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## Waves under location uncertainty

Valentin Resseguier, Bertrand Chapron, Fabrice Collard

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# Waves under location uncertainty

**Valentin Resseguier,**  
Bertrand Chapron, Fabrice Collard



# Motivations

- Mimic interaction wave / (small-scale) current
  - Small-scale currents are hardly observed or simulated
  - Waves see them during very short time
- Quantification of associated modelling errors
  -  Ensemble forecasts and data assimilation
- Improve the Scalian's wave simulator
- Study influence of swell / current interactions onto satellite altimetry

# Part I

# Scalian



# SCALIAN

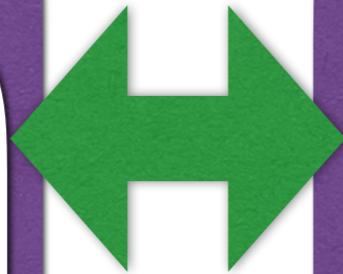
## L@b

(~ 15 peoples)

Research, R&T, R&D

### Expertise:

- Geophysical fluid dyn.
- Signal, data assimilation
- Machine Learning
- Multi-agents systems
- Drones



## CEN « Simulation » (~ 70 people)

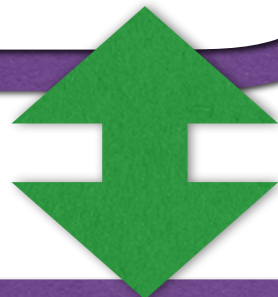
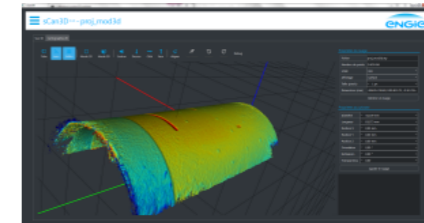
R&D and engineering

### Expertise:

- Radar, optronics, sonar
- Geophysical fluid dyn.
- Mechanical and thermal

### Business:

- Scientific softwares
- Simulations, HPC
- VR & AR



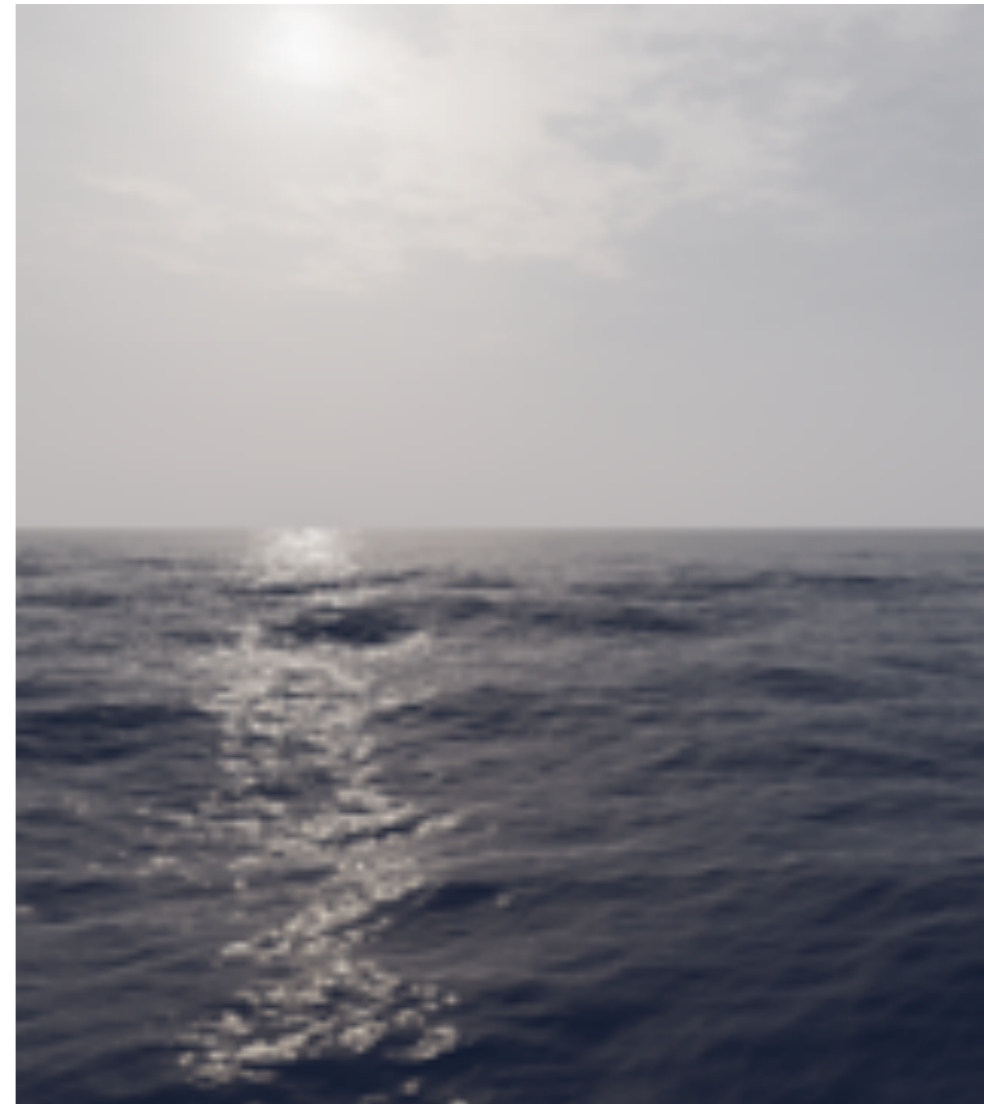
## Other Business Units

~ 2400 people



# SeaMotion-radar simulator

- Simple (mainly linear dynamics) and fast simulation of surface gravity waves
- Coupled with radar simulation
- Lot of physical phenomena taking into account
- Used to study satellite altimetry measurements



# Part II

## Waves LU

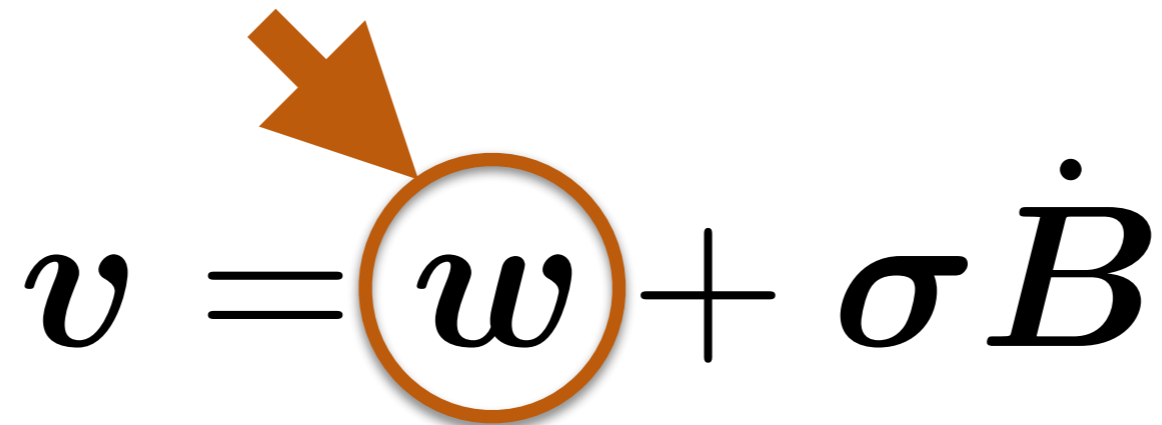
LU : Adding  
random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$



# LU : Adding random velocity

Resolved  
large scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$


# LU : Adding random velocity

Resolved  
large scales

White-in-time  
small scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

# LU : Adding random velocity

Resolved  
large scales

White-in-time  
small scales

$$v = w + \sigma \dot{B}$$

# LU : Adding random velocity

Large scales:

$w$

Small scales:

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Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Resolved large scales

White-in-time small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius and Rozovskii, 2004  
Flandoli, 2011

**Memin**, 2014  
Resseguier et al. 2017 a, b, c  
Chapron et al. 2017  
Cai et al. 2017

**Holm**, 2015  
Holm and Tyranowski, 2016  
Arnaudon et al., 2017

Cotter and al 2017  
Crisan et al., 2017  
Gay-Balmaz & Holm 2017  
Cotter and al 2018 a, b

# Dispersion ratio

At the first order  
in steepness :

Large scales:

$$\omega$$

Small scales:

$$\sigma \dot{B}$$

Variance  
tensor:

$$\mathbf{a} = \mathbf{a}(\mathbf{x}, \mathbf{x}) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Wave:

$$ae^{\frac{i}{\epsilon} \phi}$$

Doppler  
frequency:

$$\omega_0 =$$

$$\sqrt{g\|k\|}$$

# Dispersion ratio

At the first order  
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Small scales:

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Wave:

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Doppler  
frequency:

$$\omega_0 =$$

$$\sqrt{g \|k\|}$$

$$\frac{D\phi}{Dt} = \omega_0(k) = \sqrt{g \|k\|}$$

# Dispersion ratio

At the first order  
in steepness :

Large scales:

$$\omega$$

Small scales:

$$\sigma \dot{B}$$

Variance  
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$$\mathbf{a} = \mathbf{a}(\mathbf{x}, \mathbf{x}) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

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Doppler  
frequency:

$$\omega_0 =$$

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Stochastic  
transport

$$\frac{D\phi}{Dt}$$

$$= \omega_0(k) = \sqrt{g \|k\|}$$

# Dispersion ratio

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in steepness :

Large scales:

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Small scales:

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Variance  
tensor:

$$\mathbf{a} = \mathbf{a}(\mathbf{x}, \mathbf{x}) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler  
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

Stochastic  
transport

$$\frac{D\phi}{Dt}$$

Frequency without currents

$$\omega_0(k) = \sqrt{g \|k\|}$$



# Case 0: Stationary deterministic velocity

Random ray

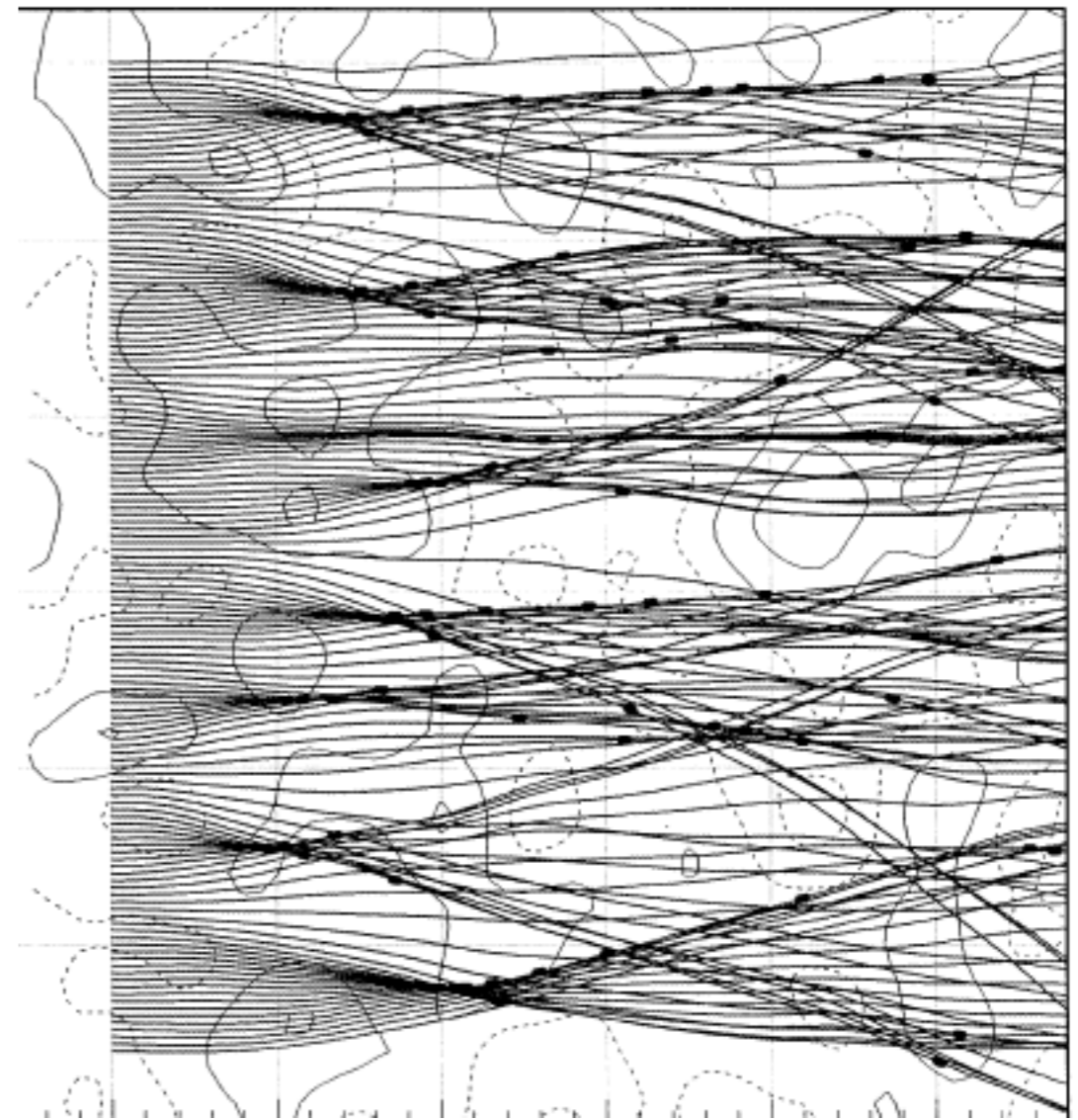
$$\dot{X}_R = u_g$$

Stretching of phase

$$\dot{k} = -\nabla w^T k$$

Conservation of  
action

$$\text{cst.} = A = \frac{a^2}{\omega_0}$$



White & Fornberg 1997

Large scale  
group velocity:

$$\boldsymbol{w} + \nabla_{\boldsymbol{k}} \omega_0$$

Case 0:

Stationary deterministic velocity

Random ray

$$\dot{\boldsymbol{X}}_R = \boldsymbol{u}_g$$

Stretching of phase

$$\dot{\boldsymbol{k}} = -\nabla \omega^T \boldsymbol{k}$$

Conservation of  
action

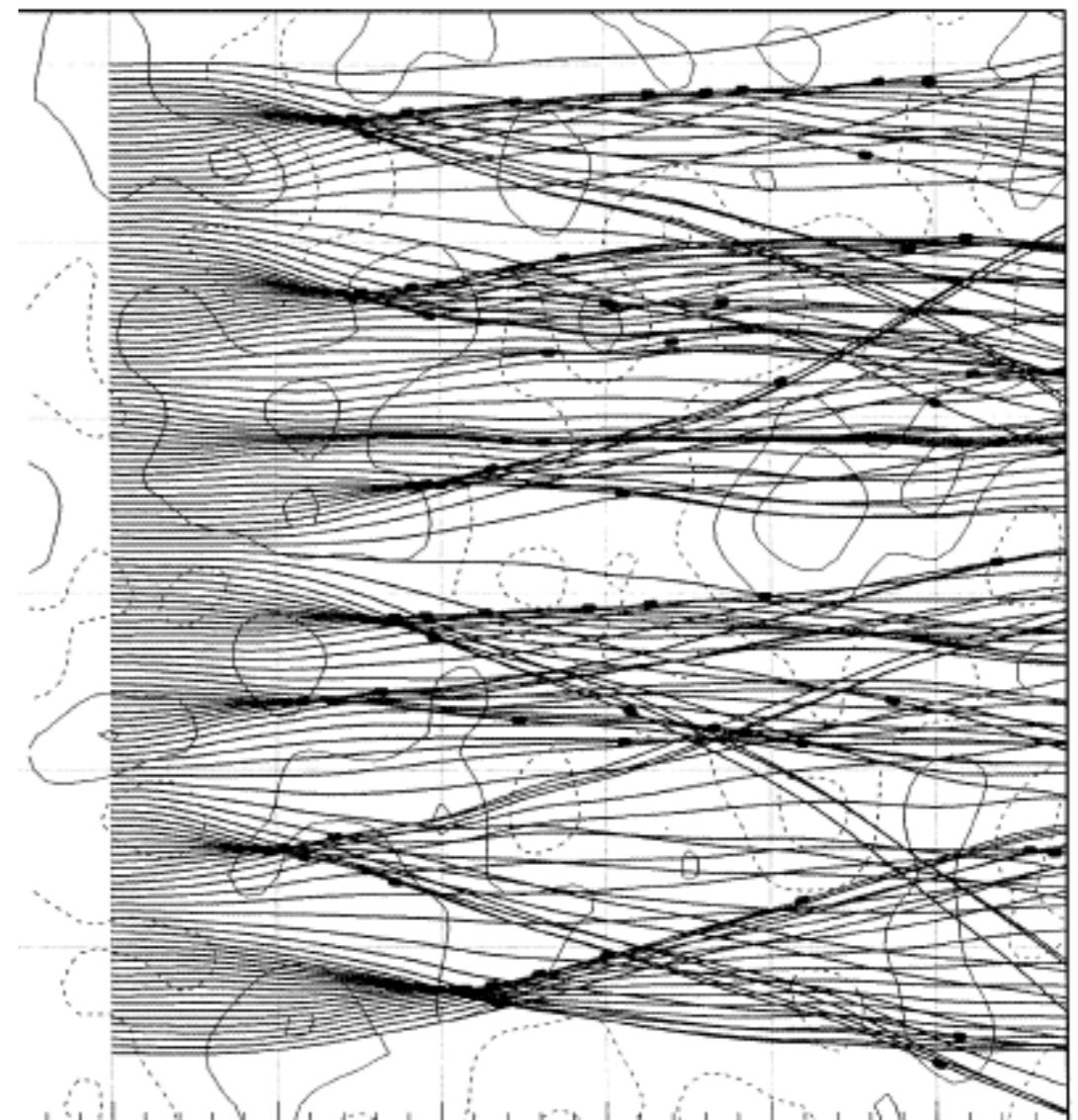
$$\text{cst.} = A = \frac{a^2}{\omega_0}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler  
frequency:

$$\omega_0 = \sqrt{g \|\boldsymbol{k}\|}$$



White & Fornberg 1997

Large scale group velocity:

$$\mathbf{w} + \nabla_{\mathbf{k}} \omega_0$$

Simple linear case 1:  
stationary deterministic,  
divergence-free and  
linear in x large-scale velocity

Stretching of phase

$$\dot{\mathbf{k}} = -\nabla \omega^T \mathbf{k}$$

Ratio  
effective rotation / strain rate

$$r = \frac{\omega}{\sigma_w}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

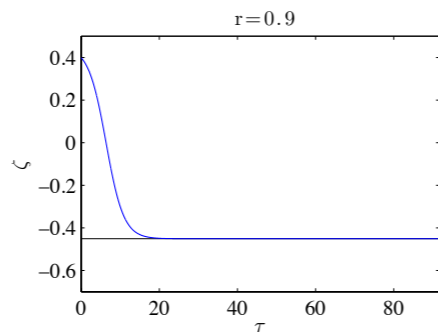
Doppler frequency:

$$\omega_0 =$$

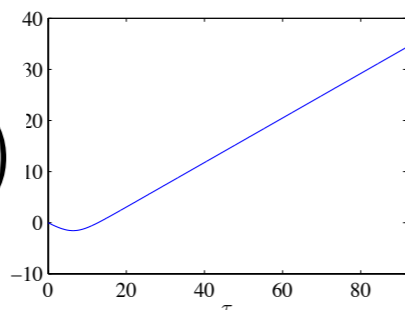
$$\sqrt{g \|\mathbf{k}\|}$$

$$|r| < 1$$

Hyperbolic

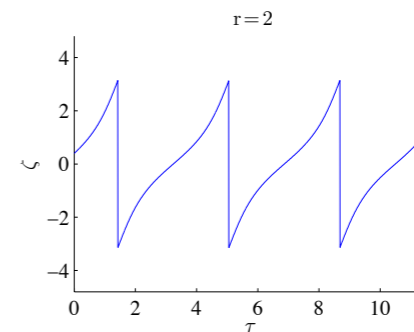


$\ln(\|\mathbf{k}\|)$

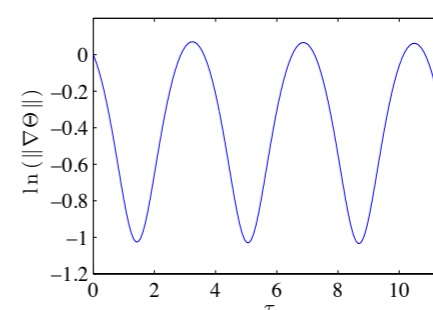


$$|r| > 1$$

Elliptic



$\ln(\|\mathbf{k}\|)$



$$\frac{D\zeta}{D\tau} = r - \cos(\zeta)$$

$$\frac{D}{D\tau} \ln(\|\mathbf{k}\|) = -\sin(\zeta)$$

Large scale group velocity:

$$\boldsymbol{w} + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\sigma d\boldsymbol{B} (\sigma d\boldsymbol{B})^T\}}{dt}$$

Wave:

$$\boldsymbol{a} e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|\boldsymbol{k}\|}$$

# Simple linear case 2:

no large-scale current  
+ homogeneous and divergence-free  
small-scale velocity

$$\beta(\boldsymbol{x}) e^{i(\omega_0 t + \boldsymbol{k} \cdot \sigma B_t)}$$

Wave with  
Brownian phase

Stochastic stretching of phase

$$\dot{\boldsymbol{k}} = -\nabla(\sigma \dot{B})^T \boldsymbol{k} \quad \text{Log-normal wave-number}$$

Conservation of action

$$\text{cst.} = A = \frac{a^2}{\omega_0} \quad \text{Log-normal amplitude}$$

Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

## Simple linear case 3:

divergence-free and linear-in-x large-scale current  
+ homogeneous and divergence-free  
small-scale velocity

Stochastic stretching  
of phase

$$\dot{k} = -\nabla (w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Large scale  
group velocity:

$$w + \nabla_k \omega_0$$

Small scale  
group velocity:

$$\sigma \dot{B}$$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$ae^{\frac{i}{\epsilon} \phi}$$

Doppler  
frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

## Simple linear case 3:

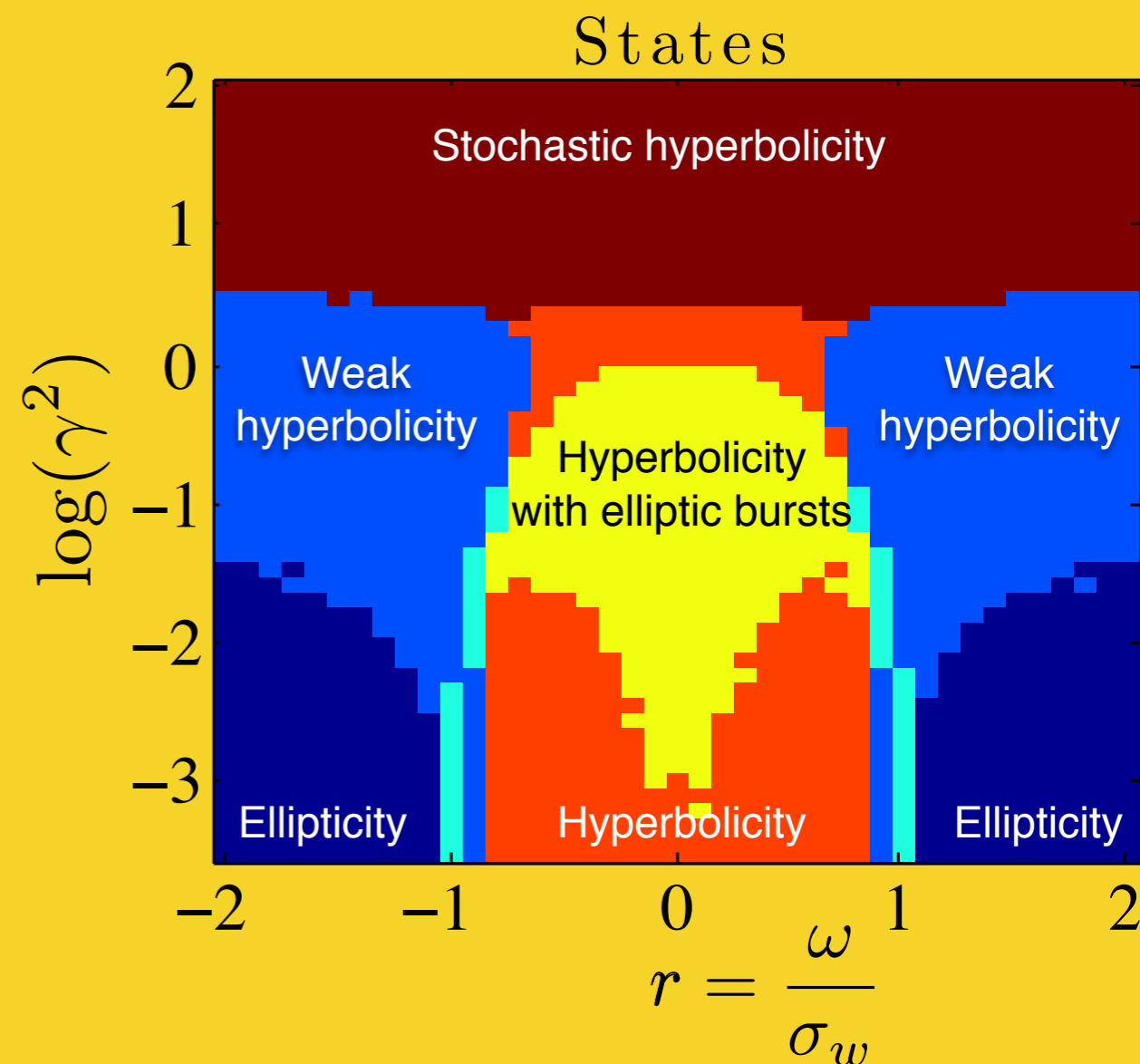
divergence-free and linear-in-x large-scale current  
+ homogeneous and divergence-free  
small-scale velocity

Stochastic stretching  
of phase

$$\dot{k} = -\nabla (w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$



Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

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divergence-free and linear-in-x large-scale current  
+ homogeneous and divergence-free  
small-scale velocity

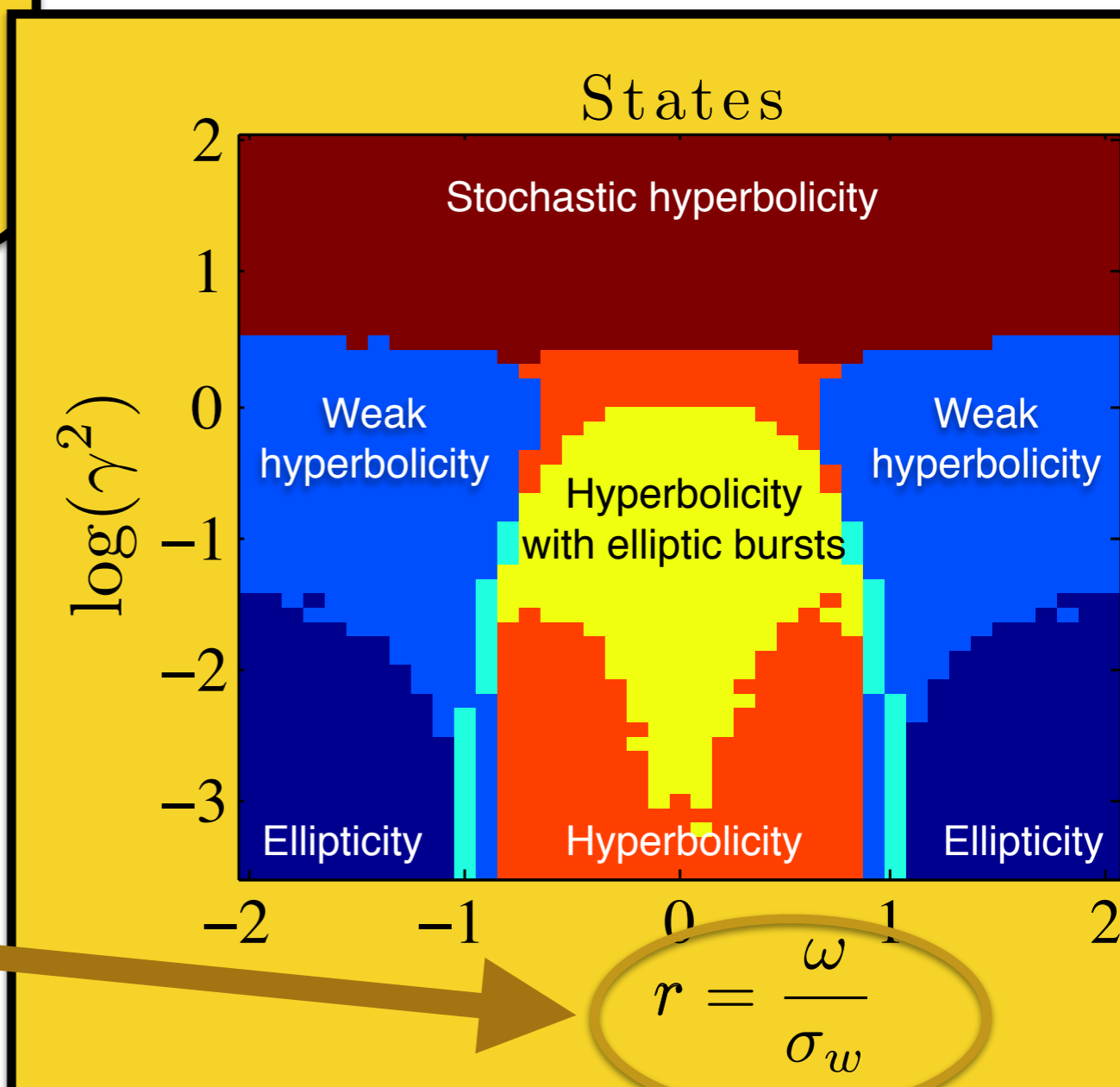
Stochastic stretching  
of phase

$$\dot{k} = -\nabla (w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Ratio  
effective rotation / strain rate



Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

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divergence-free and linear-in-x large-scale current  
+ homogeneous and divergence-free small-scale velocity

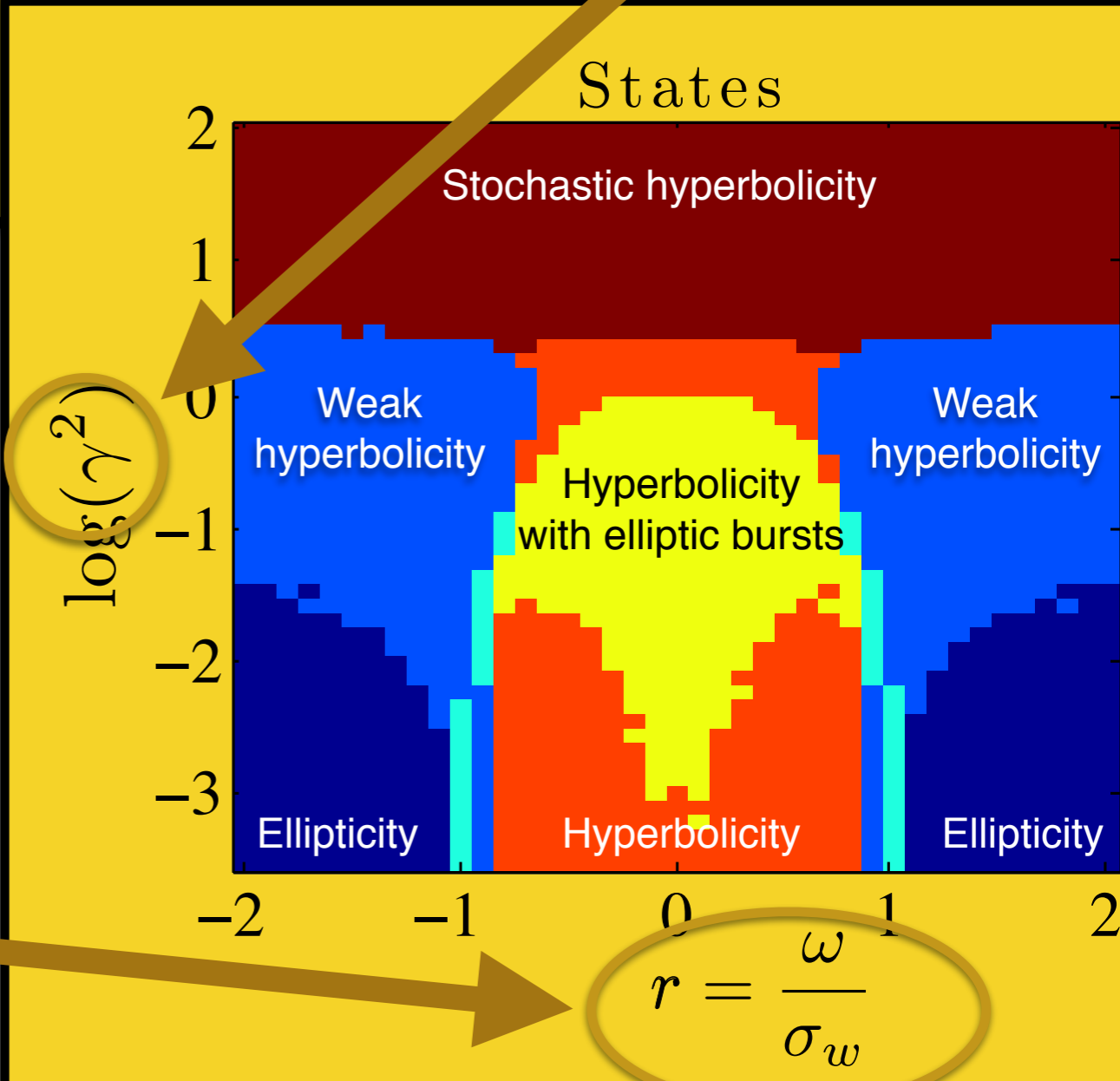
Stochastic stretching of phase

$$\dot{k} = -\nabla (w + \sigma \dot{B})^T k$$

Action

$$\text{cst.} = \frac{a^2}{\omega_0}$$

Depends on small scale's statistics





Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

# General case

$$\dot{X}_R = u_g \quad \text{Random ray}$$

Stochastic stretching of phase

$$\dot{k} = \left( -\nabla (w + \sigma \dot{B})^T + (\nabla \sigma^T)^2 \right) k \quad \left| \quad \text{Wave-vector} \right.$$

Conservation of action

$$\dot{A} = -\nabla \cdot u_g A \quad \left| \quad A = \frac{a^2}{\omega_0} \quad \text{Amplitude} \right.$$

Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

# General case

$$\dot{X}_R = \mathcal{U}_g \quad \text{Random ray}$$

Stochastic stretching of phase

$$\dot{k} = \left( -\nabla (w + \sigma \dot{B})^T + (\nabla \sigma^T)^2 \right) k \quad \text{Wave-vector}$$

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$$\dot{A} = -\nabla \cdot u_g A \quad \left| \quad A = \frac{a^2}{\omega_0} \quad \text{Amplitude}$$

Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

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$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$a e^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

# General case

$$\dot{X}_R = \mathcal{U}_g \quad \text{Random ray}$$

Stochastic stretching of phase

$$\dot{k} = \left( -\nabla (w + \sigma \dot{B})^T + (\nabla \sigma^T)^2 \right) k \quad \text{Wave-vector}$$

Function of  $x$

Conservation of action

$$\dot{A} = -\nabla \cdot u_g A \quad \left| \quad A = \frac{a^2}{\omega_0} \quad \text{Amplitude}$$

Large scale group velocity:

$$w + \nabla_k \omega_0$$

Small scale group velocity:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Wave:

$$ae^{\frac{i}{\epsilon} \phi}$$

Doppler frequency:

$$\omega_0 = \sqrt{g \|k\|}$$

# General case

Non-linear coupling

$$\dot{X}_R = \underbrace{u_g}_{\text{Function of } k} \quad \text{Random ray}$$

Stochastic stretching of phase

$$\dot{k} = \underbrace{\left( -\nabla (w + \sigma \dot{B})^T + (\nabla \sigma^T)^2 \right)}_{\text{Function of } x} k \quad \left| \quad \text{Wave-vector} \right.$$

Conservation of action

$$\dot{A} = -\nabla \cdot u_g A \quad \left| \quad A = \frac{a^2}{\omega_0} \quad \text{Amplitude} \right.$$

# Conclusion

# Conclusion and outlooks

- Method of characteristics : low computational costs  
We can simulate a lot of realisations, and initial conditions
- Understanding the dynamics with increasing complexity
- Tests against statistical physics results and satellite measurements.  
Test case : Wave spectra variations due to Agulhas courant
- Integration into the SeaMotion Radar
- Which are the swell / current effects on satellite measurements?
- Outlook : stochastic Wave Watch III