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HYBRID (BOLTED/BONDED) JOINTS APPLIED TO AERONAUTIC PARTS: ANALYTICAL ONE-DIMENSIONAL MODELS OF A SINGLE-LAP JOINT

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Abstract:

The load transfer in hybrid (bolted/bonded) single-lap joint is complex due to the association of two different transfer modes (discrete and continuous) through elements with different stiffness. Analytical methods exist for these two different modes, when considered separately. In this paper two one-dimensional elastic analytical models are presented for the determination of the load transfer in single lap configuration. The first one is developed by using the integration of the local equilibrium equations. From this first method an elastic-plastic approach is presented. The second one uses the Finite Element Method, introducing a new element called "bonded-bar". These models are robust, easy to use and provide the same results. They allow to analyze the load transfer and to evaluate different geometric and mechanical parameters' influence. Thus they represent the first step for the design of a hybrid joint able to replace its bolted equivalent used on aircraft.

Key words: hybrid (bolted/bonded) joint, single-lap joint, load transfer, analytical analysis, Finite Element Method

Nomenclature

- $E^{(j)}$: Young modulus of the adherent j in MPa
- G : Coulomb modulus of the adhesive in MPa
- $u_i^{(j)}$: Longitudinal displacement in mm of the adherent j in the bay i
- b : Transversal pitch in mm
- d_i : Abscissa of the fastener i (d : edge distance in mm; s : longitudinal pitch in mm)
- e : Thickness of the adhesive in mm
- $e^{(j)}$: Thickness of the adherent j
- L : Length of the lap mm
- L_g : Length of the left bar not bonded in mm
- L_d : Length of the right bar not bonded in mm
- x_p : Length of the plastic region in mm
- T_p : Plastic adhesive shear stress in MPa

N : Normal force in N
 T : Adhesive shear stress in MPa

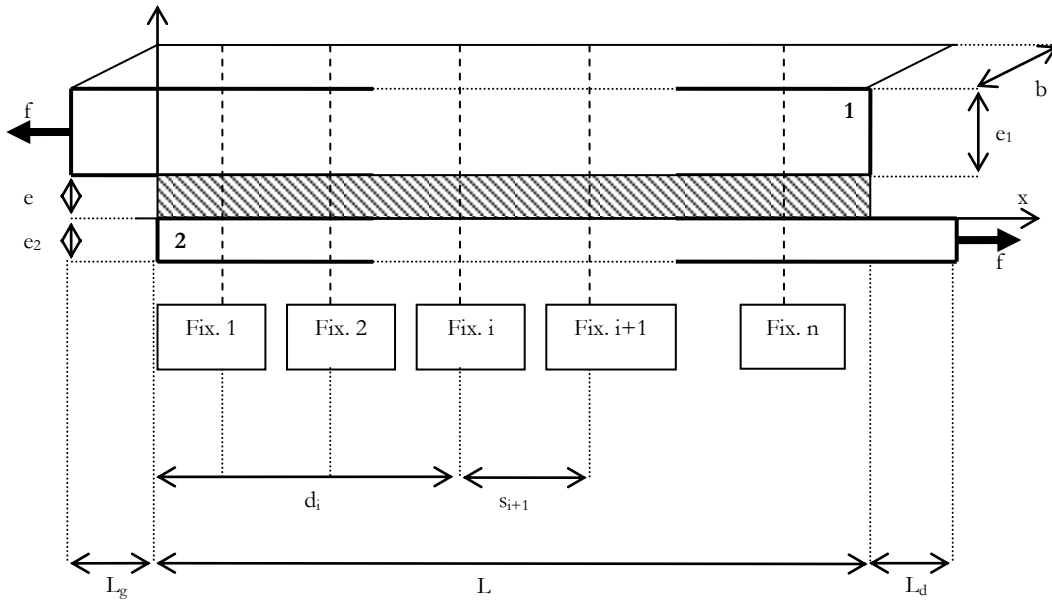


Figure 0. Nomenclature

1 Introduction

The joints under study are joints of civil aircraft. The longitudinal joints of the fuselage are investigated. These longitudinal joints of fuselage are composed of aluminium sheets and titanium bolts. The developed method, which is presented in this paper, has to apply to the other joints on aircraft.

This paper deals with load transfer in hybrid single-lap joints. Hybrid joints are bolted/bonded joints, then associating a discrete transfer mode with a continuous one, each one belonging to its own stiffness. The bolted joint (discrete transfer mode) generates a high overstress around the holes of the fasteners which is prejudicial to the fatigue resistance. The bonded joint (continuous transfer mode) allows a better distribution of the transfer; however it presents a plastic accommodation which is prejudicial to the static strength in the long term. In the domain of aircraft structure assembly, the hybrid joining could be interesting because it could reduce the load transferred by the fasteners in order to improve the fatigue life, while ensuring static strength under extreme loads. The idea is to design the hybrid joint in order to share the load between the adhesive and the fasteners in a suitable way. That's why the influence of the joint geometry and the material properties on the load transfer is investigated by means of developing efficient designing tools. Analytical approaches are thus privileged.

Analytical methods exist for the two elementary transfer modes. The second and the third parts of this paper deal respectively with the analytical model of bolted joints design and with the analytical model of the bonded joints design, which will be used again within the analytical approaches of hybrid (bolted/bonded) joints. In the fourth and the fifth part two elastic one-dimensional analytical models are presented for the determination of the load transfer of hybrid joints in single-lap configuration. Even if the adhesive shear stress can be deduced from the load transfer, the results presented in the sixth part illustrate the load transfer obtained from these models. In the seventh part, a perfectly elastic-plastic behaviour of the adhesive is introduced.

2 Analytical model for bolted joints

The load transfer in a bolted joint is a discrete transfer mode. It means that between each bolt (i.e.: on each bay) the transferred load is constant. In [1] the author calculates the load transferred by the fasteners using an analogy with an electric meshing and modelling the fasteners by springs, which work by shearing (Figure 1). The behaviour of a fastener in a joint is a difficult problem and the determination of its flexibility provides numerous studies and formulations ([2], [3], Douglas, Boeing). The behaviour of a fastener can be defined by a curve force-displacement of the joint. The linear part of this curve gives the rigidity of the fastener (Figure 2), quoted C_u .

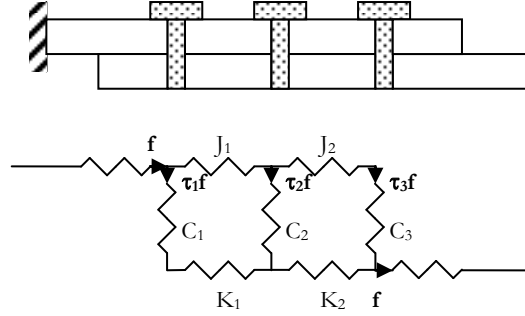


Figure 1. Electric meshing of bolted joint

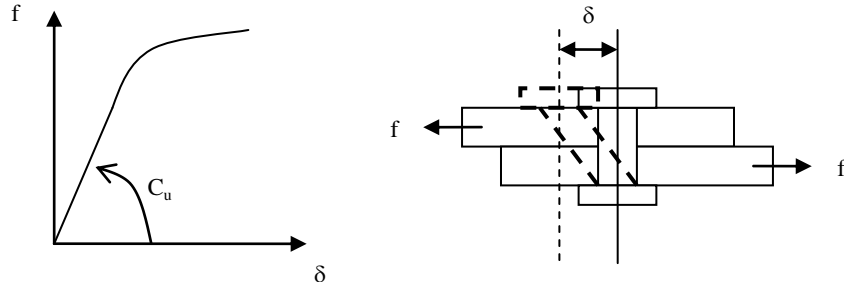


Figure 2. Behaviour of a fastener

The idea of electric meshing of the bolted joint model will be used later in this paper. The whole fasteners are assumed to have the same rigidity.

3 Analytical model for bonded joints

In [4], Hart-Smith analyses the stress distribution in a bonded double-lap joint, without taking into account the bending of the adherents and the adhesive peeling stress, since the eccentricity of the load path is not influent in double-lap configuration. The author realized the local equilibrium of an elementary length of the adherent. Considering the case of the single-lap configuration (Figure 3), the equilibrium equations are:

$$\frac{dN^{(2)}(x)}{bdx} = T(x) \quad \text{and} \quad \frac{dN^{(1)}(x)}{bdx} = -T(x) \quad (1) \text{ and } (2)$$

The elastic behaviour of the adhesive gives the following equation:

$$T(x) = \frac{G}{e} (u^{(2)}(x) - u^{(1)}(x)) \quad (3)$$

Whereas the hypothesis of elastic adherents provides:

$$\frac{du^{(j)}(x)}{dx} = \frac{N^{(j)}(x)}{be^{(j)}E^{(j)}} \quad \text{for } j=1,2 \quad (4)$$

The author gets then the following differential equation of the second order with constant factors:

$$\frac{d^2T(x)}{dx^2} - \eta^2 T(x) = 0 \quad \text{where: } \eta^2 = \frac{G}{e} \left(\frac{1}{e^{(1)}E^{(1)}} + \frac{1}{e^{(2)}E^{(2)}} \right) \quad (5) \text{ and } (6)$$

This bonded joint model will be used later in this paper.

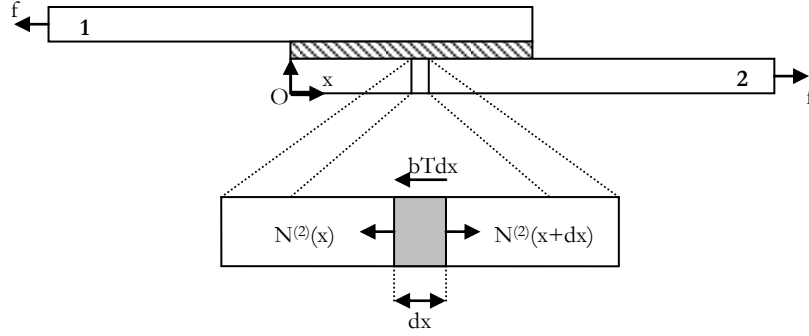


Figure 3. Bonded single-lap joint and local equilibrium

4 First analytical model for hybrid joints

The goal is to combine both approaches in order to get a load transfer, which is continuous by parts. A similarly approach was developed for the calculation of stepped joints in [5]. The conditions of longitudinal force-equilibrium for a differential element dx in the bay i within the joint are (1) and (2). By differentiating (1) and with (3), it comes:

$$\frac{d^2N_i^{(2)}(x)}{bdx^2} = \frac{G}{e} \left(\frac{du_i^{(2)}(x)}{dx} - \frac{du_i^{(1)}(x)}{dx} \right) \quad (7)$$

Using (4) and the equation of the general equilibrium (f is the applied load in N):

$$N_i^{(1)}(x) + N_i^{(2)}(x) = f \quad (8)$$

it comes for the bay i (cf. Figure 4) the following differential equation:

$$\frac{d^2N_i^{(2)}(x)}{dx^2} - \eta^2 N_i^{(2)}(x) = \gamma f \quad \text{where: } \gamma = -\frac{G}{ee^{(1)}E^{(1)}} \quad (9) \text{ and } (10)$$

Consequently the number of equations is equal to the number of bays i . In order to solve (9) on each bay, it is required to express the boundaries conditions.

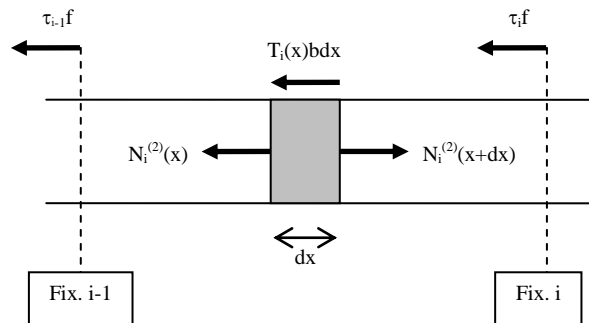


Figure 4. Bay number i of the hybrid joint

The developed approach is based on the following hypotheses:

- an elastic behaviour of materials (adherents, adhesive, fasteners);
- normal stress in the adherents (no bending);
- shear stress in the adhesive (no peeling).

It is assumed that the adhesive thickness is constant along the lap-joint and that the fasteners have the same rigidity. The mechanical and geometric parameters are free.

The solution of the equation (9) is:

$$N_i^{(2)}(x) = A_i e^{-\eta x} + B_i e^{\eta x} - \gamma \eta^{-2} f \quad (11)$$

For n fasteners (thus $n+1$ bays), there are $2n+2$ unknowns, which are determined thanks to the boundaries conditions.

The first condition is no tensile load at the start of the adherent 2, whereas the second condition corresponds to a complete load transfer at the end of the lap:

$$N_1^{(2)}(0) = 0 \Leftrightarrow A_1 + B_1 = \gamma \eta^{-2} f \quad (12)$$

$$N_{n+1}^{(2)}(L) = f \Leftrightarrow A_{n+1} e^{-\eta L} + B_{n+1} e^{\eta L} = (1 + \gamma \eta^{-2}) f \quad (13)$$

By considering the fasteners, the equation which corresponds to the load transfer ratio τ_i at the fastener i between both bays i and $i+1$ is:

$$N_{i+1}^{(2)}(d_i) = N_i^{(2)}(d_i) + \tau_i f \quad (14)$$

Moreover the fasteners are simulated by springs, the rigidity of which is C_u (in N.mm^{-1}), thus:

$$\tau_i f = C_u (u_i^{(2)}(d_i) - u_i^{(1)}(d_i)) \quad (15)$$

Thus with (1), (3) and (11):

$$\tau_i f = \varphi (-A_i e^{-\eta d_i} + B_i e^{\eta d_i}) \quad \text{where: } \varphi = \frac{e \eta C_u}{b G} \quad (16) \text{ and } (17)$$

And with (14) and (16), n additional equations come:

$$r e^{-\eta d_i} A_i + q e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} - e^{\eta d_i} B_{i+1} = 0 \quad (18)$$

$$\text{with: } r = 1 - \varphi \quad \text{and} \quad q = 1 + \varphi \quad (19) \text{ and } (20)$$

Finally, the continuity of the adhesive shear stress provides the n last equations:

$$T_i(d_i) = T_{i+1}(d_i) \Leftrightarrow e^{-\eta d_i} A_i - e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} + e^{\eta d_i} B_{i+1} = 0 \quad (21)$$

Thus, a linear system, the size of which is $2n+2$, is obtained:

$$\begin{cases} A_1 + B_1 = \gamma \eta^{-2} f \\ A_{n+1} e^{-\eta L} + B_{n+1} e^{\eta L} = (1 + \gamma \eta^{-2}) f \\ r e^{-\eta d_i} A_i + q e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} - e^{\eta d_i} B_{i+1} = 0 \\ e^{-\eta d_i} A_i - e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} + e^{\eta d_i} B_{i+1} = 0 \end{cases}, i \in [1; n] \quad (22)$$

The resolution of this system gives the parameters A_i and B_i and allows to build the whole functions, which characterize the one-dimensional behaviour of the hybrid joint.

In the particular case of a constant pitch s between the fasteners and a constant distance between the end of the joint and the fastener d (longitudinal edge distance), the linear system (22) may be simplified in a linear system the size of which is 2.

Changing the parameters like:

$$\begin{cases} \alpha_i = e^{-\eta d_i} A_i \\ \beta_i = e^{\eta d_i} B_i \\ \alpha_{n+1} = e^{-\eta L} A_{n+1} \\ \beta_{n+1} = e^{\eta L} B_{n+1} \end{cases}, i \in [1; n] \quad (23)$$

and setting:

$$U_i = \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}, i \in [1; n+1] \quad (24)$$

the linear system (22) becomes:

$$U_i - \frac{1}{2} \Gamma_X U_{i+1} = 0, i \in [1, n] \quad (25)$$

$$\text{where: } \Gamma_X = \begin{pmatrix} (2+\varphi)e^{\eta X} & -\varphi e^{-\eta X} \\ \varphi e^{\eta X} & (2-\varphi)e^{-\eta X} \end{pmatrix}, \text{ and } X = d \text{ for } i = n+1 \text{ else } X = s \quad (26)$$

The matrix Γ_X is diagonalisable and $Sp_{\Gamma_X} = \{x_1, x_2\}$. In the base of diagonalisation (25) becomes:

$$V_i - \frac{1}{2} \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix} V_{i+1} = 0, i \in [1, n-1] \quad (27)$$

A solution is searched like:

$$V_i = \begin{pmatrix} A \tilde{\alpha}^i \\ B \tilde{\beta}^i \end{pmatrix} \quad (28)$$

Then, it comes:

$$\begin{cases} \tilde{\alpha} = \frac{2}{x_1} \\ \tilde{\beta} = \frac{2}{x_2} \end{cases} \quad (29)$$

Only A and B parameters have to be determined.

5 Second analytical model for hybrid joints (bonded-bar element)

The Finite Element Method is used in the second approach. The single-lap joint is meshed in 1D-elements (Figure 5). Simple elements are used: bar elements (element 1 and 7 on Figure 5), springs (element 5 and 6) and new elements called “bonded-bars” (element 2, 3 and 4). These new elements are 1D-elements since only the displacements in the direction of the load are taken into account. However they have four nodes (Figure 6) allowing to differentiate the displacement of each adherent.

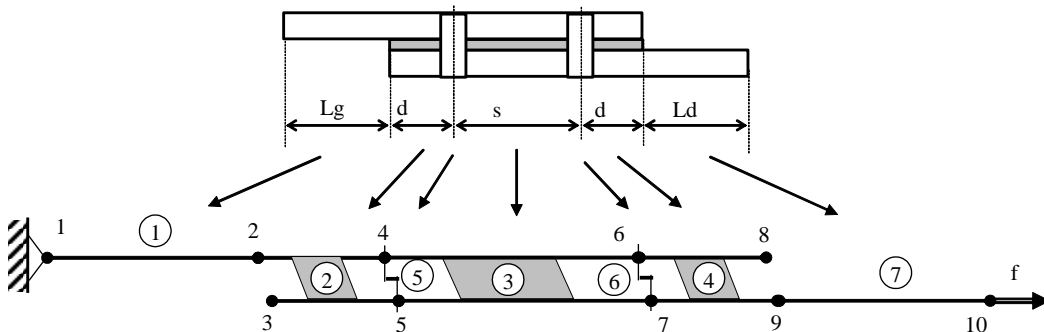


Figure 5. Structure meshing

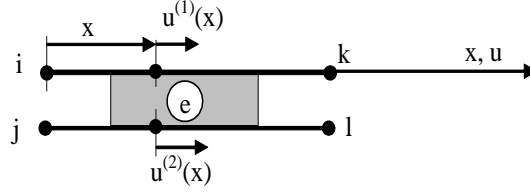


Figure 6. Bonded-bar element

The rigidity matrix of the bonded-bar element has to be determined. The length of the bonded-bar element is quoted Δ . The subscript i is not useful here.

Thanks to the equations (1) to (4), the following system of differential equations is obtained:

$$\begin{cases} \frac{d^2 u^{(1)}}{dx^2} + \frac{G}{e E^{(1)} e^{(1)}} (u^{(2)} - u^{(1)}) = 0 \\ \frac{d^2 u^{(2)}}{dx^2} - \frac{G}{e E^{(2)} e^{(2)}} (u^{(2)} - u^{(1)}) = 0 \end{cases} \quad (30)$$

and it is solved (by addition and subtraction for example) as:

$$\begin{cases} u^{(1)} = 0.5 \left[-\theta (E e^{-\eta x} + F e^{\eta x}) + Cx + D \right] \\ u^{(2)} = 0.5 \left[\omega (E e^{-\eta x} + F e^{\eta x}) + Cx + D \right] \end{cases} \quad (31)$$

where:

$$\psi = \frac{G}{e} \left(\frac{1}{E^{(1)} e^{(1)}} - \frac{1}{E^{(2)} e^{(2)}} \right), \quad \theta = 1 + \frac{\psi}{\eta^2} \quad \text{and} \quad \omega = 1 - \frac{\psi}{\eta^2} \quad (32) \text{ and } (34)$$

The following boundaries conditions:

$$\begin{cases} u^{(1)}(0) = u_i \\ u^{(2)}(0) = u_j \\ u^{(1)}(\Delta) = u_k \\ u^{(2)}(\Delta) = u_l \end{cases} \quad (35)$$

leads to:

$$E = \frac{(u_j - u_i) e^{\eta \Delta} - (u_l - u_k)}{2sh(\eta \Delta)} \quad F = \frac{(u_l - u_k) - (u_j - u_i) e^{-\eta \Delta}}{2sh(\eta \Delta)} \quad C = \frac{u_l \theta + u_k \omega - u_j \theta - u_i \omega}{\Delta} \quad D = u_j \theta + u_i \omega \quad (36) \text{ to } (39)$$

The rigidity matrix of the bonded-bar element is defined by:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \\ u_l \end{bmatrix} = \begin{bmatrix} Q_i \\ Q_j \\ Q_k \\ Q_l \end{bmatrix} \quad (40)$$

It follows that:

$$\begin{aligned} \frac{\partial Q_i}{\partial u_i} &= k_{11} \\ \frac{\partial Q_i}{\partial u_j} &= k_{12} \\ \frac{\partial Q_i}{\partial u_k} &= k_{13} \\ \frac{\partial Q_i}{\partial u_l} &= k_{14} \\ &\dots \end{aligned} \quad (41)$$

The matrix components can be obtained using the boundaries conditions. Indeed, the normal loads in the adherents are obtained from (4) and (31); the forces Q_i , Q_j , Q_k , and Q_l which the nodes i , j , k and l exert on the element are:

$$\begin{cases} Q_i = -N_1(0) \\ Q_j = -N_2(0) \\ Q_k = N_1(\Delta) \\ Q_l = N_2(\Delta) \end{cases} \quad (42)$$

Finally, the rigidity matrix of the bonded-bar element is:

$$K_{bonded-bar} = \omega h \begin{pmatrix} ct + \mu & 1 - ct & -cs - \mu & 1 - cs \\ 1 - ct & ct + \mu^{-1} & 1 - cs & -cs - \mu^{-1} \\ -cs - \mu & 1 - cs & ct + \mu & 1 - ct \\ 1 - cs & -cs - \mu^{-1} & 1 - ct & ct + \mu^{-1} \end{pmatrix} \quad (43)$$

where:

$$h = 0.5 \frac{E^{(2)} e^{(2)} b}{\Delta}, \quad cs = \eta \Delta \cosh(\eta \Delta), \quad ct = \eta \Delta \coth(\eta \Delta) \quad \text{and} \quad \mu = \frac{E^{(1)} e^{(1)}}{E^{(2)} e^{(2)}} = \frac{\omega}{\theta} \quad (44) \text{ to } (47)$$

On the other hand, the rigidity matrix of the bar elements, which describe the not bonded portions of the adherents, take the conventional form:

$$K_{bar} = \begin{pmatrix} K_b & -K_b \\ -K_b & K_b \end{pmatrix} \quad (48)$$

where:

$$K_b = \frac{E^{(1)} e^{(1)} b}{L_g} \quad (\text{element 1}) \quad \text{or} \quad K_b = \frac{E^{(2)} e^{(2)} b}{L_d} \quad (\text{element 7}) \quad (49) \text{ and } (50)$$

Similarly, the rigidity matrix of the fasteners is:

$$K_{fastener} = \begin{pmatrix} C_u & -C_u \\ -C_u & C_u \end{pmatrix} \quad (51)$$

The rigidity matrix of the whole structure can be obtained using the conventional assembly rules of the FEM, and the classical system ($F=Ku$) is solved. For n fasteners, the size of this linear system is $(2n+5)$.

In order to calculate the load transfer of the fasteners, the nodal forces are determined thanks to (4), (31), (42) and the nodal displacements calculated by the previous system:

$$\begin{cases} Q_i = -0.5 E^{(1)} e^{(1)} b [-\theta \eta (F - E) + C] \\ Q_j = -0.5 E^{(2)} e^{(2)} b [\omega \eta (F - E) + C] \\ Q_k = 0.5 E^{(1)} e^{(1)} b [-\theta \eta (F e^{\eta \Delta} - E e^{-\eta \Delta}) + C] \\ Q_l = 0.5 E^{(2)} e^{(2)} b [\omega \eta (F e^{\eta \Delta} - E e^{-\eta \Delta}) + C] \end{cases} \quad (52)$$

6 Results

Both previous approaches lead to the same results.

For a hybrid joint with 3 fasteners with a Coulomb modulus of the adhesive, the value of which is near from 0Mpa, the models allow to find the results which are given by the analytical model of a bolted joint. In the same way, the models provide the behaviour of a bonded joint, when the rigidity of the fasteners is equal to 0.

From now we will consider examples with two fasteners. The Table 1 gives the values of the mechanical and geometric parameters used for the whole following curves. The

transversal pitch is taken equal to 1mm in order to present significantly the effect of the fasteners on the curves, even if it is not realistic.

$E^{(j)}$ (Mpa)	G (Mpa)	$e^{(j)}$ (mm)	e (mm)	d (mm)	s (mm)
72000	800	2,4	0,4	9,6	19,2

Table 1. Geometric and mechanical parameters used

The following figure (Figure 7) gives the load transfer along the second adherent in a two fasteners hybrid joint; this load transfer is compared to the load transfer in a simply bonded and a simply bolted joint.

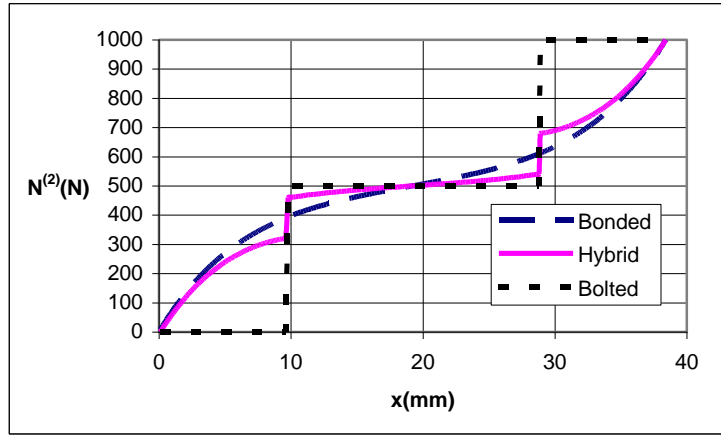


Figure 7. Load transfer ($b=1\text{mm}$, $C_u=40000\text{N/mm}$)

This figure shows that the load transfer in a hybrid joint sets between the load transfer in a bonded joint and a bolted joint. Both steps correspond to the transfer by the fasteners and the aspect in \sinh corresponds to the load transferred by the adhesive.

The following figure (Figure 8) represents the load transfer for different rigidities of the fasteners.

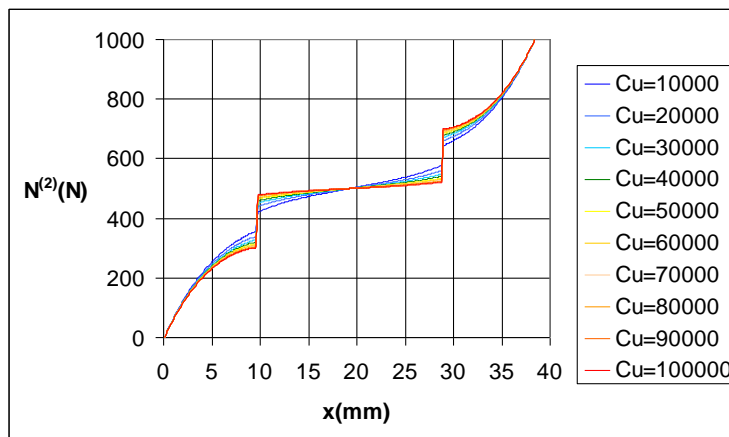


Figure 8. Load transfer as a function of C_u ($b=1\text{mm}$)

This figure shows that the load transfer is mainly performed at the bonded ends of the joint. Then, compared to a simply bolted joint, the bonding decreases highly the load transferred by the fasteners.

Moreover the parametric study performed thanks to these models fits the trends given in [6] (experimental and numeric study of hybrid joints in single-lap configuration) that is to say the load transferred by the bolts increases when:

- the Young modulus of adherents increases;
- the thickness of the adherents increases;
- the length of the lap decreases;
- the Coulomb modulus of the adhesive decreases;
- the longitudinal pitch decreases.

Thanks to these models, it is possible to add that the load transferred by the fastener increases when:

- the rigidity of the fasteners increases;
- the transversal pitch decreases;
- the edge distance decreases.

It is noticeable that the load transferred by the fasteners is nearing 0 when the edge distance increases.

Finally, using the Finite Element code SAMCEF, a two-dimensional model in plane strain is developed. The hypotheses are the same than the one of the analytical approaches. Three superposed layers simulate the joint adherent-adhesive-adherent. These elements are quadrangle elements of degree 2. Each fastener is simulated by quadrangle elements of degree 2. As it is shown on the Figure 9, there is a good correlation between the numerical and analytical approaches.

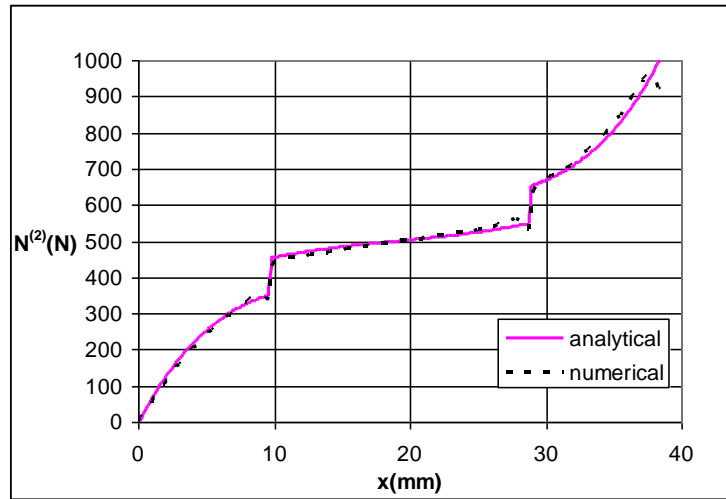


Figure 9. Comparison between numerical and analytical approaches ($b=1\text{mm}$, $C_u=10000\text{N/mm}$)

7 Considering the elastic-plastic behaviour of the adhesive

In the previous models, the adhesive is perfectly elastic. However, perfectly elastic-plastic behaviour of the adhesive can be computed using the first approach. It is assumed that both edge distances are equal. Moreover, it is assumed that the adhesive has a plastic behaviour only into the edge distances, since the load transfer is more important along both bonded ends. This elastic-plastic approach is inspired by [4].

The length, along which the adhesive has a plastic behaviour, is quoted x_p :

$$0 \leq x_p \leq d \quad (53)$$

It comes:

$$\forall x \leq x_p, T_1(x) = T_p \quad \text{and} \quad \forall x \in [L - x_p; L], T_{n+1}(x) = T_p \quad (54) \text{ and } (55)$$

The local equilibrium equations are the same as (1) and (2). Hence, by integrating:

$$\forall x \leq x_p, N_1^{(2)}(x) = T_p b x \quad \text{and} \quad \forall x \in [L - x_p; L], N_{n+1}^{(2)}(x) = f - T_p b(L - x) \quad (56) \text{ and } (57)$$

Both following equations replace (12) and (13):

$$N_1^{(2)}(x_p) = T_p b x_p \quad \text{and} \quad N_{n+1}^{(2)}(L - x_p) = f - T_p b x_p \quad (58) \text{ and } (59)$$

Finally with (11) along the elastic region, the linear system (22) in elastic-plastic behaviour of the adhesive becomes:

$$\begin{cases} A_1 e^{-\eta x_p} + B_1 e^{\eta x_p} = \gamma \eta^{-2} f + T_p b x_p \\ A_{n+1} e^{-\eta(L-x_p)} + B_{n+1} e^{\eta(L-x_p)} = (1 + \gamma \eta^{-2}) f - T_p b x_p, i \in [1; n] \\ q e^{-\eta d_i} A_i + r e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} - e^{\eta d_i} B_{i+1} = 0 \\ e^{-\eta d_i} A_i - e^{\eta d_i} B_i - e^{-\eta d_i} A_{i+1} + e^{\eta d_i} B_{i+1} = 0 \end{cases} \quad (60)$$

The method consists in an iterative resolution of the elastic problem. More precisely, for the first iteration x_p is taken equal to 0. Then (60) is solved until the adhesive shear stress in the elastic region is lower to T_p .

The following figure (Figure 10) represents the ratio of the bolt load transfer with the elastic-plastic adhesive (τ_p) divided by the ratio of the bolt load transfer with the elastic adhesive (τ_e) when the load increase.

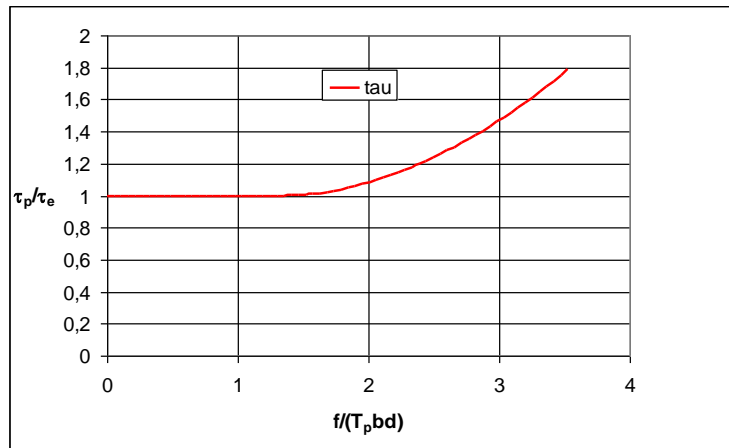


Figure 10. Bolt load transfer with elastic-plastic adhesive ($b=19,2\text{mm}$, $C_u=50000\text{N/mm}$)

This curve shows that the bolt load transfer in elastic-plastic case is around twice as big as the ratio of the bolt load transfer in elastic case, when the adhesive is plasticized all along the edge distance.

In that case, (3) and (15) allows to write:

$$\tau_p f = C_u \frac{e}{G} T_p \quad (61)$$

This last equation provides the three most important parameter of the design of a hybrid joint:

- the edge distance (d);
- the rigidity of the fastener (C_u);
- the relative rigidity of the adhesive (G/e).

8 Conclusions

Two one-dimensional elastic analytical models are developed and presented in this paper. They allow to analyse easily (Figure 11) the influence of the mechanical and geometric parameters on the load transfer in a hybrid joint. These models are robust and simple to use. These models can be regarded as a design tool. Moreover, an original approach, inspired by the Finite Element Method, is presented and validated. Finally, an extension to the perfectly elastic-plastic behaviour of the adhesive is proposed.

Three ways are under consideration to continue. The first way is the development of a two-dimensional model taking into account the bending of the adherent due to the eccentricity of the load path, and the adhesive peeling stress. The second way is the numerical simulation. A three-dimensional model is required to represent accurately and to understand better the behaviour of the hybrid joint. Finally, the last way is the test experience. Static tests using instrumented bolts ([6]) are launched in order to validate and calibrate these models. In particular, the rigidity of the fastener does not seem to be determined accurately, using to the different existing formulations, which does not take into account the hybrid configuration of joint.

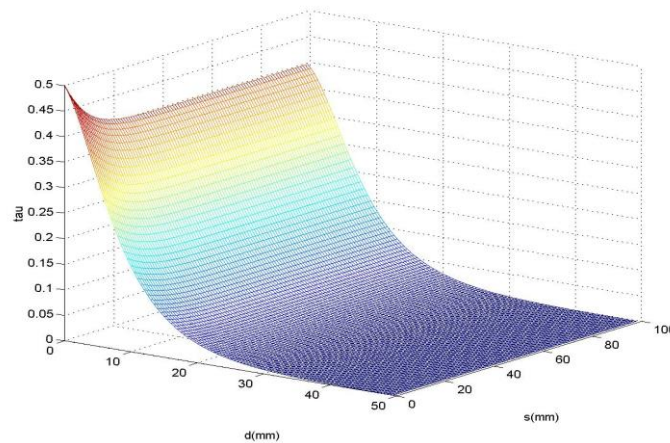


Figure 11. Influence of d and s on the bolt load transfer ($b=1\text{mm}$, $C_u=50000\text{N/mm}$)

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