d	S_{sqr}	S_{HS}	S_d	S_{sqr}
Gaussian	(1)	5.4	5.8	4.8	69	82	80	69	58	69	78	62	81
	(2)	4.6	6.4	4.9	54	63	41	37	32	38	76	64	54
	(3)	—	—	—	—	—	—	—	—	—	46	30	48
	(4)	5.0	5.1	5.8	64	74	72	61	53	62	72	56	73
	(5)	—	—	—	—	—	—	—	—	—	77	60	77
t_5	(1)	3.6	5.7	4.2	26	32	35	30	26	35	38	41	44
	(2)	3.3	6.5	3.4	22	31	18	14	17	16	38	41	23
	(3)	—	—	—	—	—	—	—	—	—	16	16	20
	(4)	4.0	6.4	4.8	23	32	30	25	25	31	30	33	34
	(5)	—	—	—	—	—	—	—	—	—	36	38	40

Table S4

Size-adjusted empirical power (in %) for the same settings as in Table S3.

Distrib.	Observ. pattern	Covariance configuration								
		B			C			D		
		S_{HS}	S_d	S_{sqr}	S_{HS}	S_d	S_{sqr}	S_{HS}	S_d	S_{sqr}
Gaussian	(1)	66	79	81	67	56	69	78	60	81
	(2)	54	59	42	38	29	38	78	60	55
	(3)	—	—	—	—	—	—	47	26	49
	(4)	64	73	69	61	52	59	72	56	71
	(5)	—	—	—	—	—	—	77	59	74
t_5	(1)	32	29	39	36	23	38	44	38	48
	(2)	23	24	20	18	14	18	40	33	26
	(3)	—	—	—	—	—	—	20	12	23
	(4)	29	26	31	31	19	32	37	27	35
	(5)	—	—	—	—	—	—	43	32	41

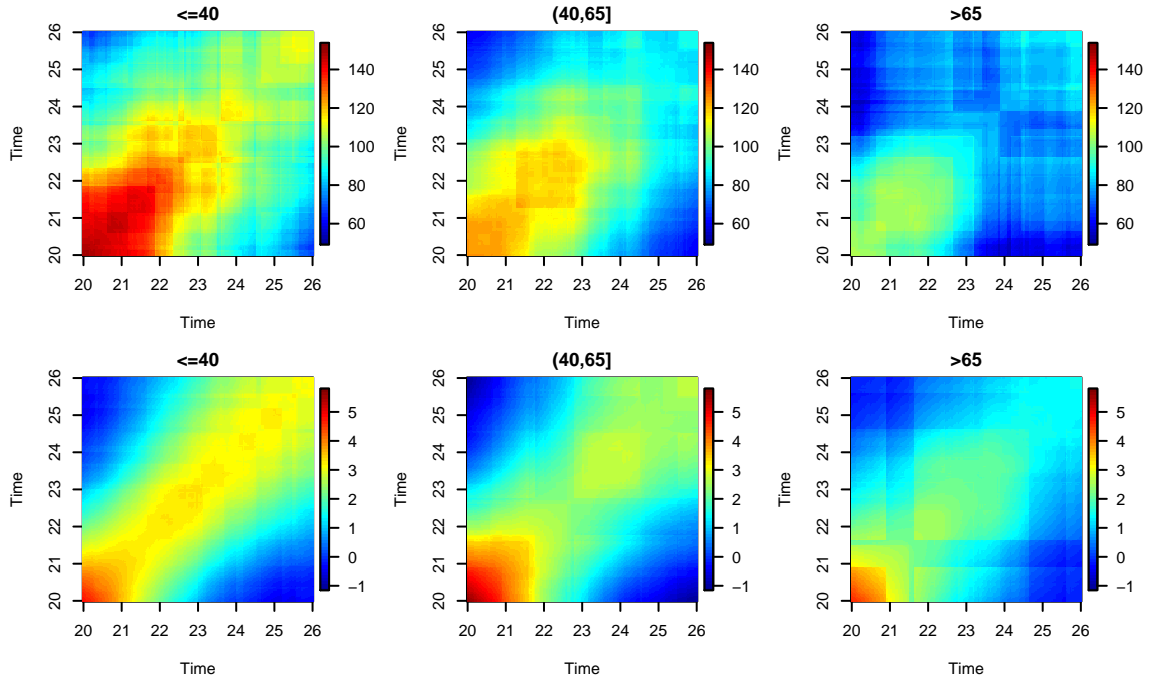


Fig. S1. Estimated covariance functions of heart rate profiles (top row) and of their derivatives (bottom row) in age groups.

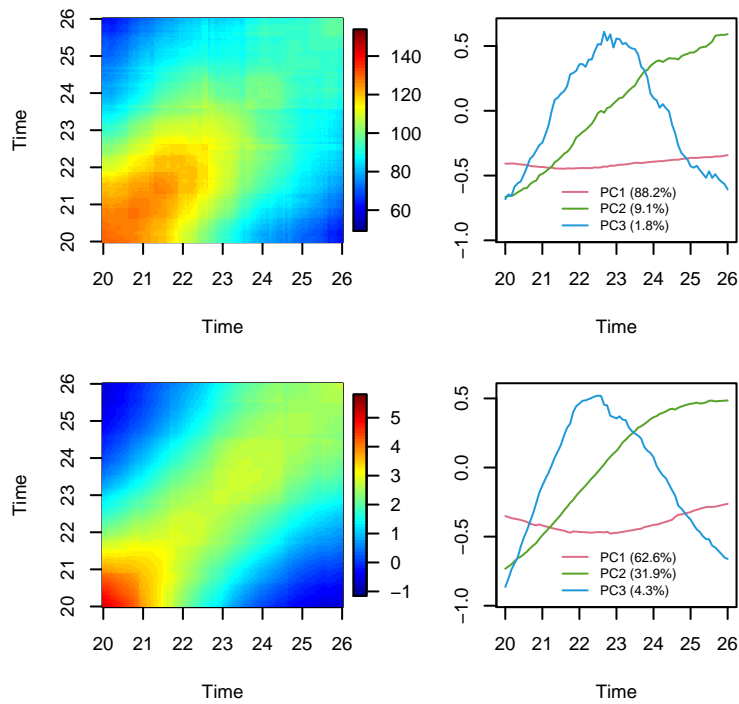


Fig. S2. The null estimate of the covariance function (left column) and its three leading principal components (right column) for heart rate profiles (top row) and for their first derivative (bottom row).

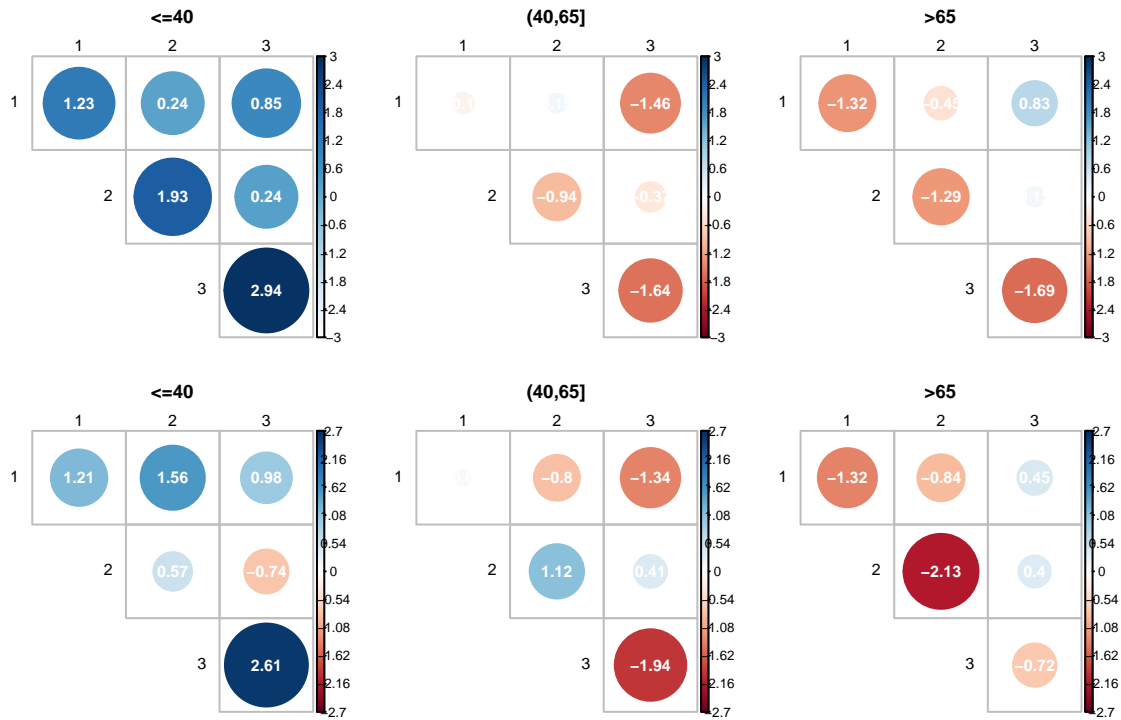


Fig. S3. Standardized components of the score vector for testing equal covariances contrasting age groups against the null for heart rate profiles (top row) and for their derivatives (bottom row).

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References

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