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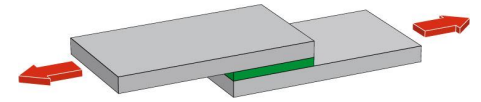
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# Simplified stress analysis of bonded joints using the macro-element technique

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- 1. SOGETI HIGH TECH**
- 2. FRAME**
- 3. MECHANICAL ANALYSIS**
- 4. MACRO-ELEMENT TECHNIQUE**
- 5. CURRENT CAPABILITIES**
- 6. RELEVANCE**
- 7. APPLICATION**

# Content

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## SOGETI HIGH TECH

# A Capgemini group subsidiary

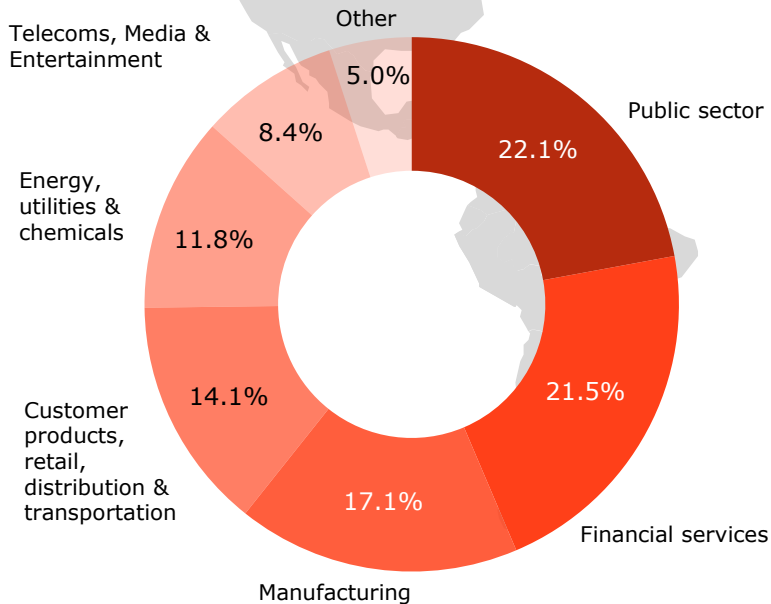
In 44 countries

Revenue of 10.1 billion euros

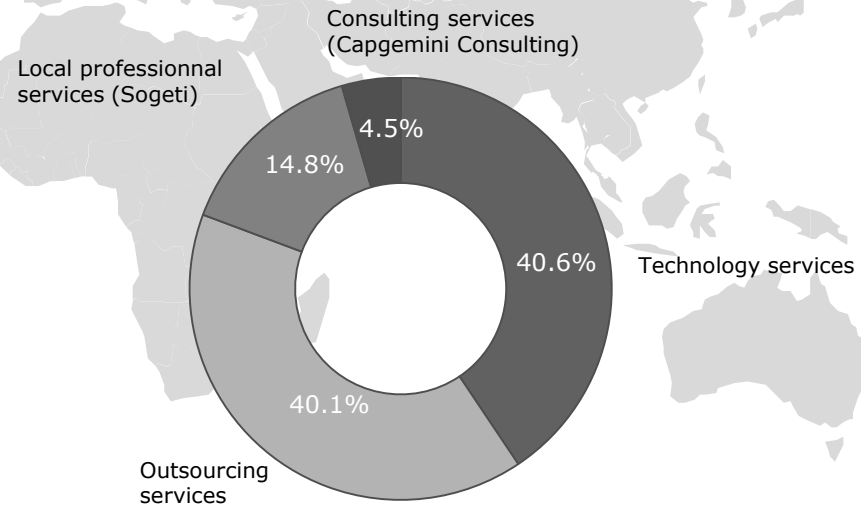
131.000 people with more than 50,000 pin offshore\*

Capgemini, founded in 1967, is one of the world foremost providers of consulting, technology and outsourcing services.

## Revenue by industry



## Revenue by business



Capgemini Consulting

Capgemini  
CONSULTING. TECHNOLOGY. OUTSOURCING

SOGETI

Prosodie

EURIWARE

\*offshore = Inde, Vietnam, Maroc, Guatemala, Pologne,...

# Sogeti : Engineering and Technology Consulting Services in the world

Nearly **20.000** employees on **100** locations in **15** countries



▲ Sogeti High Tech business development

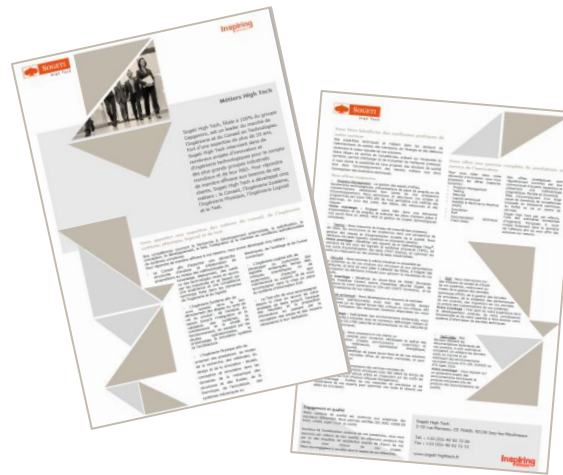
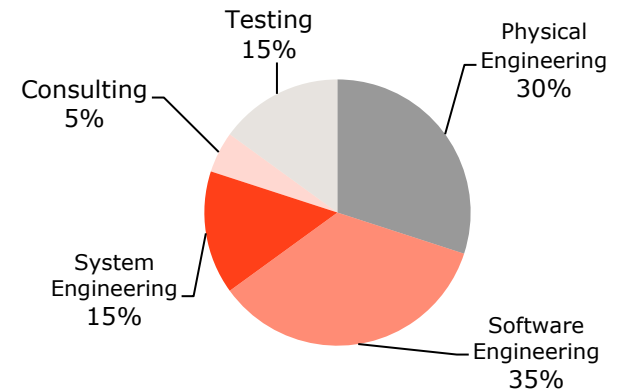
# Sogeti High Tech, specialist in Engineering and R&D

## • Leader in Engineering and Technology Consulting Services

- A 25-year long expertise in services for industrial companies
- Seven main markets :
  - Aeronautics
  - Energy
  - Life Sciences
  - Railway
  - Space
  - Defense
  - Telecoms & media

## • Five business lines

- Consulting
- Systems Engineering
- Physics Engineering
- Software Engineering
- Testing



# Sogeti High Tech's locations

- 3000 employees in France
- 300 employees in Germany
- 19 locations
- Rank n°1 in the Aeronautics & Space sectors
- 5th rank in the French market





# Content

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**FRAME**

### JoSAT (Joint Stress Analysis Tool)

#### ✓ ID SHEET:

- internal research project
- started in 2008
- self-funding
- workload: 7400 days at the end of June 2015

#### ✓ DRIVEN LINE

- *research theme*: joining technologies
- *2 research axes*: axis **bonding** and axis **bolting**
- *objectives*: **better understanding of the mechanical behavior** of bonded joints and bolted joints
  1. to develop a **simplified mechanical analysis tool**
  2. to **better control** these joining technologies

### Partnerships

#### ✓ ISAE (Institut Supérieur de l'Aéronautique et de l'Espace, Toulouse):

- signed in 2009
- prolonged in 2012 up to 2017



THEMES	WHAT?
COMPOSITE MATERIALS	2 PhD Theses 1 MS Thesis
JOINING TECHNOLOGIES	1 PhD Thesis 6 MS Theses



#### ✓ BORDEAUX INP / ENSEIRB-MATMECA

- our Center of Competences is trusted to teach their students

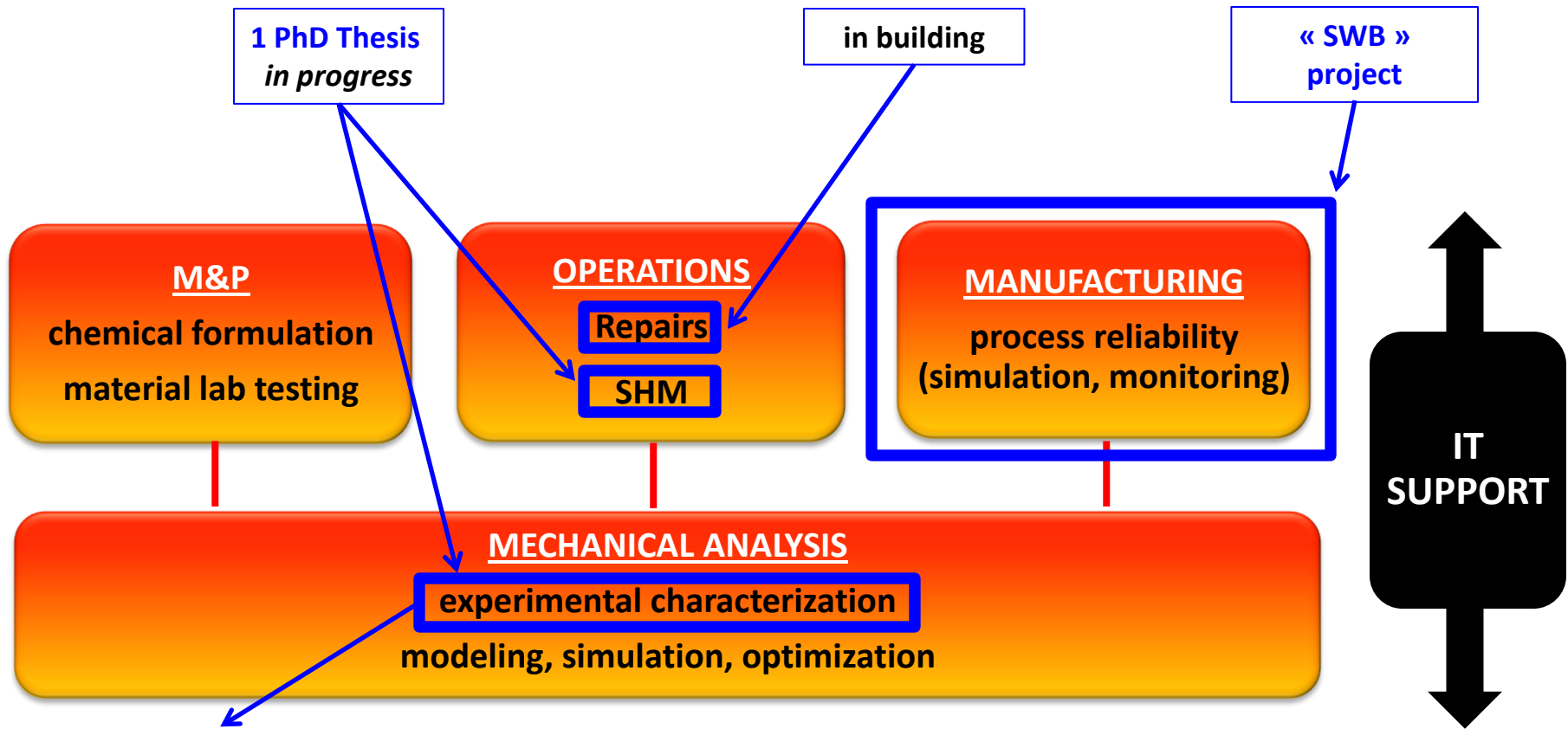
the Mechanics of Assemblies (<https://www.enseirb-matmeca.fr/syllabus1415/index.php?&module=MS312&langage=EN>)

#### ✓ AN ADHESIVE SUPPLIER

- to be signed but collaborative activities already in progress

## Rationale of Development

Stress can support M&P, Manufacturing and Operations

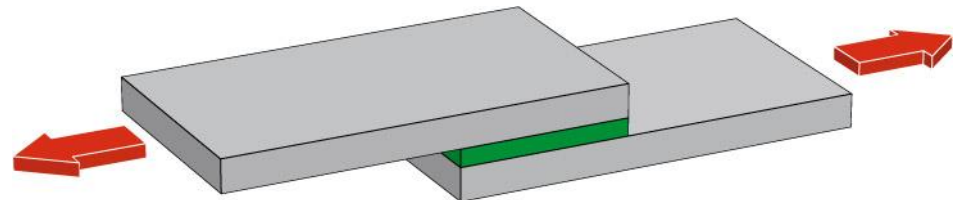


Presentation on the 3<sup>rd</sup> July 2015 at Adhesive Bonding Conference in Porto

## MECHANICAL ANALYSIS

### Strength Prediction

- ✓ Strength prediction consists in **comparing computed criteria with allowable**.
- ✓ The definition of criteria can be based on:
  - experimental and theoretical investigations on the failure mechanisms
  - in-service feedbacks
- ✓ Criteria requires input data , provided by **mechanical analysis**.
- ✓ Allowable are obtained from experimental characterization.



# Mechanical Analysis

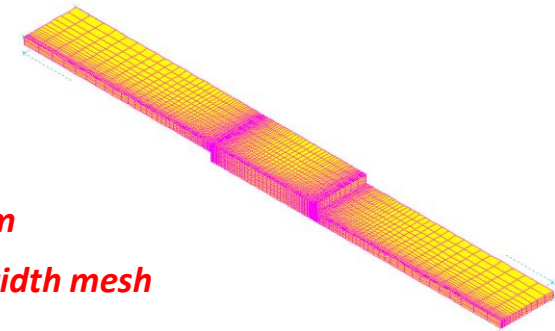
*How?*

## Finite Element (FE) Analysis

- ✓ FE method can address the mechanical analysis of bonded joints, to provide input data to criteria.
- ✓ Nevertheless, FE analysis:
  - is **time consuming**
  - and demands **highly skilled engineers** to be suitably applied
- ✓ The relative difference in thickness between the adhesive and the adherends, and the mesh requirements conducts to develop models with **a very high number of degrees of freedom**

### Example of single-lap bonded joint in 3D:

- **adherend thickness: 2 mm / adhesive thickness: 0.2 mm**
  - **10 cubic elements in adhesive thickness = 0.02 mm each**
  - **transition ratio of 1 imposed at the adhesive interface, an element size of 0.02 mm**
- ⇒ **potentially 100 elements in the adherend thickness, to be multiplied by length, width mesh parameters**



### A Mathematical Issue

- ✓ Various analytical closed-form solutions exist, based on simplifying hypotheses on the kinematics and the number of adhesive stress tendon components to be considered, leading to accurate mechanical behavior approximation.
- ✓ But **the application field appears as restricted**, even for practical problems (ex: steel to aluminium joint including bending and normal displacement)
- ✓ To enlarge the application field, **mathematical procedures** shall be used to solve the set of governing differential equations (deduced from hypotheses taken)

**The macro-element technique is a mathematical procedure**



## MACRO-ELEMENT TECHNIQUE

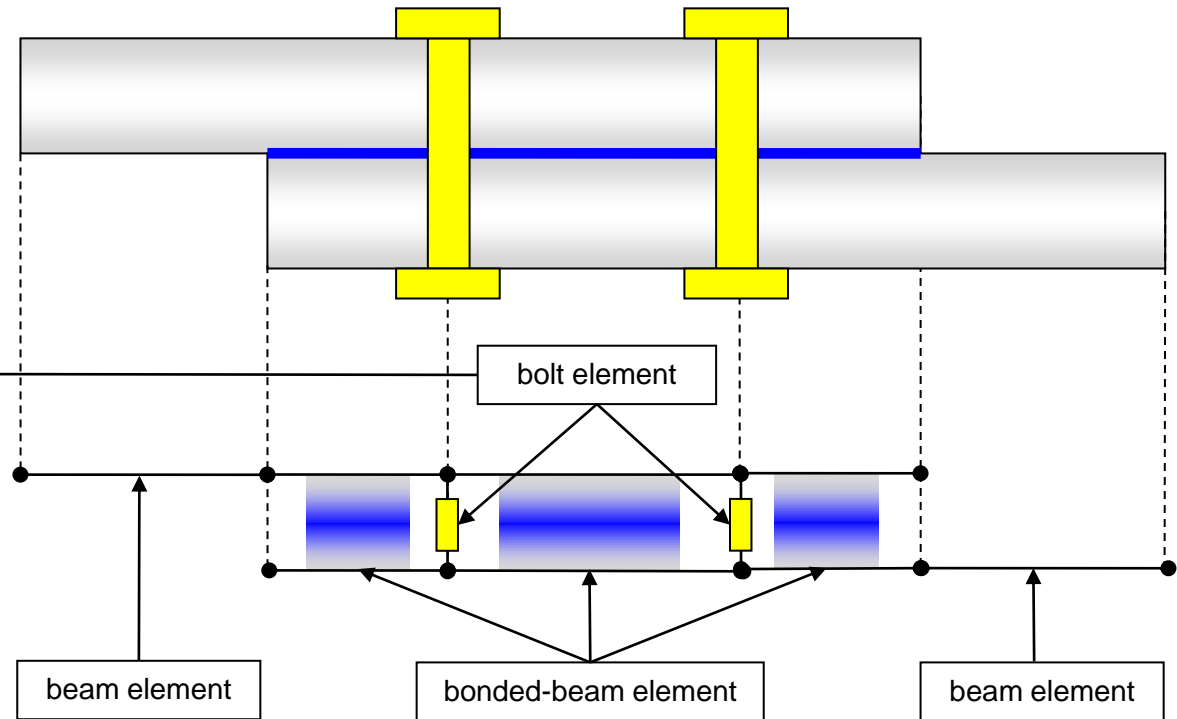
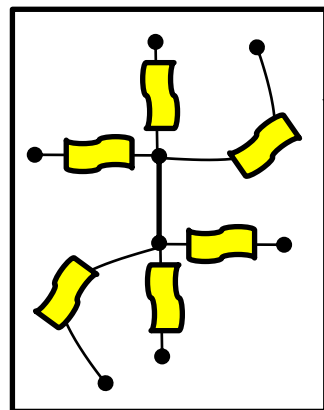
# Macro-Element Technique

Origin

## Simplified Analysis of Hybrid Joints

- ✓ First developments between 2004 and 2006 in the frame of [PAROISSIEN's PhD \[1\]](#), to simplify the stress analysis of **hybrid (bolted/bonded) joints**
- ✓ **Significant extension of the application field** since 2008 by SOGETI HIGH TECH in the frame of JoSAT

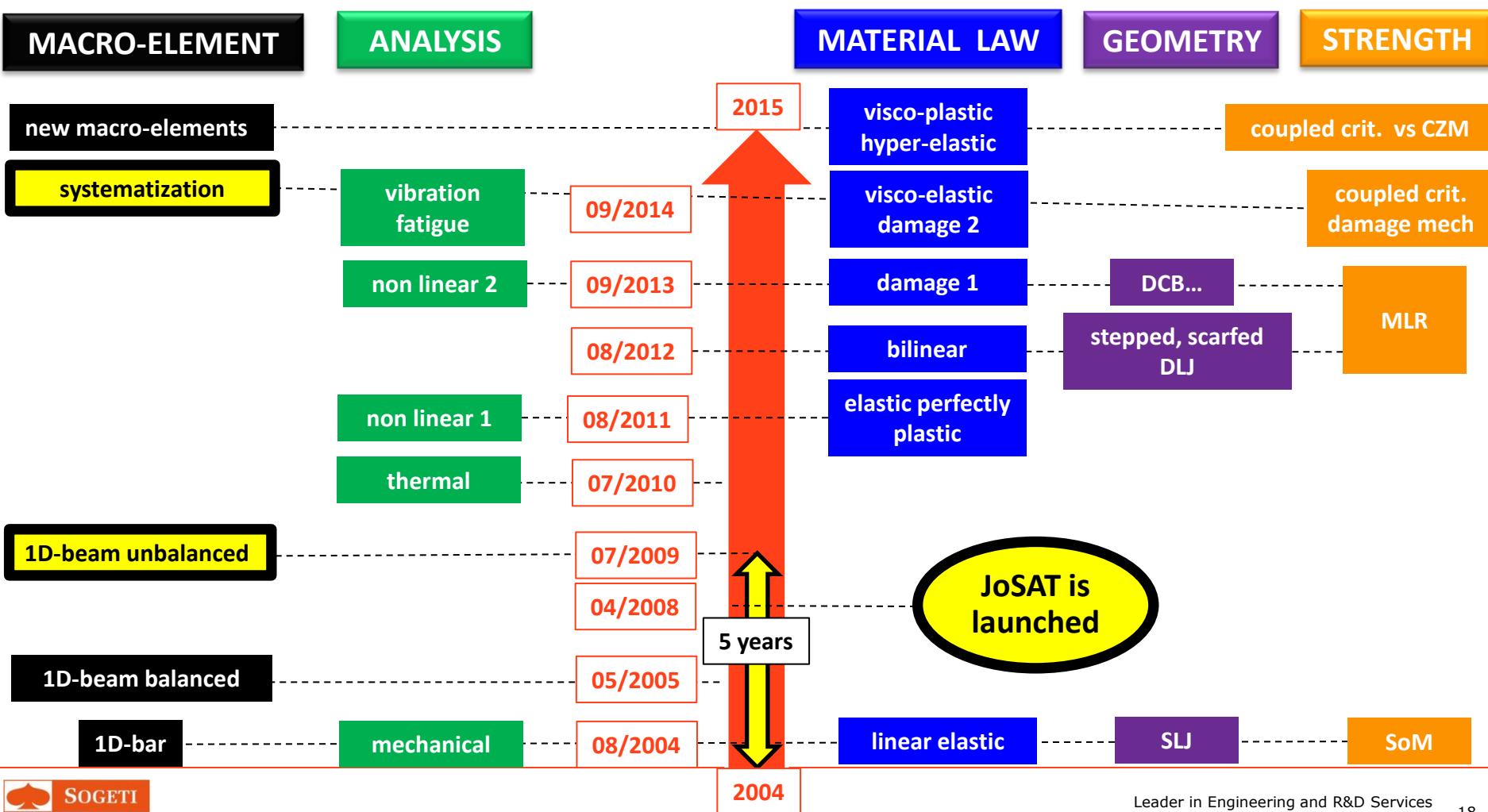
Idea from:  
Prof. Marc SARTOR  
INSA Toulouse



# Macro-Element Technique

## Chronology

From 2004 up to now

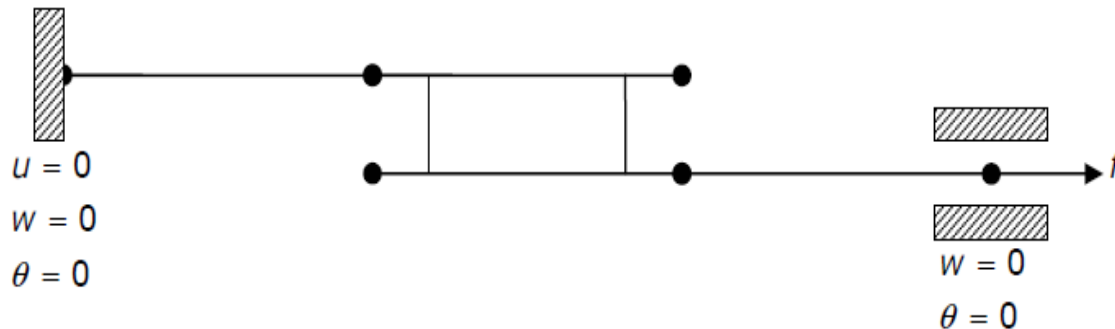


# Macro-Element Technique

How?

## Inspired by FE Method

- ✓ **1<sup>st</sup> STEP: MESHING THE JOINT**, in beam (or bar) elements and macro-element.
  - **Only 1 macro-element** is needed **for 1 entire overlap**
- ✓ **2<sup>nd</sup> STEP: ASSEMBLY OF THE STIFFNESS MATRIX ( $K$ )** for the joint
  - **KEY POINT:** the stiffness matrix of the macro-element (see next slide)
- ✓ **3<sup>rd</sup> STEP: APPLICATION OF BOUNDARY CONDITIONS** (load and prescribed displacement)
- ✓ **4<sup>th</sup> STEP: MINIMIZATION OF POTENTIAL ENERGY**
  - leading to a linear system to be solved:  $F=KU$



### Example:

*single-lap bonded joint in-plane loaded membrane + bending*

=> the solution consists in inverting a **13x13 linear system only!**

# Macro-Element Technique

## Stiffness Matrix (Bonding)

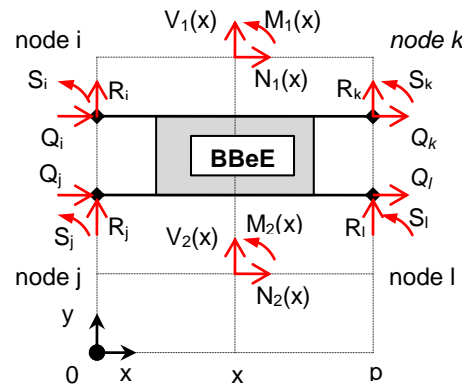
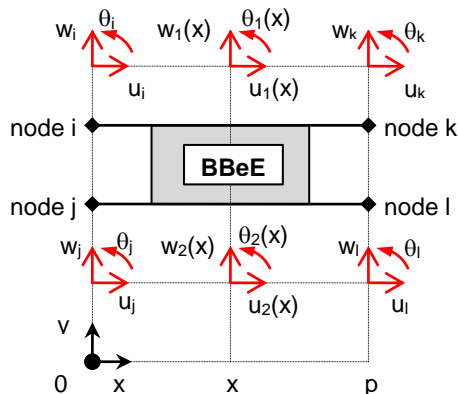
### Principle

✓ (semi-)analytical formulation based on the set of governing differential equations:

- local equilibrium equations
- constitutive equations

There is **not any hypotheses on the shape of interpolation functions.**

The shape of interpolation functions is the shape of solutions of the system of governing differential equations



$$K_{BBe} = \begin{pmatrix} \left[ \frac{\partial Q_\sigma}{\partial u_\tau} \right] & \left[ \frac{\partial Q_\sigma}{\partial w_\tau} \right] & \left[ \frac{\partial Q_\sigma}{\partial \theta_\tau} \right] \\ \left[ \frac{\partial R_\sigma}{\partial u_\tau} \right] & \left[ \frac{\partial R_\sigma}{\partial w_\tau} \right] & \left[ \frac{\partial R_\sigma}{\partial \theta_\tau} \right] \\ \left[ \frac{\partial S_\sigma}{\partial u_\tau} \right] & \left[ \frac{\partial S_\sigma}{\partial w_\tau} \right] & \left[ \frac{\partial S_\sigma}{\partial \theta_\tau} \right] \end{pmatrix}, \sigma, \tau = i, j, k, l$$

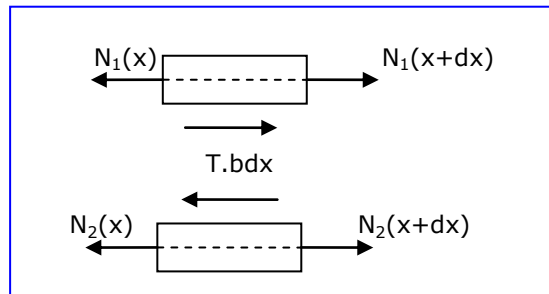
# Macro-Element Technique

## 1D-Bar Stiffness Matrix (Bonding)

### Hypotheses

✓ linear local equilibrium :

- VOLKERSEN [2]

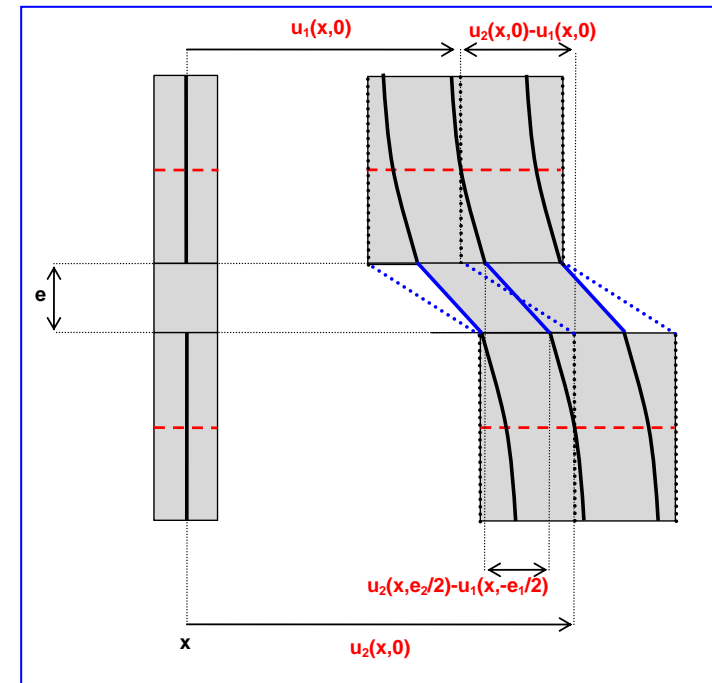


✓ adherend as linear bars:

- including thermal expansion
- linear variation of the adherend shear stress with the thickness as TSAÏ, OPLINGER and MORTON [3]

✓ adhesive layer as shear springs continuously distributed:

- adhesive thickness constant along the overlap
- adhesive shear stress and shear stress supposed constant in the adhesive thickness



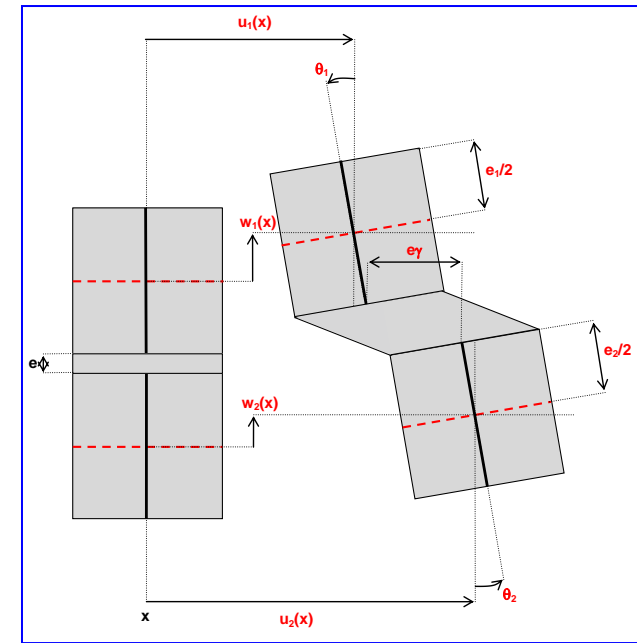
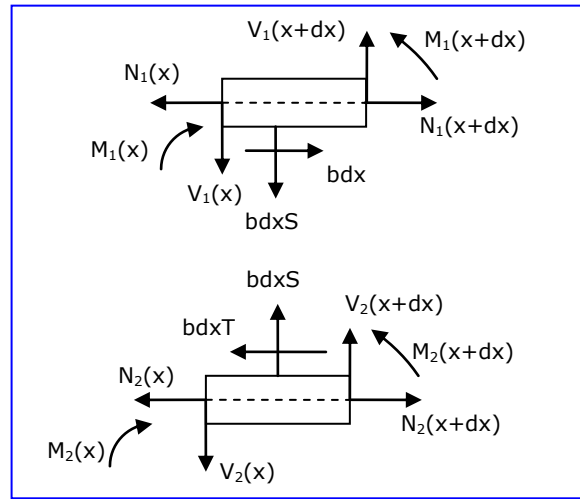
# Macro-Element Technique

## 1D-Beam Stiffness Matrix (Bonding)

### Hypotheses

✓ linear local equilibrium available:

- GOLAND & REISSNER [4]
- HART-SMITH [5]



✓ adherend as linear Euler-Bernoulli beam:

- in the classical laminated theory
- including thermal expansion
- linear variation of the adherend shear stress with the thickness as TSAÏ, OPLINGER and MORTON [3]

✓ adhesive layer as shear and peel springs continuously distributed:

- adhesive thickness constant along the overlap
- adhesive shear stress and shear stress supposed constant in the adhesive thickness

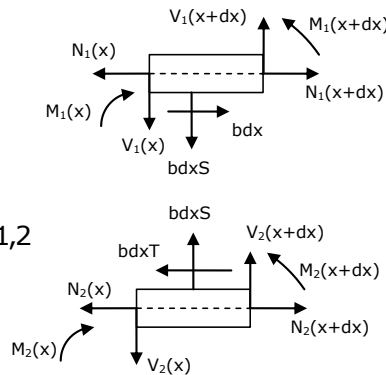
# Macro-Element Technique

## 1D-Beam Stiffness Matrix (Bonding)

### Equations (1)

#### local equilibrium (GOLAND & REISSNER)

$$\begin{cases} \frac{dN_j}{bdx} = (-1)^j T \\ \frac{dV_j}{bdx} = (-1)^{j+1} S \\ \frac{dM_j}{dx} + V_j + \frac{1}{2}(e_j + e) b T = 0 \end{cases}, \quad j = 1, 2$$



#### constitutive equations shear springs, peel springs

$$\begin{cases} T = \frac{G}{e} \left( u_2 - u_1 - \frac{1}{2} e_1 \theta_1 - \frac{1}{2} e_2 \theta_2 \right) \\ S = \frac{E}{e} (w_1 - w_2) \end{cases}$$

#### constitutive equations Euler-Bernoulli laminated beam

$$\begin{cases} N_j = A_j \frac{du_j}{dx} - B_j \frac{d^2 w_j}{dx^2} \\ M_j = -B_j \frac{du_j}{dx} + D_j \frac{d^2 w_j}{dx^2}, \quad j = 1, 2 \\ \theta_j = \frac{dw_j}{dx} \end{cases}$$

SYSTEM OF COUPLED DIFFERENTIAL EQUATIONS

resolution

Expression for displacements then for internal forces

HARD WORK



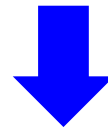
# Macro-Element Technique

## 1D-Beam Stiffness Matrix (Bonding)

### Equations (2)

*Computation of nodal forces and nodal displacements*

$$\mathbf{U} = \begin{pmatrix} u_i \\ u_j \\ u_k \\ u_l \\ w_i \\ w_j \\ w_k \\ w_l \\ \theta_i \\ \theta_j \\ \theta_k \\ \theta_l \end{pmatrix} = \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \\ w_1(0) \\ w_2(0) \\ w_1(\Delta) \\ w_2(\Delta) \\ \theta_1(0) \\ \theta_2(0) \\ \theta_1(\Delta) \\ \theta_2(\Delta) \end{pmatrix} = \mathbf{MC} \quad \text{and} \quad \mathbf{F} = \begin{pmatrix} Q_i \\ Q_j \\ Q_k \\ Q_l \\ R_i \\ R_j \\ R_k \\ R_l \\ S_i \\ S_j \\ S_k \\ S_l \end{pmatrix} = \begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \\ -V_1(0) \\ -V_2(0) \\ V_1(\Delta) \\ V_2(\Delta) \\ -M_1(0) \\ -M_2(0) \\ M_1(\Delta) \\ M_2(\Delta) \end{pmatrix} = \mathbf{NC}$$



*Stiffness Matrix*

$$\mathbf{K} = \mathbf{NM}^{-1}$$

## CURRENT CAPABILITIES

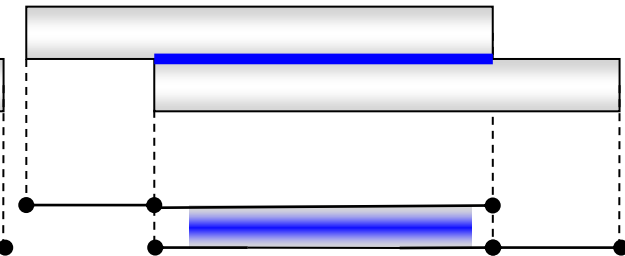
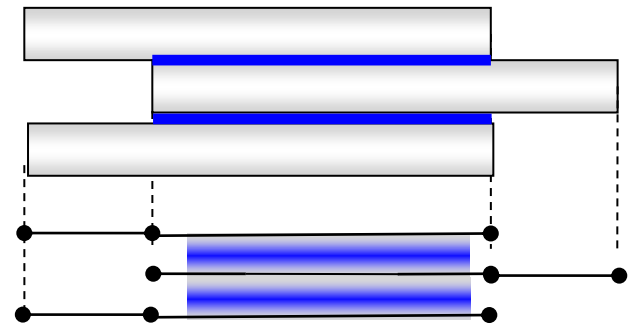
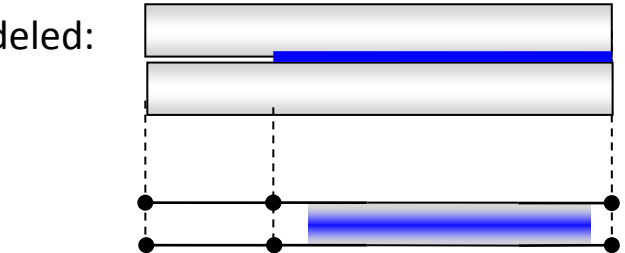
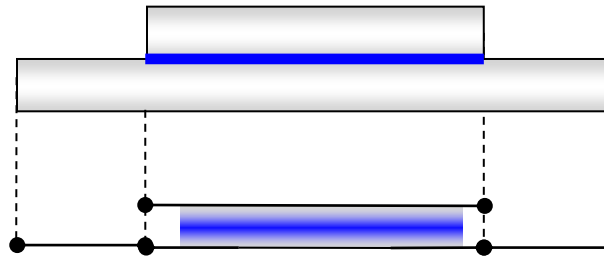
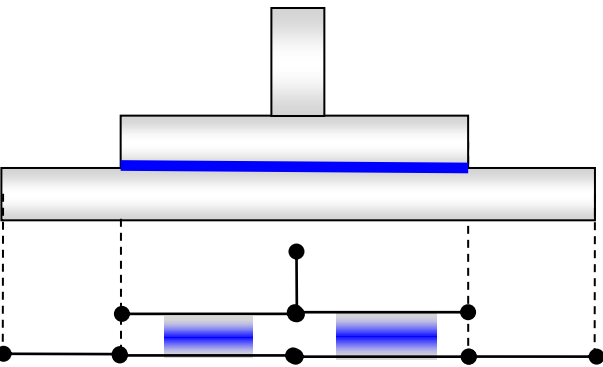
# Current Capabilities

*Input*

## Geometry

✓ AS A “LEGO GAME”, various geometrical configurations can be modeled:

- squared-end single-lap as the nominal configuration
- tapered-end single-lap configuration
- double-lap configuration
- fracture mechanics coupons (ENF, DCB, MMB)
- patch or stiffened configuration **[POSSIBLE]**
- etc.... + including fasteners



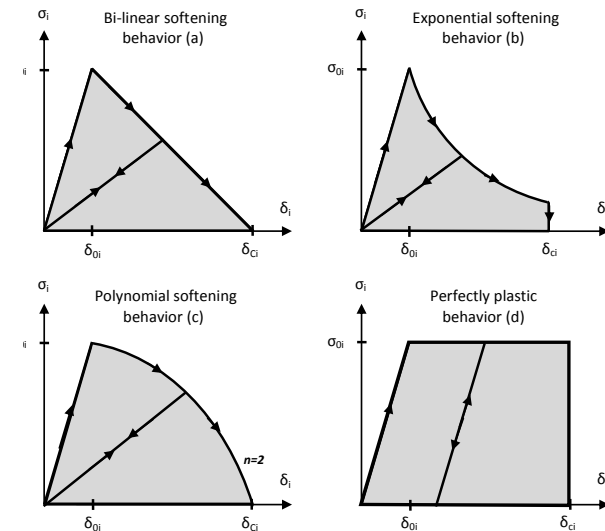
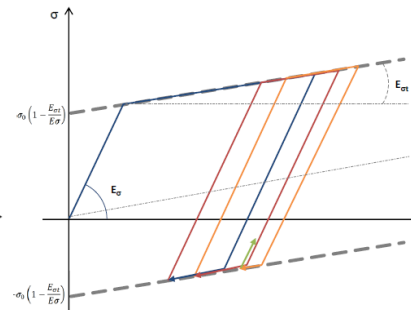
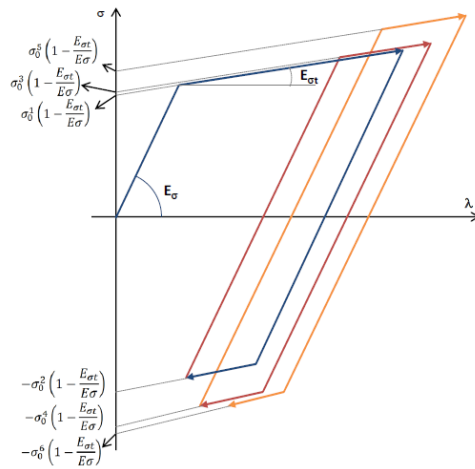
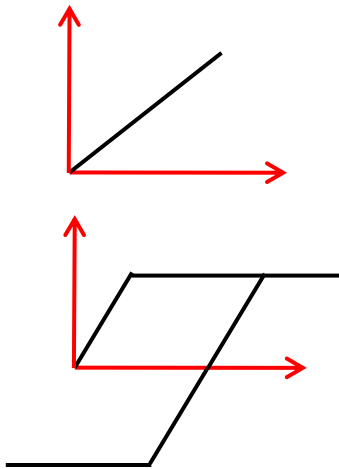
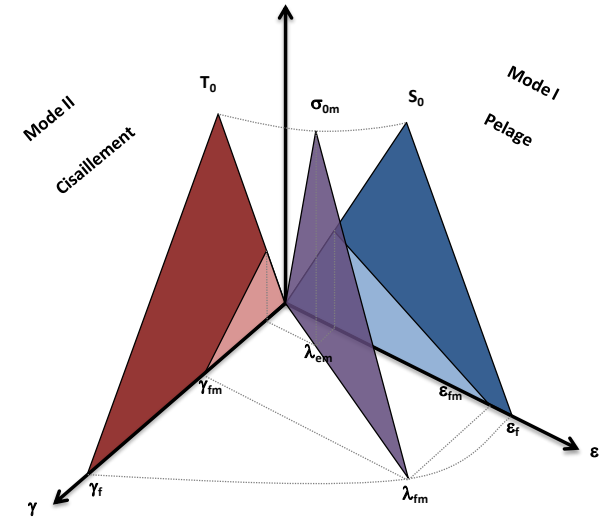
# Current Capabilities

Input

## Adhesive Material

✓ VARIOUS ADHESIVE MATERIAL CAN BE SUPPORTED:

- linear elastic
- elastic perfectly plastic [6, 8]
- bilinear (isotropic, kinematic, mixed hardening) [6, 8]
- damage evolution law with various shapes and mixed mode [9]
- visco-elastic including time-temperature dependency



### Adherend Material

#### ✓ EULER-BERNOULLI BEAM

- in the frame of the classical laminated theory
- balanced and unbalanced cases
- linear elastic

#### ✓ LINEAR ELASTIC BEHAVIOR IS NOT A RESTRICTION

- the non-linear algorithm already developed to support non-linear adhesive material
- non linear adherend material could be then implemented

# Current Capabilities

*Input*

## Loading

- ✓ **STATIC [6-9]**
  - loading in force or in displacement
- ✓ **HYDRO-THERMAL**
  - uniform variation of temperatures [7] and/or of moisture rate
- ✓ **FATIGUE**
  - basing on progressive damage approach [10, 11] [algo ok, approach under assessment]
- ✓ **VIBRATION**
  - mass matrix implemented
  - free modes assessment

### Results Directly Available

#### ✓ DISTRIBUTION AT ANY POINTS:

- displacements in the adherends ( $u, v, \theta$ )
- internal forces in the adherends (normal force, shear force, bending moment)
- forces in the fasteners (bolt transfer rate)
- shear stress and peel stress along the overlap
- **[EASILY FAISABLE]** stress and strain in the adherends can be easily computed from internal forces

**The tool provides input for the computation of strength criteria**

# Content

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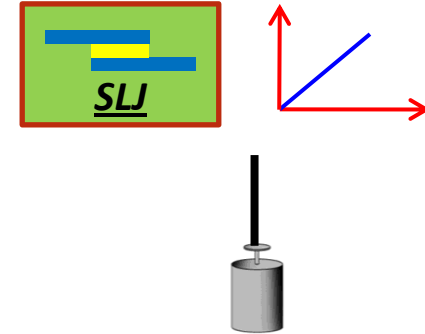
**RELEVANCE**



# Relevance

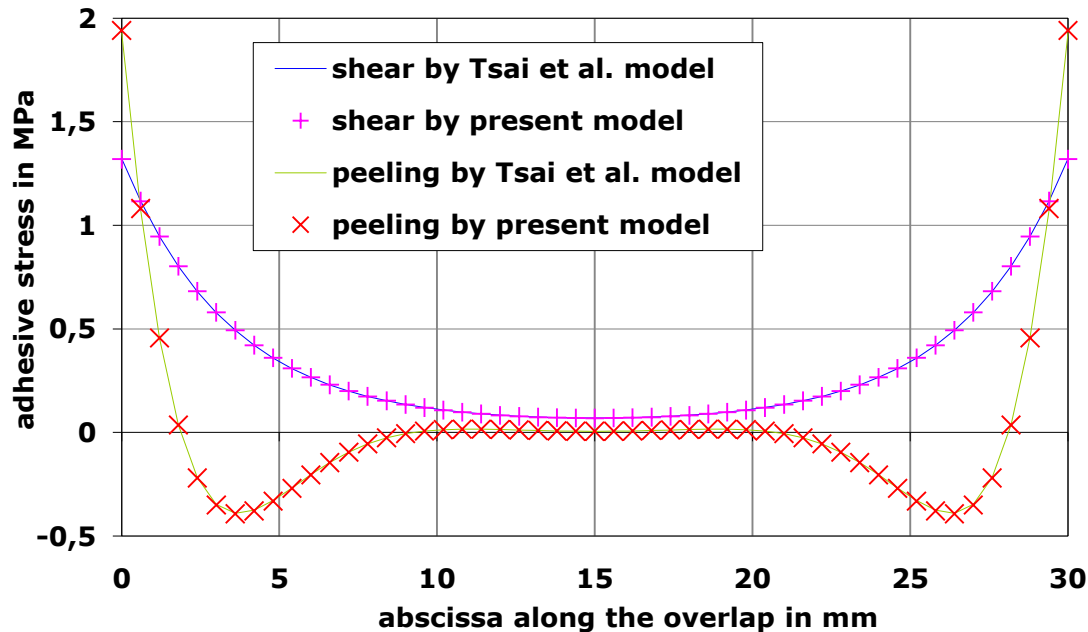
## Validation

### Against Available Literature [6]



✓ GOLAND & REISSNER [4] stress analysis improved by TSAI et al. [3]

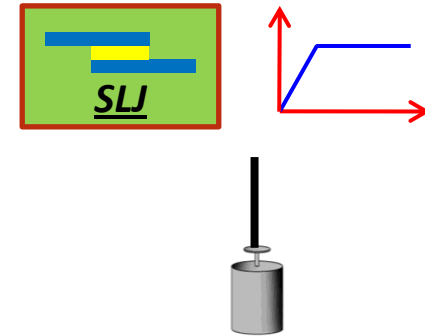
- linear elastic adhesive
- 1D-beam kinematics, simply supported, in-plane mechanically loaded



# Relevance

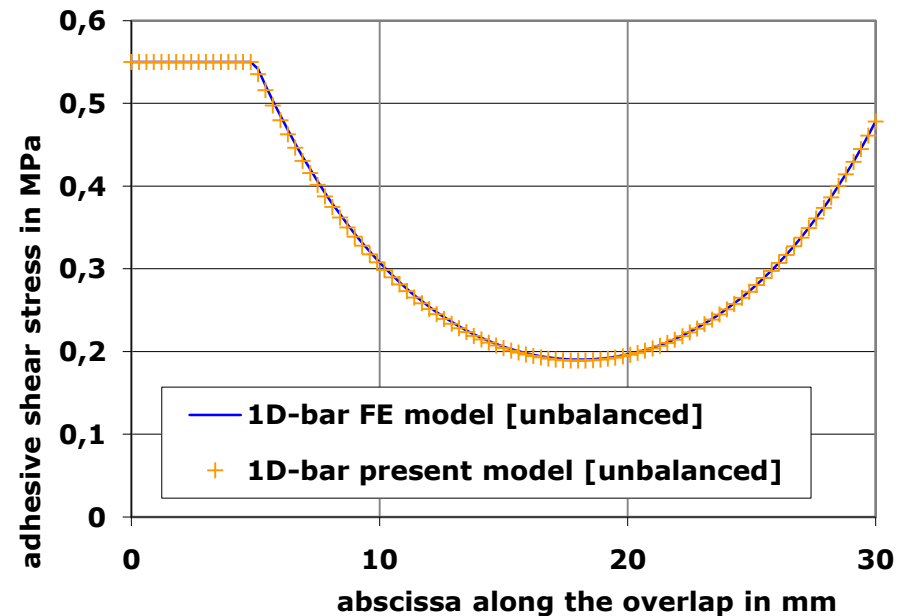
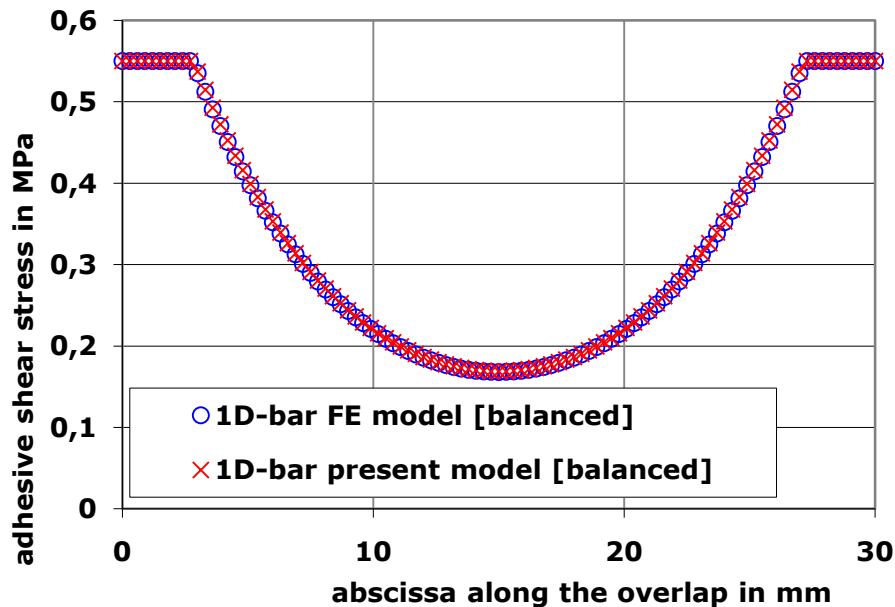
## Validation

### Against Degraded FE Model [6]



✓ Degraded FE model, composed by bars and springs

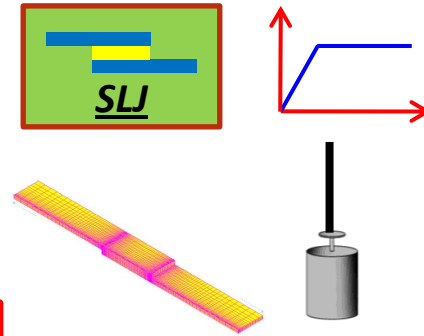
- elastic perfectly plastic adhesive, after maximal stress yield criterion
- 1D-bar kinematics, in-plane mechanically loaded



# Relevance

## Assessment

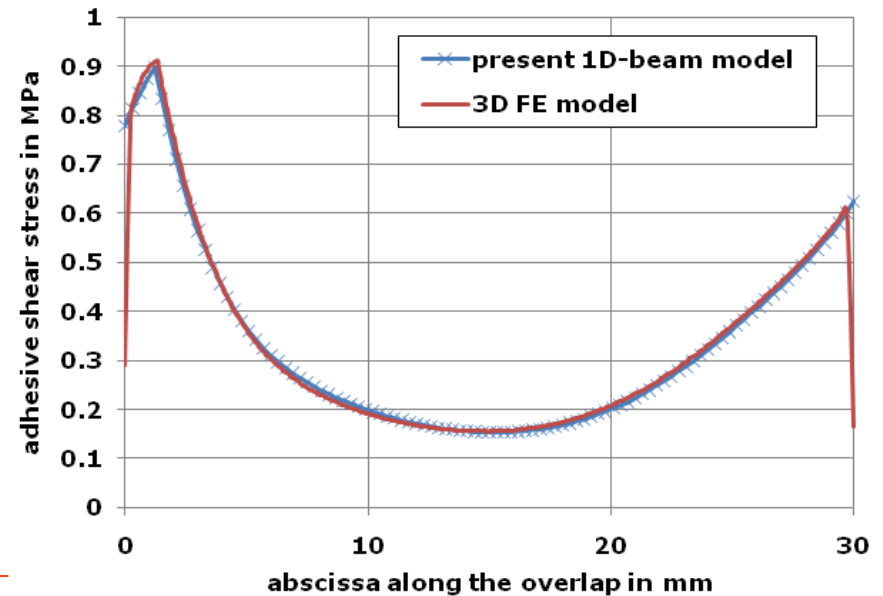
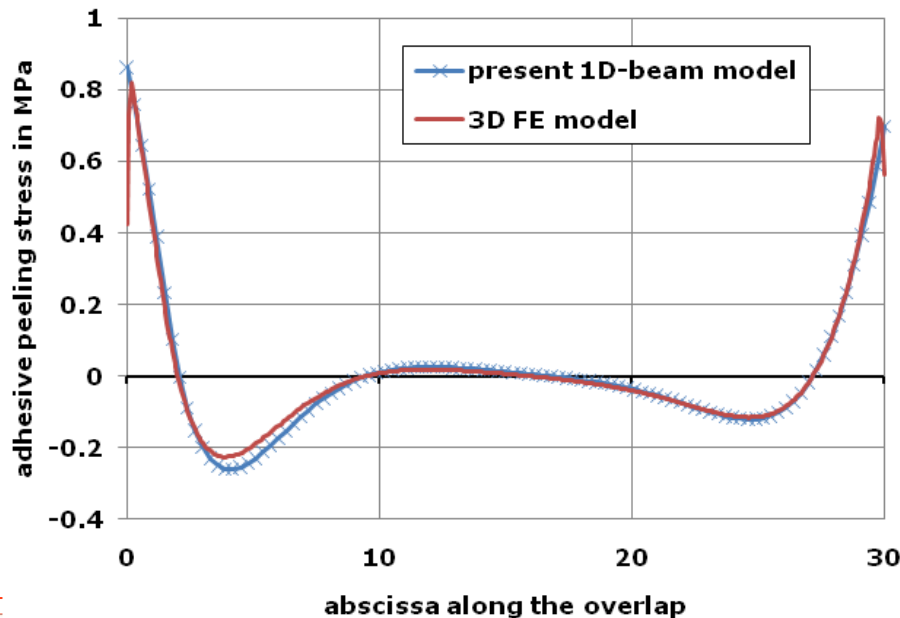
### Against Refined FE Model [6,8]



#### ✓ Against refined 3D FE models

- elastic perfectly plastic adhesive after Von Mises yield criterion
- 1D-beam kinematics, unbalanced, in-plane mechanically loaded

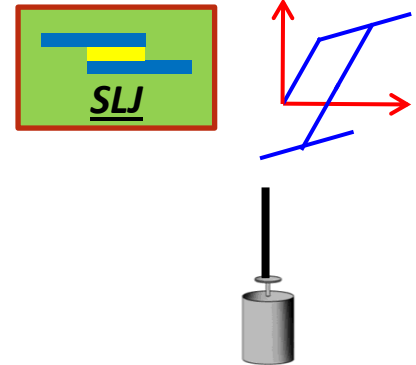
**CPU TIME SAVINGS: 50 times faster than FE model**



### With SCILAB Code

✓ **Clamped single-lap bonded joint:**

- bilinear elasto-plastic
- kinematic, isotropic and mixed hardening
- mechanically loaded
- loading then unloading



# Relevance

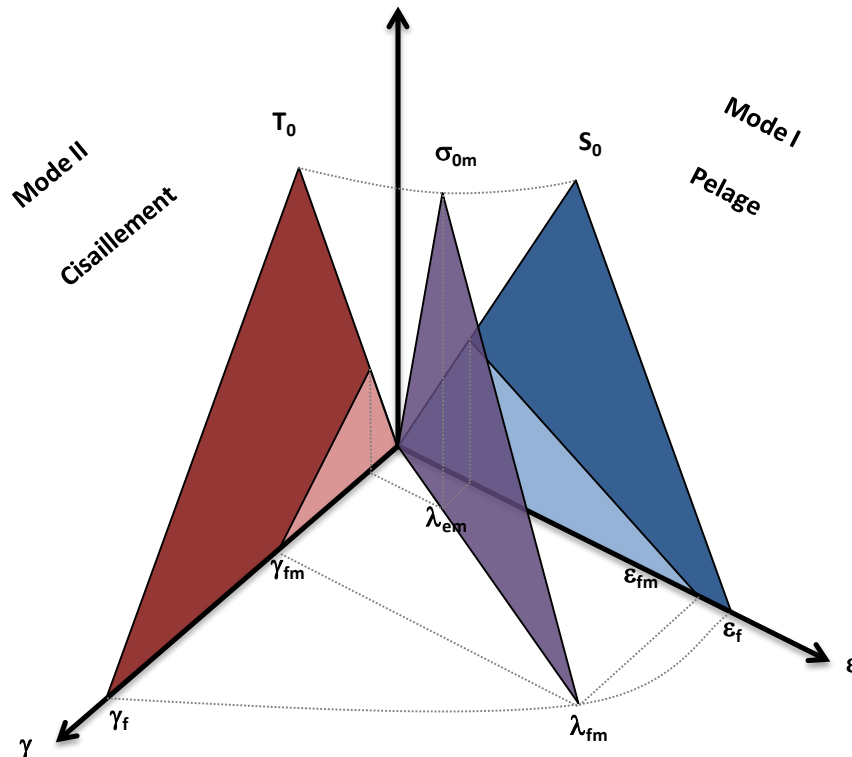
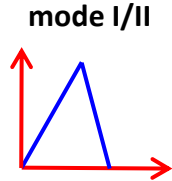
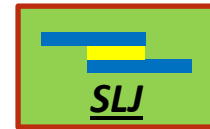
## Demonstration

### Early IHM

✓ Clamped single-lap bonded joint in-plane loaded:

- bilinear damage evolution
- mixed mode I/II
- loading then unloading

**DEMO**



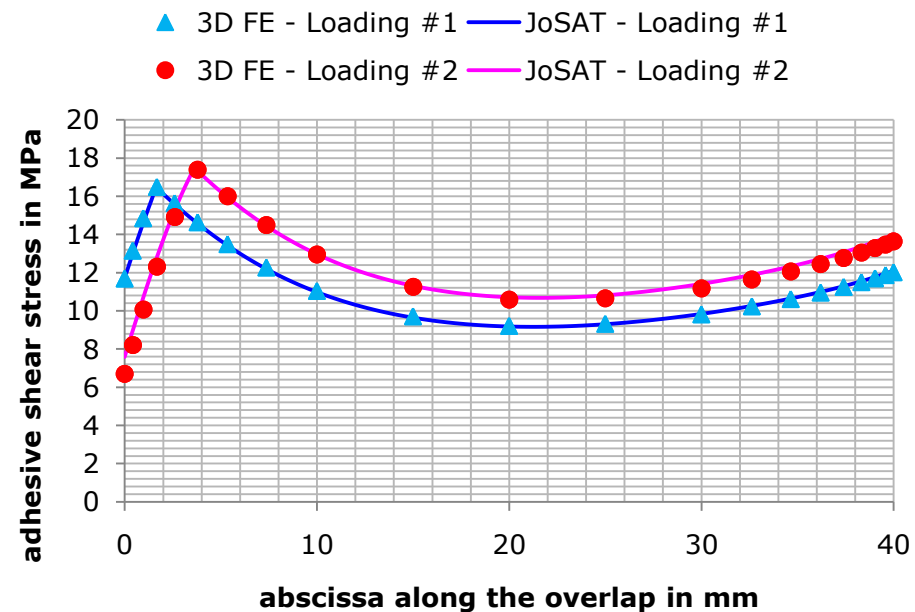
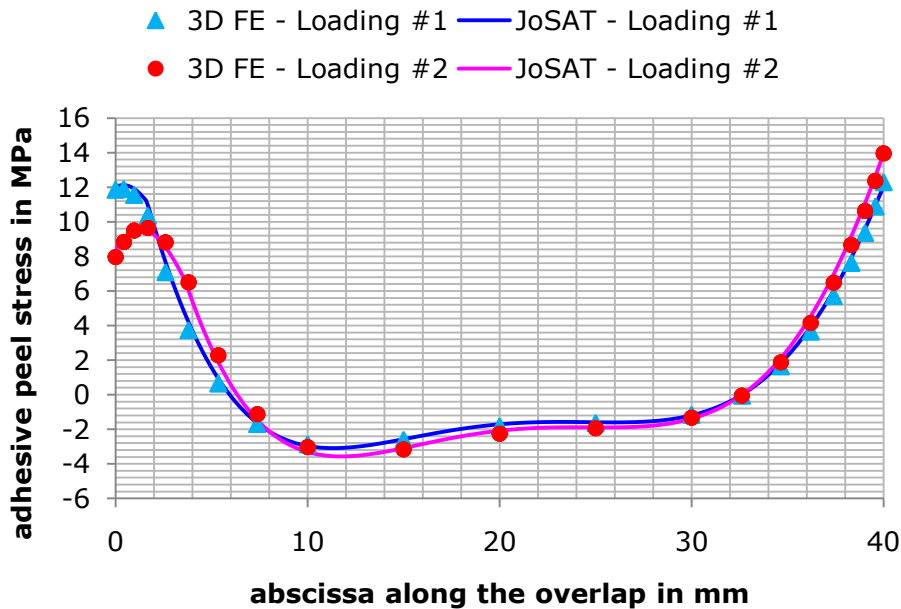
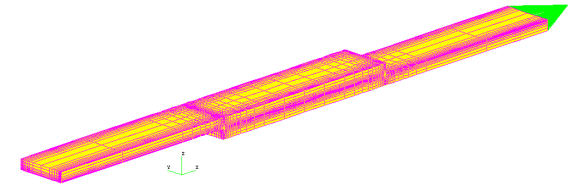
# Relevance

## Demonstration

### Against Refined FE models

#### ✓ Against refined 3D FE models

- bilinear adhesive damaging evolution law
- 1D-beam kinematics, unbalanced, in-plane mechanically loaded



**VERY GOOD AGREEMENT WITH 3D FE PREDICTIONS**

# Content

A thick red line that starts on the left, dips down into a small curve, then runs horizontally across the top of the page, and finally curves up on the right side.

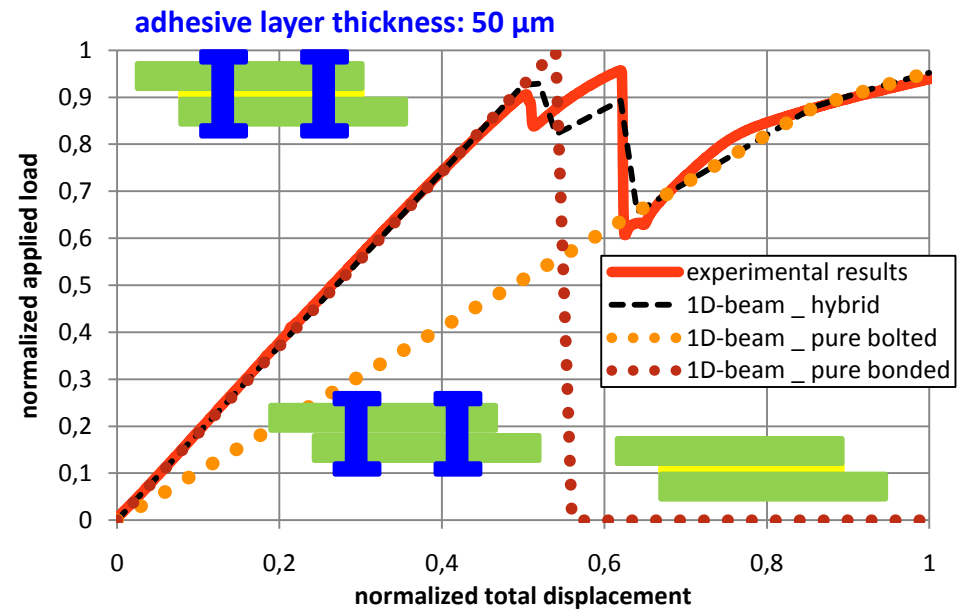
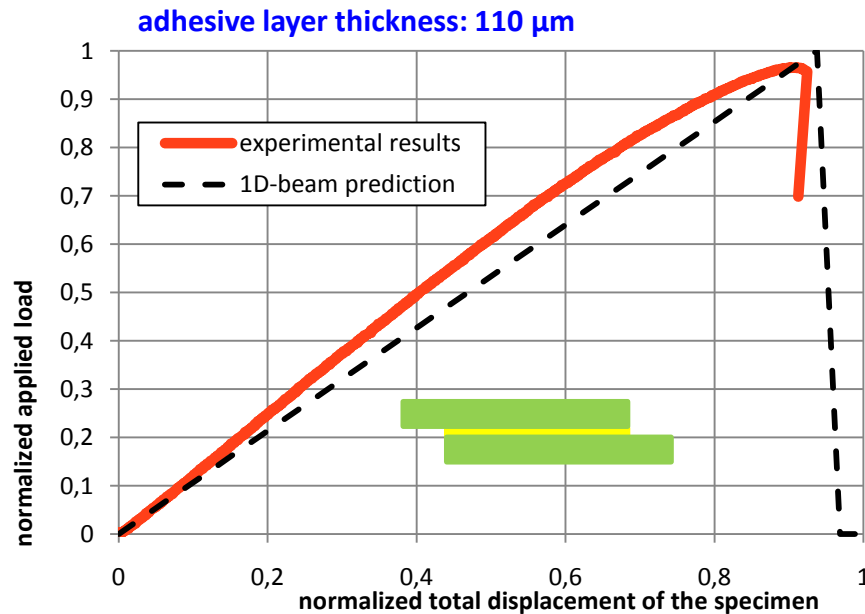
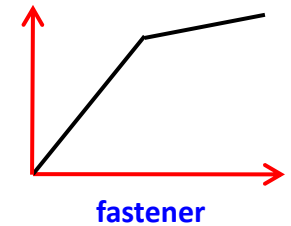
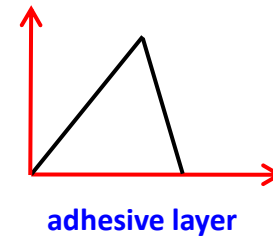
## APPLICATION

## Experimental Test

### Hybrid (Bolted / Bonded) Joints

#### ✓ Identification of material parameters [IN PROGRESS]

- through an optimization platform
- to analyze failure mechanisms

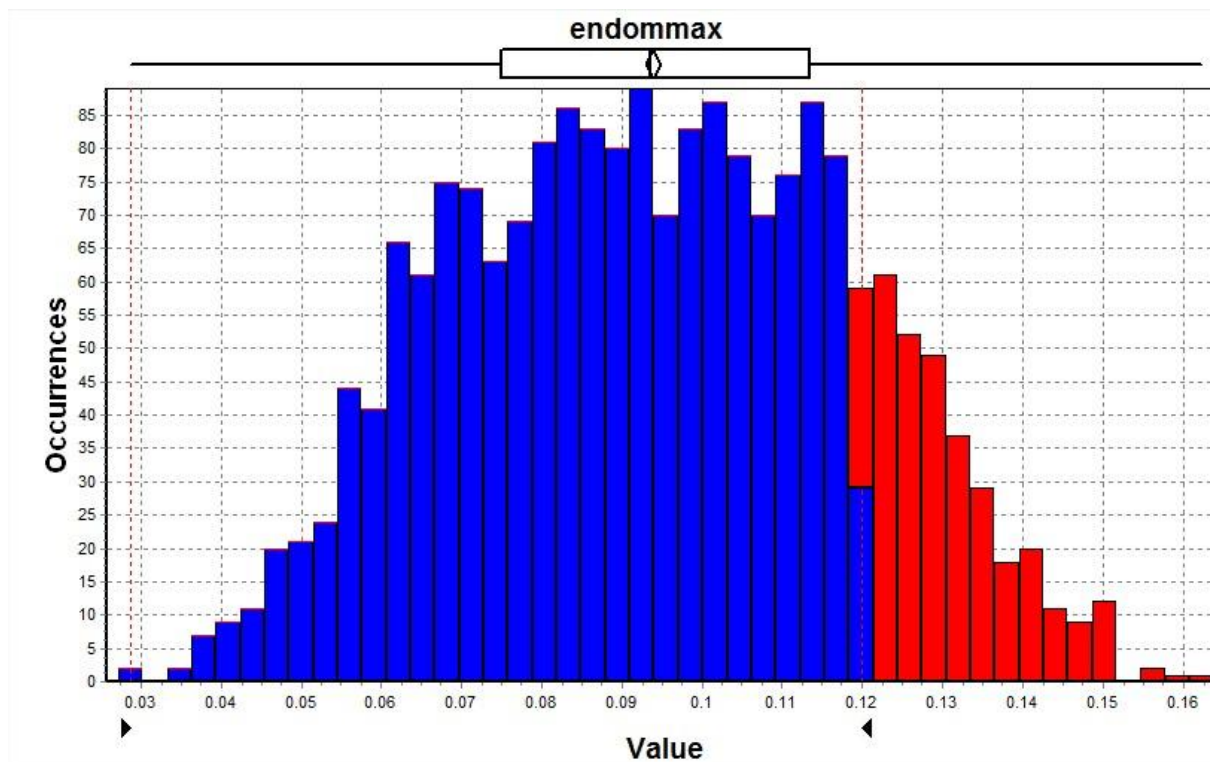




## Process Reliability

✓ Simulation of the reliability of the manufacturing process

- Considering manufacturing knowledge
- Simulation with Monte-Carlo analysis



Moments		
Mean:		0.09401
Std Dev:		0.02479
Std Err Mean:		0.00055
Upper 95% Mean:		0.0951
Lower 95% Mean:		0.09292
N:		2000
Variance:		0.00061
Skewness:		0.01648
Kurtosis:		-0.68315
CV:		0.26372

Quantiles		
100.0%	maximum	0.16232
99.5%		0.14999
97.5%		0.14007
90%		0.12683
75%	quartile	0.1134
50%	median	0.0936
25%	quartile	0.07496
10%		0.06186
2.5%		0.04842
0.5%		0.03993
0.0%	minimum	0.0287

Reliability		
Lower Bound:	<input type="text" value="-Infinity"/>	
Upper Bound:	<input type="text" value="0.12"/>	
Reliability:		0.834

Plot Type		

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