



HAL
open science

Space-time error correcting codes

Massinissa Lalam, Karine Amis Cavalec, Dominique Leroux

► **To cite this version:**

Massinissa Lalam, Karine Amis Cavalec, Dominique Leroux. Space-time error correcting codes. *IEEE Transactions on Wireless Communications*, 2008, 7 (5), pp.1472-1476. 10.1109/TWC.2008.060866 . hal-01893767

HAL Id: hal-01893767

<https://hal.science/hal-01893767>

Submitted on 29 Jun 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

Space-Time Error Correcting Codes

Massinissa Lalam, Karine Amis, and Dominique Leroux

Abstract—A new space-time block code family for two transmitters, called the space-time error correcting codes (STECCs), is presented. As it is built from any linear forward error correcting code (FEC), a STECC is able to correct errors while achieving high spectral efficiencies. The key principle is the FEC linearity which is exploited to transmit linear combinations of FEC codewords to create a space-time redundancy. The proposed code exhibits interesting performance on an ISI-free block fading channel. This result is all the more true as the number of FEC codewords considered in the combination is high.

Index Terms—Multiple-input multiple-output, space-time error correcting code.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) systems have been intensively studied since the mid 90's and can be sorted into different families depending on the transmission aspect they try to optimize. The first family deals with the transmission quality by exploiting the space-time diversity. It is mainly composed of the *space-time block codes* (STBCs) and especially orthogonal ones (OSTBCs), as introduced by Alamouti [1] and generalized by Tarokh *et al.* [2]. They offer good performance but a poor spectral efficiency when the number of transmit antennas becomes larger than two.

The second family increases the transmission throughput and takes advantage of the multiplexing gain. It is mainly composed of the *layered space-time* (LST) architectures [3]–[5], which are close to a spatial multiplexing of the modulation symbols. The spectral efficiency is high, especially when the number of antennas is larger than two, but their detection is more difficult. The third family is composed of STBCs trying to achieve an optimum compromise between diversity (transmission quality) and multiplexing gain (transmission throughput). It contains among others the *golden codes* [6] and the *TAST* codes [7], which are full-rate and full-diversity.

However, these coding schemes do not possess error correction capability. Therefore, FEC techniques are added to keep a given transmission quality level. The most common one is the *space-time bit interleaved coded modulation* (ST-BICM) [8], [9]. Few space-time coding schemes include the FEC properties in their design. In [10], Reed-Solomon (RS) codes [11] are used for their symbol protection property. Each RS codeword is composed of Galois field symbols and the RS code is chosen so that one Galois field symbol matches

one symbol sent through one transmit antenna. However, this approach is more efficient if one Galois field symbol matches all the symbols sent through the antennas [12]. This kind of structures tend to incorporate the FEC in the design of space-time codes, but the space and time diversities are still independently exploited. A way to include FEC techniques in the design of space-time codes is to use space-time trellis codes [13]–[15] that exhibit good performance but whose decoding complexity is always an exponential of the modulation order.

In this letter, we introduce a new family of MIMO coding schemes for two transmit antennas that includes an error protection at the center of its design: the *space-time error correcting codes* (STECCs). These codes are built on any linear FEC. The first antenna transmits K FEC codewords and the second one their linear combinations. A space-time redundancy is thus created. The receiver exploits the particular space-time structure as well as the FEC linearity to estimate the information bits.

This letter is organized as follows. Part II presents the STECCs construction. Part III describes the MIMO transmission model. Part IV deals with the corresponding receiver structure. Part V describes the performance on a Rayleigh block flat fading channel while part VI concludes the letter.

II. DEFINITION

A. Construction

A STECC is a space-time block code able to correct errors due to the transmission. It is built from any linear FEC. The key principle is the space-time redundancy generation using the FEC linearity.

Let $C(n, k)$ be a linear FEC, where n denotes the code length, k its dimension and \mathcal{C} the codeword set. We consider K codewords $\{\mathbf{c}_i\}_{1 \leq i \leq K} \in \mathcal{C}^K$ and denote $\mathbf{c}_{\neq i} = \bigoplus_{j=1; j \neq i}^K \mathbf{c}_j \in \mathcal{C}$ the codeword of $C(n, k)$ obtained by linear combination, where \oplus stands for the addition in the set \mathcal{C} . For $n_t = 2$ transmit antennas, we form the $n_t \times Kn$ matrix:

$$\mathbf{C} = \begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \dots & \mathbf{c}_K \\ \mathbf{c}_{\neq 1} & \mathbf{c}_{\neq 2} & \dots & \mathbf{c}_{\neq K} \end{pmatrix} \quad (1)$$

After an M -order binary to symbol conversion (BSC), we obtain the straightforward $n_t \times L_K$ symbol matrix \mathbf{S} , where $M = 2^{m_b}$ and $L_K = Kn/m_b$. Each row of the symbol matrix \mathbf{S} is sent through a different antenna. The matrix \mathbf{S} represents a STBC codeword.

We denote $C_{1-K}^{C(n, k)}$ this STBC built over the FEC $C(n, k)$ and name it a *space-time error correcting code* (STECC). The transmitter scheme is straightforward and given by Fig 1 for $C_{1-K}^{C(n, k)}$ and an M -order BSC. The overall coding rate of the STECC is the product of the space-time structure coding rate $R_s = 0.5$ (K original codewords of $C(n, k)$ for a total of $2K$

The authors are with the Institut TELECOM / TELECOM Bretagne, TAMCIC (UMR 2872 CNRS), Dpt Signal & Communications, Technopôle Brest Iroise, CS 83818 - 29238 Brest Cedex 3, France (e-mail: massinissa.lalam@telecom-bretagne.eu).

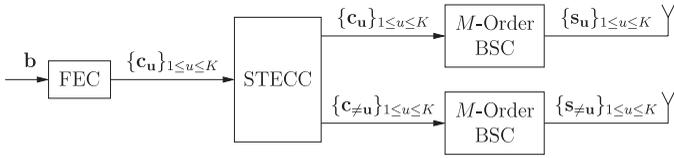


Fig. 1. Transmitter scheme for the $C_{1-K}^{C(n,k)}$ STECC ($n_t = 2$ transmit antennas).

codewords sent) and the $C(n, k)$ FEC coding rate $R_c = k/n$. It is given by:

$$R_{STECC} = \frac{k}{2n} \quad (2)$$

B. Puncturing

The puncturing technique is often used for convolutional codes to increase their FEC coding rate R_c by erasing some bits of the codeword, leaving the decoding algorithm in charge of recovering them. We propose to extend this concept to our structure by erasing entire FEC codewords without loss of useful information to increase the space-time coding rate R_{STECC} .

A FEC codeword belonging to \mathbf{C} (see (1)) can always be found thanks to other FEC codewords of \mathbf{C} by linear combination. For the STECC $C_{1-3}^{C(n,k)}$, the FEC codewords of the third column are obtained with linear combination of codewords from the two previous columns. By erasing them, we obtain a punctured STECC denoted $C_{1-3,Punct}^{C(n,k)}$ whose overall coding rate is $R_{STECC} = 0.75R_c$ instead of $0.5R_c$:

$$\begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \\ \mathbf{c}_2 \oplus \mathbf{c}_3 & \mathbf{c}_3 \oplus \mathbf{c}_1 & \mathbf{c}_1 \oplus \mathbf{c}_2 \end{pmatrix} \rightarrow \begin{pmatrix} \mathbf{c}_1 & \mathbf{c}_2 \\ \mathbf{c}_2 \oplus \mathbf{c}_3 & \mathbf{c}_3 \oplus \mathbf{c}_1 \end{pmatrix} \quad (3)$$

Puncturing must be smartly done so as to recover erased FEC codewords by linear combination of the remaining ones. For instance, puncturing the two last columns of $C_{1-4}^{C(n,k)}$ will not allow us to recover \mathbf{c}_3 or \mathbf{c}_4 but only $\mathbf{c}_3 \oplus \mathbf{c}_4$.

III. TRANSMISSION MODEL

We consider a MIMO transmission with n_t transmit antennas and n_r receive antennas. We assume the usual Rayleigh block flat fading channel. The received signal is given by:

$$\mathbf{R} = \mathbf{H}\mathbf{S} + \mathbf{n} \quad (4)$$

where \mathbf{R} is the $n_r \times L_K$ complex received matrix, \mathbf{S} is the $n_t \times L_K$ complex transmitted matrix, \mathbf{H} is the $n_r \times n_t$ complex channel matrix and \mathbf{n} is the $n_r \times L_K$ complex AWGN matrix. Assuming a unitary average gain on each transmit-receive antenna link, the signal to noise ratio (SNR) on each receive antenna is:

$$\text{SNR} = m_b n_t R_{STECC} \frac{E_b}{N_0} \quad (5)$$

where E_b is the energy per useful bit and N_0 is the noise power spectral density.

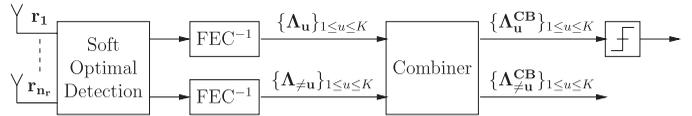


Fig. 2. Receiver with optimal detection for $C_{1-K}^{C(n,k)}$ (n_r receive antennas).

IV. RECEIVER STRUCTURE

The channel state information is supposed to be perfectly known at the receiver. For the STECC $C_{1-K}^{C(n,k)}$, the optimal receiver structure must find the K codewords $\{\hat{\mathbf{c}}_1, \dots, \hat{\mathbf{c}}_K\}$ of $C(n, k)$ that minimize the distance $\|\mathbf{R} - \mathbf{H}\mathbf{S}\|^2$. By doing so, it jointly performs the detection and the decoding with an unrealistic $\text{Card}(\mathcal{C})^K$ complexity. To retrieve the information in a practical case, we split the process into two successive operations: a detection stage and a decoding stage as shown in Fig. 2.

A. Detection

Instead of detecting $\hat{\mathbf{S}}$ at once, we use the space-time correlation introduced between the $2K$ modulation symbols associated to the modulated codewords $\{\mathbf{s}_i\}_{1 \leq i \leq K}$ and $\{\mathbf{s}_{\neq i}\}_{1 \leq i \leq K}$. Let l_c be the number of modulation symbols associated to one FEC codeword ($L_K = Kl_c$). For $1 \leq u \leq l_c$, $\{s_i(u)\}_{1 \leq i \leq K}$ determines $\{s_{\neq i}(u)\}_{1 \leq i \leq K}$. We define the optimal detector adapted to the STECC as the one which finds $\{\hat{s}_1(u), \dots, \hat{s}_K(u)\}$, and consequently $\{\hat{s}_{\neq 1}(u), \dots, \hat{s}_{\neq K}(u)\}$, such as:

$$\{\hat{s}_1(u), \dots, \hat{s}_K(u)\} = \underset{\{s_1, \dots, s_K\} \in \mathcal{S}_M^2}{\text{argmin}} d_u^2(s_1, \dots, s_K) \quad (6)$$

with

$$d_u^2(s_1, \dots, s_K) = \sum_{i=1}^K \left\| \mathbf{R}_{(i-1)l_c+u} - \mathbf{H}_{(i-1)l_c+u} \begin{pmatrix} s_i \\ s_{\neq i} \end{pmatrix} \right\|^2$$

where $\mathbf{R}_{(i-1)l_c+u}$ is the received vector and $\mathbf{H}_{(i-1)l_c+u}$ is the MIMO channel matrix at the time $t = (i-1)l_c + u$, with $1 \leq i \leq K$ and $1 \leq u \leq l_c$. This detector is called the *STECC detector* and its complexity is M^K . Note that soft output such as log-likelihood ratios (LLRs) and their approximations [16] can be easily derived from the previous calculation.

B. Decoding

1) *Principle*: As we cannot jointly decode the K original codewords of $C(n, k)$ (complexity of $\text{Card}(\mathcal{C})^K$), we successively decode the $2K$ sequences coming from the detector. We consider a soft-input soft-output (siso) decoder delivering LLRs which will exploit the time-diversity. To benefit from the space-diversity as well, we combine its output.

2) *Combining*: The combining stage will compute new LLRs by exploiting the space-time redundancy linking every bit due to the STECC structure. For the sake of clarity, we detail the LLR computation for one bit of an original FEC codeword, but the principle remains the same for one bit of a combined FEC codeword. For $1 \leq i \leq K$, let:

- $c_i(u)$ be the u^{th} bit of \mathbf{c}_i and $\Lambda_i(u)$ be its associated LLR,

- $c_{\neq i}(u) = \bigoplus_{j=1, j \neq i}^K c_j(u)$ be the u^{th} bit of $\mathbf{c}_{\neq i}$ and $\Lambda_{\neq i}(u)$ be its associated LLR.

For simplicity, we omit the index notation (u) in the following. The updated LLR associated to c_i conditionally to the previous LLRs is given by:

$$\begin{aligned} \Lambda_i^{CB} &= \ln \frac{\Pr(c_i = 1 \mid \{\Lambda_j\}_{1 \leq j \leq K}, \{\Lambda_{\neq j}\}_{1 \leq j \leq K})}{\Pr(c_i = 0 \mid \{\Lambda_j\}_{1 \leq j \leq K}, \{\Lambda_{\neq j}\}_{1 \leq j \leq K})} \quad (7) \\ &= \ln \frac{\sum_{c_i = \delta_j} \prod_{j=1}^K \Pr(c_j = \delta_j \mid \Lambda_j) \Pr(c_{\neq j} = \delta_v \mid \Lambda_{\neq j})}{\sum_{c_i = 0} \prod_{j=1}^K \Pr(c_j = \delta_j \mid \Lambda_j) \Pr(c_{\neq j} = \delta_v \mid \Lambda_{\neq j})} \\ &= \ln \frac{\sum_{c_j = \delta_j} e^{\sum_{j=1}^K (2\delta_j - 1) \frac{\Lambda_j}{2} + (2 \bigoplus_{v=1}^K \delta_v - 1) \frac{\Lambda_{\neq j}}{2}}}{\sum_{c_j = \delta_j} e^{\sum_{j=1}^K (2\delta_j - 1) \frac{\Lambda_j}{2} + (2 \bigoplus_{v=1}^K \delta_v - 1) \frac{\Lambda_{\neq j}}{2}}} \quad (8) \end{aligned}$$

C. Complexity

The M^K complexity of the STECC detector represents the bottleneck of the previous receiver. However, this issue has been addressed in [17] where a turbo-like iterative receiver is proposed. The detection complexity increases linearly with the modulation set order while performance in terms of bit error rate (BER) keeps roughly the same. In this letter, the receiver is the one described above and we focus only on the performance in terms of BER achieved by the STECC family to exhibit its merit.

V. SIMULATION RESULTS

We consider QPSK-modulation over a Rayleigh block flat fading channel constant over τ modulation symbols and $n_r = 2$ receive antennas. For the FEC, we use either the binary block code $BCH(15, 7)$ [18] or the 8-state convolutional code $(13_8, 15_8)$ [19].

A. The STECC family

We consider the STECC $C_{1-K}^{BCH(15,7)}$ with various values of K and $\tau = 2$. The BER versus E_b/N_0 at the output of the combiner stage is plotted in Fig 3. For $E_b/N_0 \leq 2$ dB, the performance gets better with higher K . At a BER of 2×10^{-4} , the improvement is 1.5 dB between the $K = 2$ and the $K = 4$ curves. This is due to the number of FEC codewords which are correlated with each other and which defines the depth of the space-time correlation. A long STECC is thus useful to reduce the minimum SNR value that guarantees a given BER. However, the optimal detector complexity grows exponentially with the parameter K . For $K \geq 4$ such receiver structure cannot be used in practice and a lower-complexity receiver such as the one described in [17] has to be preferred.

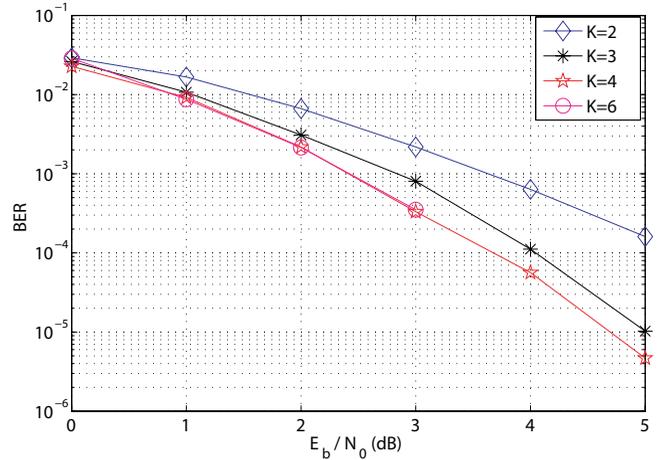


Fig. 3. $C_{1-K}^{BCH(15,7)}$ $K = \{2, 3, 4, 6\}$, QPSK, $n_t = 2$, $n_r = 2$, $\tau = 2$ - BER vs E_b/N_0 .

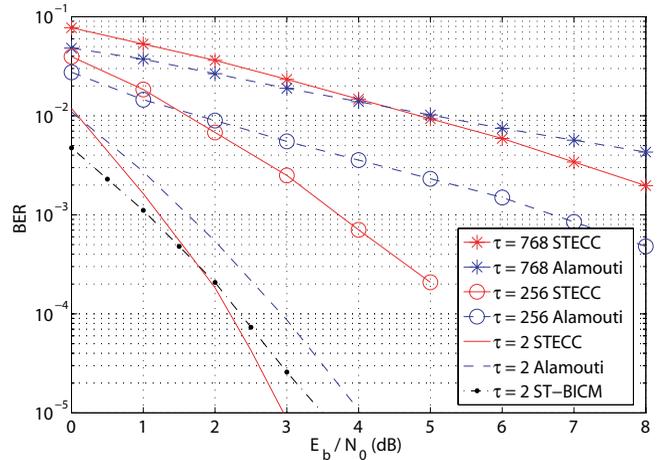


Fig. 4. $C_{1-3}^{CC(13,15)}$ vs ST-BICM($CC(11, 13, 15, 17)$) vs ($CC(13, 15)$, Alamouti), QPSK, $n_t = 2$, $n_r = 2$, $\tau = \{768, 256, 2\}$ - BER vs E_b/N_0 .

B. Performance

We use the STECC $C_{1-3}^{CC(13,15)}$ with various values of τ . Each FEC codeword has a 512-bit length and is pseudo-randomly interleaved. For comparison, two MIMO coding schemes with the same spectral efficiency and the same number of useful bits ($\approx K256 = 768$) are considered: an Alamouti encoding [1] associated with the same FEC and for $\tau = 2$ an ST-BICM [8] using the 8-state convolutional code $CC(11_8, 13_8, 15_8, 17_8)$ of coding rate $R_c = 0.25$. Convolutional codes are decoded using the siso BCJR algorithm [20]. The ST-BICM scheme is decoded with three iterations of a turbo-like receiver with an interference canceller optimized according to the minimum mean square error criterion [21]. The BER versus E_b/N_0 curves are plotted in Fig 4.

1) *Quasi-static and block fading channels:* For $\tau = 768$ (channel constant over one entire space-time codeword) and $\tau = 256$ (channel constant over one FEC codeword of the STECC), the STECC $C_{1-3}^{CC(13,15)}$ gives better results than the Alamouti scheme for E_b/N_0 greater than 4.4 dB and 1.3 dB respectively. A longer FEC is more sensible to a quasi-static channel and in these cases the convolutional code can not

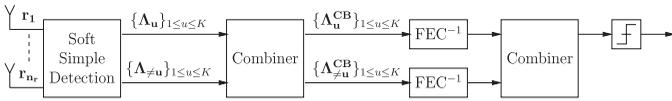


Fig. 5. Receiver with simplified detection for $C_{1-K}^{C(n,k)}$ (n_r receive antennas).

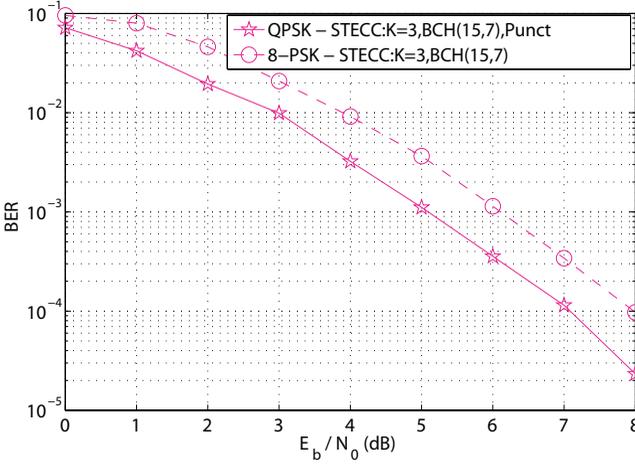


Fig. 6. $(C_{1-3}^{BCH(15,7)}, \text{QPSK})$ vs $(C_{1-3}^{BCH(15,7)}, \text{8-PSK})$, $n_t = 2$, $n_r = 2$, $\tau = 2$ - BER vs E_b/N_0 .

significantly improve the Alamouti detection output when the channel is severe. On the contrary the STECC benefits from its design of a better space-time redundancy which is exploited to improve the performance as shown in the figure.

2) *Fast fading channels*: For $\tau = 2$ and at a BER of 10^{-5} the STECC $C_{1-3}^{CC(13,15)}$ outperforms both reference schemes with a better BER versus E_b/N_0 slope curve, and thus a better asymptotical performance. The gain is almost equal to 1.1 dB and 0.5 dB in comparison with the Alamouti scheme and the ST-BICM scheme respectively.

C. Puncturing

One drawback of the STECC family is its one-half coding rate. However, puncturing described in Section II can be applied to increase this space-time coding rate. We use a QPSK with the punctured STECC $C_{1-3}^{BCH(15,7)}$ for a spectral efficiency of 1.4 bit/channel use ($m_b n_t R_{STECC}$). To keep the same spectral efficiency, we consider an 8-PSK associated with the non-punctured STECC $C_{1-3}^{BCH(15,7)}$. In both cases we replace the STECC detector of complexity M^K with a sub-optimal association of a classical soft *ML detector* of complexity M^{n_t} followed by a combiner stage to reduce the simulation process without loss of generality as shown in Fig 5. The missing LLRs are initialized to zero. The combiner stages will recover them by using (8).

The BER versus E_b/N_0 at the output of each receiver is plotted in Fig 6. The punctured STECC (solid curve) performs better than the non-punctured STECC (dashed curve) with a gain of 1 dB at a BER of 10^{-3} . It is more efficient to use a punctured STECC instead of a higher modulation to improve the spectral efficiency.

VI. CONCLUSION

This letter has presented a new family of space-time codes for two transmit antennas: the space-time error correcting codes (STECCs). They combine in an easy way K codewords from a linear forward error correcting code (FEC). The deep space-time correlation created between the $2K$ modulation symbol vectors and the FEC properties are exploited at the receiver side to improve the transmission quality.

In the example we used for two receive antennas, the STECC outperforms a traditional scheme using the Alamouti encoding and the same FEC or a ST-BICM structure with the same spectral efficiency. It presents better asymptotical performance and thus a better space-time diversity exploitation but also a higher complexity. However, it has been shown in [17] that the use of a turbo-like iterative receiver significantly reduces the complexity while achieving the same level of performance in terms of error rates.

Finally, the STECC structure is flexible enough to use puncturing upon it. For a spectral efficiency increase, this technique gives better results than the use of an higher modulation order. Moreover, it brings minor changes in the receiver structures: only the detection part has to be considered, as the new STECC codeword is shorter. The LLRs associated to the missing codewords are replaced by zeros, leaving the combiner stage the task to recover them.

ACKNOWLEDGEMENT

The authors would like to thank the Brittany regional council for the funding of this study. They also wish to thank the anonymous reviewers for their helpful comments and suggestions, which improved the presentation of this paper.

REFERENCES

- [1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, pp. 1451–1458, Oct. 1998.
- [2] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.
- [3] G. J. Foschini, "Layered space time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Techn. J.*, Autumn 1996.
- [4] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using v-blast space-time communication architecture," *Electron. Lett.*, vol. 35, Jan. 1999.
- [5] H. Gamal and A. Hammons, "A new approach to layered space-time coding and signal processing," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2321–2334, Sept. 2001.
- [6] M. Damen, A. Tewfik, and J. Belfiore, "A construction of a space-time code based on number theory," *IEEE Trans. Inform. Theory*, vol. 48, pp. 753–760, Mar. 2002.
- [7] H. E. Gamal and M. Damen, "Universal space-time coding," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1097–1119, May 2003.
- [8] A. Tonello, "On turbo equalization of interleaved space-time codes," in *Proc. IEEE Vehicular Technology Conference, VTC'01 Fall*, vol. 2, pp. 887–891, Oct. 2001.
- [9] S. Chtourou, R. Visoz, and A. Berthet, "A class of low complexity iterative equalizers for space-time bicm over mimo block fading multipath awgn channel," in *Proc. IEEE International Conference on Communications, ICC'04*, vol. 1, pp. 618–624, June 2004.

- [10] P. A. Martin and D. P. Taylor, "High-throughput error correcting space-time block codes," *IEEE Commun. Lett.*, vol. 8, pp. 458–460, Nov. 2003.
- [11] I. Reed and G. Solomon, "Polynomial codes over certain finite fields," *J. Soc. Ind. Appl. Math.*, vol. 8, pp. 300–304, June 1960.
- [12] M. Lalam, K. Amis, and D. Leroux, "On the use of reed-solomon codes in space-time coding," in *Proc. IEEE 16th International Symposium on Personal Indoor and Mobile Radio Communications, PIMRC'05*, vol. 1, pp. 31–35, Sept. 2005.
- [13] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.
- [14] G. Caire and G. Colavolpe, "On low-complexity space-time coding for quasi-static channels," *IEEE Trans. Inform. Theory*, vol. 46, pp. 1400–1416, June 2003.
- [15] T. Meixia and R. Cheng, "Diagonal block space-time code design for diversity and coding advantage over flat fading channels," *IEEE Trans. Signal Processing*, vol. 52, pp. 1012–1020, Apr. 2004.
- [16] P. Robertson, E. Villebrun, and P. Hoher, "A comparison of optimal and sub-optimal map decoding algorithms operating in the log domain," in *Proc. IEEE International Conference on Communications, ICC'95*, vol. 2, pp. 1009–1013, June 1995.
- [17] M. Lalam, K. Amis, and D. Leroux, "Iterative decoding of space-time error correcting codes," in *Proc. IEEE 18th International Symposium on Personal Indoor and Mobile Radio Communications, PIMRC'07*, pp. 1–5, Sept. 2007.
- [18] R. Bose and D. RayChaudhury, "On a class of error correcting binary group fields," *Inform. Contr.*, vol. 3, pp. 68–79, Mar. 1960.
- [19] A. Michelson and A. Levesque, *Error-Control Techniques For Digital Communication*. Wiley-Interscience, 1985.
- [20] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, Mar. 1974.
- [21] C. Laot, R. L. Bidan, and D. Leroux, "Low-complexity mmse turbo equalization : A possible solution for edge," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 965–974, May 2005.