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To cite this version:

Adeline Crépieux, S. Sahoo, T. Q. Duong, Redouane Zamoum, M. Lavagna. Non-symmetrized quantum noise in a Kondo quantum dot. AIP Advances, 2018, 8 (10), pp.101326. 10.1063/1.5043076 . hal-01896077

HAL Id: hal-01896077

https://hal.science/hal-01896077

Submitted on 16 Oct 2018

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Citation: AIP Advances **8**, 101326 (2018); doi: 10.1063/1.5043076
View online: https://doi.org/10.1063/1.5043076
View Table of Contents: http://aip.scitation.org/toc/adv/8/10
Published by the American Institute of Physics

AIP ADVANCES **8**, 101326 (2018)

Non-symmetrized quantum noise in a Kondo quantum dot

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(Received 5 June 2018; accepted 4 September 2018; published online 4 October 2018)

The fluctuations of electrical current provide information on the dynamics of electrons in quantum devices. Understanding the nature of these fluctuations in a quantum dot is thus a crucial step insofar as this system is the elementary brick of quantum circuits. In this context, we develop a theory for calculating the quantum noise at finite frequency in a quantum dot connected to two reservoirs in the presence of interactions and for any symmetry of the couplings to the reservoirs. This theory is developed in the framework of the Keldysh non-equilibrium Green function technique. We establish an analytical expression for the quantum noise in terms of the various transmission amplitudes between the reservoirs and of some effective transmission coefficient which we define. We then study the noise as a function of the dot energy level and the bias voltage. The effects of both Coulomb interactions in the dot and asymmetric couplings with the reservoirs are characterized. © *2018 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).* https://doi.org/10.1063/1.5043076

I. INTRODUCTION

The understanding of noise in quantum systems is a fundamental issue when one wants to control the transfer of charges in an accurate way. The efforts in that direction in the last ten years are numerous both from the experimental side[1–7] and from the theoretical side.[8–18] Some of the main issues raised by the works on noise in quantum systems are the following: (i) Is the measured noise the symmetrized one or the non-symmetrized one? (ii) Can we have over bias noise at low temperature? (iii) How is the noise affected by the presence of Coulomb interactions? (iv) Does the asymmetry in the couplings between the system and the reservoirs change the noise? The answer to the first point is known: the measured noise will be the symmetrized noise for active (classical) detector whereas it will be the non-symmetrized one for passive detector.[19–22] The second point is the subject of several studies.[23–25] To answer to the third and fourth points, we develop a theory for calculating the noise at finite frequency in a quantum dot (QD) coupled to two reservoirs, in the presence of Coulomb interactions in the dot and asymmetry in the couplings to the reservoirs. By using the Keldysh non-equilibrium Green function technique, we establish an analytical expression for the noise in terms of the transmission amplitudes between the reservoirs and of some effective transmission coefficients which will be defined. The result that we obtain for the noise can be considered as the analog of the Meir-Wingreen formula[26] for the current. Moreover, a physical interpretation is given on the basis of the transmission of one electron-hole pair to one of the reservoirs, where it emits an energy corresponding to the measurement frequency after recombination. The results for the noise as a function of the dot energy level and voltage show a zero value until $|eV| = h\nu$, where ν is the frequency, followed by a signal which strongly

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depends on the presence of Coulomb interactions in the dot and on the asymmetry of the couplings to the reservoirs. These findings are compared to measurements recently performed in a Kondo carbon nanotube QD.[4,7]

II. FINITE-FREQUENCY NOISE

We consider a single level interacting QD coupled to a left (L) and a right (R) reservoirs as depicted on Fig. 1. The couplings between the QD and the reservoirs are denoted $\Gamma_{L,R}$ and can be arbitrary. The asymmetry factor is defined as $a = \Gamma_L/\Gamma_R$. When the QD is in a steady state and the flat wideband limit is considered, we show that the finite-frequency non-symmetrized noise is given by the expression:[27]

$$\mathcal{S}_{\alpha\beta}(\nu) = \frac{e^2}{h} \sum_{\gamma\delta} \int_{-\infty}^{\infty} d\varepsilon M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \nu) f_\gamma^e(\varepsilon) f_\delta^h(\varepsilon - h\nu), \tag{1}$$

where $f_\gamma^e(\varepsilon) = 1/(1 + \exp(\varepsilon - \mu_\gamma)/k_B T)$ is the distribution function for electrons with energy ε in the γ reservoir, and $f_\delta^h(\varepsilon - h\nu) = 1 - f_\delta^e(\varepsilon - h\nu)$, the distribution function for holes with energy $\varepsilon - h\nu$ in the δ reservoir. The indices α, β, γ and δ can take either the L value when it relates to the left reservoir, or the R value when it relates to the right reservoir. The expressions for the matrix elements entering in Eq. (1) and denoted as $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \nu)$ are given in Table I. They depend on the transmission amplitudes $t_{\alpha\beta}(\varepsilon)$, the reflexion amplitudes $r_{\alpha\alpha}(\varepsilon)$, the transmission coefficients $\mathcal{T}_{\alpha\beta}(\varepsilon)$, and some effective transmission coefficients $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$, which are defined as:

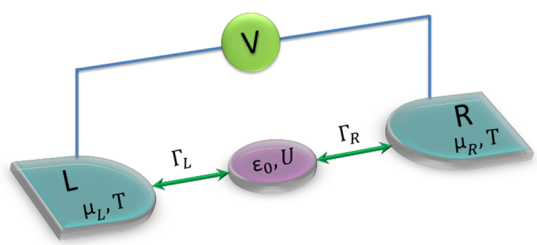

FIG. 1. Schematic view of the single level QD (in purple) coupled to biased reservoirs (in blue). $\mu_{L,R}$ are the chemical potentials of the reservoirs with $eV = \mu_L - \mu_R$, and T their temperature. The dot is characterized by its level energy ε_0 and Coulomb energy U. The coupling energies to the reservoirs, $\Gamma_{L,R}$, can be distinct as observed in many experiments.

TABLE I. Expressions of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \nu)$ involved in the Eq. (1) for the noise $S_{\alpha\beta}(\nu)$ of an interacting QD with arbitrary coupling symmetry to the reservoirs.

$M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$	$\gamma = \delta = L$	$\gamma = \delta = R$	$\gamma = L, \delta = R$	$\gamma = R, \delta = L$
$\alpha = L$ $\beta = L$	$\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon)\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon - h\nu)$ $+ \lvert t_{LL}(\varepsilon) - t_{LL}(\varepsilon - h\nu)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon - h\nu)$	$[1 - \mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon - h\nu)$	$\mathcal{T}_{LR}(\varepsilon)[1$ $-\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon - h\nu)]$
$\alpha = R$ $\beta = R$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon - h\nu)$	$\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon)\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon - h\nu)$ $+ \lvert t_{RR}(\varepsilon) - t_{RR}(\varepsilon - h\nu)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)[1 - \mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon - h\nu)]$	$[1 - \mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon)]$ $\times \mathcal{T}_{LR}(\varepsilon - h\nu)$
$\alpha = L$ $\beta = R$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon - h\nu)$ $\times [r_{LL}^*(\varepsilon)r_{LL}(\varepsilon - h\nu) - 1]$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon - h\nu)$ $\times [r_{RR}(\varepsilon)r_{RR}^*(\varepsilon - h\nu) - 1]$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon - h\nu)$ $\times r_{LL}^*(\varepsilon)r_{RR}^*(\varepsilon - h\nu)$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon - h\nu)$ $\times\ r_{RR}(\varepsilon)r_{LL}(\varepsilon - h\nu)$
$\alpha = R$ $\beta = L$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon - h\nu)$ $\times [r_{LL}(\varepsilon)r_{LL}^*(\varepsilon - h\nu) - 1]$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon - h\nu)$ $\times [r_{RR}^*(\varepsilon)r_{RR}(\varepsilon - h\nu) - 1]$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon - h\nu)$ $\times\ r_{LL}(\varepsilon)r_{RR}(\varepsilon - h\nu)$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon - h\nu)$ $\times r_{RR}^*(\varepsilon)r_{LL}^*(\varepsilon - h\nu)$

$$t_{\alpha\beta}(\varepsilon) = i\sqrt{\Gamma_\alpha \Gamma_\beta} G^r(\varepsilon), \tag{2}$$

$$r_{\alpha\alpha}(\varepsilon) = 1 - t_{\alpha\alpha}(\varepsilon), \tag{3}$$

$$\mathcal{T}_{\alpha\beta}(\varepsilon) = |t_{\alpha\beta}(\varepsilon)|^2, \tag{4}$$

$$\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon) = 2\mathrm{Re}\{t_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon), \tag{5}$$

where $G^r(\varepsilon)$ is the retarded Green function in the QD, and Γ_α is the coupling strength between the QD and the α reservoir. Eq. (1) is obtained considering the approximation in which the two-particle Green function in the dot is factorized into a product of two single-particle Green functions in the dot. From Eqs. (2)–(5), we see that once $G^r(\varepsilon)$ is known, the transmission amplitudes and coefficients are entirely determined, and consequently, the noise given by Eq. (1) can be calculated explicitly. We want to underline that the effective transmission coefficient defined in Eq. (5) takes into account the inelastic scattering contributions.[28,29] When only elastic scattering is present or/and for a non-interacting system, $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$ coincides with $\mathcal{T}_{LR}(\varepsilon)$ since in that case, we have: $2\mathrm{Re}\{t_{\alpha\alpha}(\varepsilon)\} = \mathcal{T}_{\alpha\alpha}(\varepsilon) + \mathcal{T}_{LR}(\varepsilon)$, thanks to the optical theorem.

According to Eq. (1), $\mathcal{S}_{\alpha\beta}(\nu)$ is given by the summation over ε and all possible configurations $\{\gamma, \delta\}$, of the transmission element $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \nu)$ weighted by the factor $f_\gamma^e(\varepsilon) f_\delta^h(\varepsilon - h\nu)$ corresponding to the probability of having a pair formed by an electron of energy ε in the γ reservoir and a hole of energy $\varepsilon - h\nu$ in the δ reservoir. Hence we interpret the auto-correlator $\mathcal{S}_{\alpha\alpha}(\nu)$ as the probability of transmission of an electron-hole pair from all possible configurations, to the final state for which both electron and hole are in the α reservoir, where by recombining it emits an energy $h\nu$. The additional presence of inelastic scattering does not affect this interpretation.[27] In the case when there are several possible transmission paths, as happens for $M_{\alpha\alpha}^{\alpha\alpha}(\varepsilon, \nu)$, we point out the importance of considering the quantum superposition of the transmission amplitudes for all possible transmission paths.[30]

III. KONDO QUANTUM DOT

The retarded Green function $G^r(\varepsilon)$ for the interacting single level QD is determined numerically by using a self-consistent renormalized equation-of-motion approach,[31–33] which applies to both equilibrium and non-equilibrium situations. Note that in the presence of interactions, i.e. when $U \neq 0$, $G^r(\varepsilon)$ depends on the chemical potential μ_L and μ_R. When one incorporates the expression of the Green function into Eqs. (1)–(5), we are able to calculate both the auto-correlators $\mathcal{S}_{LL}(\nu)$ and $\mathcal{S}_{RR}(\nu)$, the cross-correlators $\mathcal{S}_{LR}(\nu)$ and $\mathcal{S}_{RL}(\nu)$, and the "total" noise defined as

$$\mathcal{S}_{\mathrm{tot}}(\nu) = \frac{\mathcal{S}_{LL}(\nu) + a^2 \mathcal{S}_{RR}(\nu) - a[\mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)]}{(1+a)^2}. \tag{6}$$

This total noise corresponds to the noise which is measured in experiments.[34–36] In Fig. 2, we report the color-scale plots of $\mathcal{S}_{LL}(\nu)$, $\mathcal{S}_{RR}(\nu)$, $2\mathrm{Re}\{\mathcal{S}_{LR}(\nu)\} = \mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)$ and $\mathcal{S}_{tot}(\nu)$ as a function of both dot energy ε_0 and voltage V for four sets of parameters: (a) $U = 0$ and $a = 1$, (b) $U = 0$ and $a = 4$, (c) $U = 3$ meV and $a = 1$, and (d) $U = 3$ meV and $a = 4$. We underline that with our choice of parameters, the estimation of the Kondo temperature with the help of the Haldane formula $k_B T_K \approx \sqrt{U\Gamma/2}\exp(\pi\varepsilon_0(\varepsilon_0 + U)/2U\Gamma)$ gives $T_K \approx 4.38$ K, which is much larger than the temperature in the reservoirs ($T = 80$ mK) and larger than the frequency ($\nu = 78$ GHz ≈ 3.74 K), which ensures the QD to be in the Kondo regime when $U = 3$ meV.

We remark first that at voltage smaller in absolute value than the frequency, here $\nu = 78$ GHz ($\approx$0.32 meV), the noise is equal to zero in all graphs, as expected at low temperature (here $T = 80$ mK) for the reason that the system cannot emit energy at a frequency larger than the energy $|eV|$ provided to it. Thus, there is a central region of width equal to $2h\nu$ (delimited by two parallel horizontal dashed lines) in the $\{\varepsilon_0, eV\}$ plane inside which the noise is strongly suppressed, in full agreement with experiments performed in a carbon nanotube Kondo QD.[4,7]

Next, we turn our interest to the effect of interactions on the dependence of the cross-correlator $\mathcal{S}_{LR}(\nu)$ with ε_0 and V. We note that when interactions are present ($U \neq 0$), the real part of the

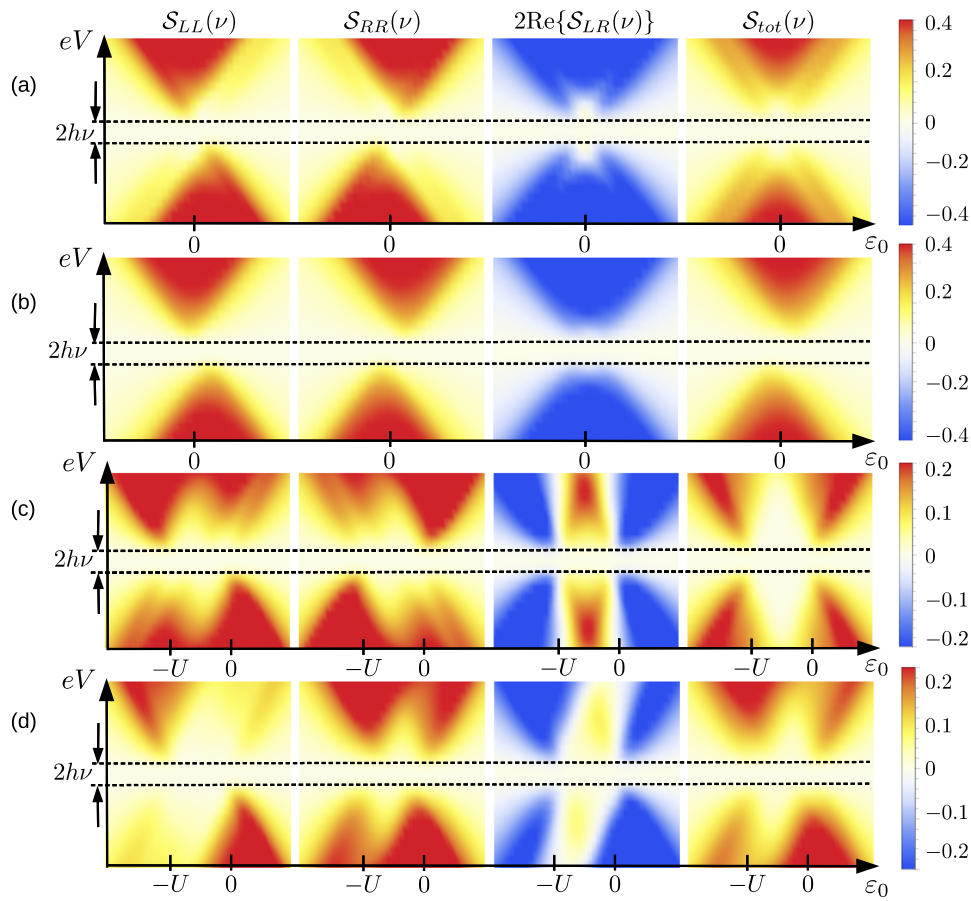

FIG. 2. Color-scale plots of $\mathcal{S}_{LL}(\nu)$, $\mathcal{S}_{RR}(\nu)$, $2\mathrm{Re}\{\mathcal{S}_{LR}(\nu)\}$ and $\mathcal{S}_{tot}(\nu)$ as a function of the dot energy level ε_0 (horizontal axis) and the bias voltage eV (vertical axis) at frequency ν = 78 GHz and temperature T = 80 mK. The chemical potentials are taken symmetrical: $\mu_L = eV/2$, $\mu_R = -eV/2$ and the interval of variation for the bias voltage eV is [-4 meV,4 meV]. The two top rows are obtained for a non-interacting QD (U = 0) within the interval $\varepsilon_0 \in$[− 3 meV,3 meV] whereas the two bottom rows corresponds to an interacting QD (U = 3 meV) within the interval $\varepsilon_0 \in$[− 6 meV,3 meV]. (a) Non-interacting U = 0; Symmetric coupling a = 1. (b) Non-interacting U = 0; Asymmetric coupling a = 4. (c) Interacting U = 3 meV; Symmetric coupling a = 1. (d) Interacting U = 3 meV; Asymmetric coupling a = 4.

cross-correlator changes its sign from negative sign (blue regions) to positive sign (yellow-red regions) when ε_0 varies (see the third column in Figs. 2(c) and 2(d)). This is not the case in the absence of interactions (see the third column in Figs. 2(a) and 2(b)). Indeed, in that case the real part of the cross-correlator stays negative (blue) as expected for carriers (here electrons) obeying a fermionic statistic. It means that when interactions are absent, the statistic of the carriers is fermionic whereas in the presence of interactions, the statistic of the carriers looks bosonic-like in some regions and fermionic-like in some others regions of the $\{\varepsilon_0, eV\}$ plane. Thus, a positive sign in the real part of the cross-correlator can be seen as the seal of the Coulomb interactions present in the QD.

We now focus on the effect of interactions on the profile of the auto-correlators $\mathcal{S}_{LL}(\nu)$ and $\mathcal{S}_{RR}(\nu)$ shown on the first and second columns in Fig. 2. We remark that the intensity of the auto-correlators is reduced when interactions are present (compare the color scale intensities in the graphs of Figs. 2(c) and 2(d) to the ones of Figs. 2(a) and 2(b)), in full agreement with the fact that the charge becomes frozen when the QD is in the Kondo regime,[37,38] leading to a reduction of the noise. We also remark the doubling of the number of red triangles in the color-scale plots of $\mathcal{S}_{LL}(\nu)$ and $\mathcal{S}_{RR}(\nu)$ and the appearance of a more complex structure when $U \neq 0$ in comparison to the U = 0 case: notably, there appears a Coulomb diamond-like structure, centered around the point of coordinates ($\varepsilon_0 = -U/2$, eV = 0) in the $\{\varepsilon_0, eV\}$ plane, inside which the noise is strongly reduced. This means that by setting

adequately the values of ε_0 and eV inside the region defined by this structure, one could reduce drastically the noise in experiments.

Finally, we discuss the effect of the coupling asymmetry on the noise color-scale plots. Whereas the dependences of the auto-correlators $\mathcal{S}_{LL}(\nu)$ and $\mathcal{S}_{RR}(\nu)$ are neither odd nor even functions of both ε_0 and eV, we note that the real part of the cross-correlator $\mathcal{S}_{LR}(\nu)$ and the total noise $\mathcal{S}_{\text{tot}}(\nu)$ are even functions of both ε_0 and eV when the couplings are symmetrical ($a = 1$) as shown in Figs. 2(a) and 2(c). This is no longer the case for asymmetric couplings ($a = 4$) as shown in Figs. 2(b) and 2(d). We also observe that in the presence of interactions the noise is strengthened in the less-coupled reservoir, here the R reservoir since the value $a = 4$ corresponds to $\Gamma_R = \Gamma_L/4$. Intuitively, this happens because the transmission of carriers from the R reservoir to the QD is weaker, and it is this transmission which mainly contributes to $\mathcal{S}_{LL}(\nu)$, than the transmission from the L reservoir to the QD, which mainly contributes to $\mathcal{S}_{RR}(\nu)$.

IV. CONCLUSION

We have developed a theory to calculate the finite-frequency noise in a non-equilibrium Kondo QD, which allows us to analyze the features observed in the evolution of the noise as a function of dot energy level and bias voltage. We have discussed the effect of the asymmetry in the couplings to the reservoirs. We predicted a change of sign in the real part of the cross-correlator when interactions are present in the QD; this is related to the fact that the statistics of the carriers are no longer fermionic. We also highlighted the appearance at $U \neq 0$ of a Coulomb diamond like structure in the auto-correlators and total noise profiles inside which the fluctuations are reduced.

ACKNOWLEDGMENTS

The authors want to thank H. Baranger, H. Bouchiat, R. Deblock, R. Delagrange, M. Guigou, F. Michelini and X. Waintal for valuable discussions. For financial support, the authors acknowledge the Indo-French Center for the Promotion of Advanced Research (IFCPAR) under Research Project No. 4704-02.

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