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# Multi-objective optimization of centrifugal pump impeller based on kriging model and multi-island genetic algorithm

Yandong Gu<sup>1\*</sup>, Ji Pei<sup>1</sup>, Shouqi Yuan<sup>1</sup>, Jinfeng Zhang<sup>1</sup>, Wenjie Wang<sup>1</sup>



## Abstract

Low-specific-speed pumps are extensively used in various fields, but the efficiency is relatively low. An optimization approach is proposed based on numerical simulation, design of experiment, approximation model and multi-island genetic algorithm. The low-specific-speed centrifugal pump IS50-32-160 is determined as the research subject, three parameters containing blade outlet width  $b_2$ , blade outlet angle  $\beta_2$ , and blade warp angle  $\varphi$  are selected as the design variables, and then 20 impellers are designed by optimal Latin hypercube sampling method. The commercial software ANSYS CFX 14.5 is applied to conduct the steady numerical simulation to obtain the efficiency and head, which are the optimization objectives. Approximation model is built between the objectives and the design variables based on the Kriging model. Finally, the best combination of impeller parameters is figured out by solving the approximation model with multi-island genetic algorithm. The results of original pump show that the performance curves obtained by experiments and simulations have a good agreement and the head deviation is 3.3% under the design condition. The optimization method improves the hydraulic efficiency by 3.2%. The internal flow in the optimized impeller are steadier and the pressure fluctuation intensity is decreased. This optimization method presented can provide references to the optimization of non-over-load design of low-specific-speed centrifugal pumps.

## Keywords

low-specific-speed centrifugal pump — multi-objective optimization — Kriging model — multi-island genetic algorithm — numerical simulation

<sup>1</sup> National Research Center of Pumps, Jiangsu University, Zhenjiang, China

\*Corresponding author: laogubulao@163.com

## Introduction

The low-specific-speed centrifugal pump, whose specific speed is between 30 and 80 ( $n_s=3.65nQ^{0.5}/H^{0.75}$ ), is widely used in the agricultural irrigations, power plants and chemical productions etc. According to the traditional design method, the impeller passage is very narrow and long in order to meet the requirements of low flow rate and high head. However, the low efficiency of low-specific-speed centrifugal pumps is urgent to increase.

The present optimization on the efficiency and head for low-specific-speed centrifugal pump mainly focuses on the empirical formulas and experiments, but it is difficult for the designers to select the coefficients of empirical formulas and time-consuming to conduct experiments<sup>[1-3]</sup>. The optimization techniques on the improvements of performance of pump are well-developed. Kim et al applied factorial design to optimize the impeller of pump and analyzed the effects of impeller's parameters on efficiency with response surface method<sup>[4,5]</sup>. A multi-objective optimization of required net positive suction head ( $NPSH_r$ ) and the efficiency of pump under design flow rate was presented by using metamodels and optimization algorithm<sup>[6-8]</sup>. Kim et al optimized the efficiency and cavitation of a mixed-flow pump by combining design of experiment with regression analysis<sup>[9]</sup>. Wang et al. coupled the multi-objective genetic algorithm with back propagation neural network to optimize a transonic axial compressor<sup>[10]</sup>. Cravero et al applied an optimization

design approach, which combines the design of experiment, response surface method and artificial neural networks, to a four-stage axial flow turbine<sup>[11]</sup>. Wang et al. built expressions of efficiencies and head based on the hydraulic loss and optimized the expression with adaptive simulated annealing algorithm<sup>[12]</sup>.

In this study, a multi-objective optimization procedure of low-specific-speed centrifugal pump is proposed to improve the efficiency under design flow rate. The optimization combines Optimal Latin Hypercube Sampling (OLHS), Kriging model, multi-island genetic algorithm with numerical simulation. Finally, the inner flow is deeply analyzed to illustrate the improvement of performance of low-specific-speed centrifugal pump.

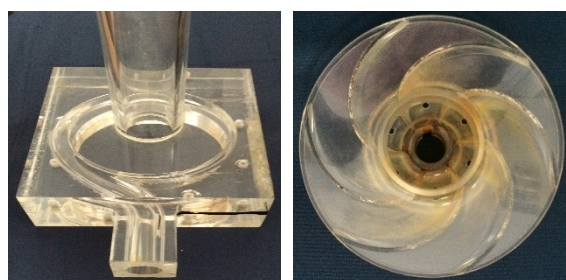
## 1. Model pump and numerical simulation

### 1.1 model pump

The low-specific-speed centrifugal pump IS50-32-160 is determined as the research subject. The pump consists of the impeller equipped with six straight blades and a spiral volute with rectangle section, all of them are made of PMMA for PIV measurements to shoot the inner unsteady flow, as shown in Fig.1. The design flowrate  $Q$  is  $6.3 \text{ m}^3/\text{h}$ , and the head is 8m. The rotating speed is 1450r/min and the specific speed  $n_s$  is 46.5. The main geometrical parameters of the model pump are showed in table 1.

The computational domains of the low-specific-speed centrifugal pump are showed in Fig.2. The software Cfturbo is used to build the impeller domain. In order to avoid the backflow

occurring and extending to the whole inlet pipe and the recirculation at the outlet pipe, the length of the suction is extended to five times as long as the inlet diameter of impeller and the pipe whose length is five times the outlet diameter of volute is added to the volute.



(a) Impeller (b) Volute

Fig 1. Model pump

Table 1. Main geometrical parameters of pump

	Geometrical parameters	Symbols	value
impeller	Inlet diameter	$D_1$	50mm
	Outlet diameter	$D_2$	160mm
	Blade outlet width	$b_2$	6mm
	Blade inlet angel	$\beta_1$	24°
	Blade outlet angel	$\beta_2$	30°
	Blade wrap angle	$\varphi$	150°
	cutwater diameter	$D_3$	170mm
volute	Inlet width	$b_3$	18mm
	Cutwater angle	$\alpha_3$	27°
	Outlet diameter	$D_4$	32mm

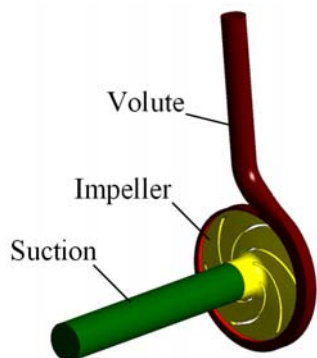


Fig 2. Computational domain of centrifugal pump

## 1.2 Mesh generation

Comparing with the unstructured mesh, structured mesh can accelerate the speed of convergence. The quality and the number of the structured mesh are very significant before conducting the numerical simulation, especially for the optimization procedure of the pump.

The structured grids of computational domains are generated by software ICEM CFD, as showed in Fig.3. The grids are refined in the near-wall flow regions and the mesh independence check was performed with several numbers of grids<sup>[13]</sup>. The number of total grids is approximately 3.1 million.

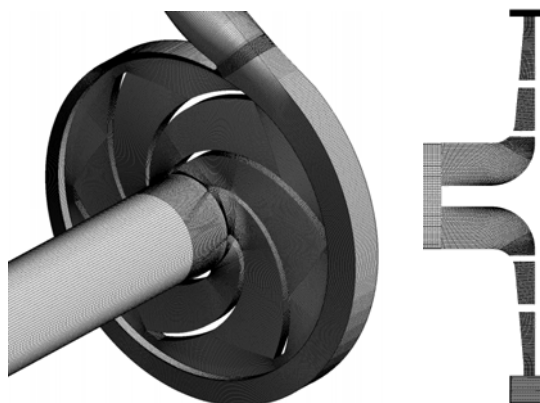


Fig 3. Structured mesh of the centrifugal pump

## 1.3 Numerical simulation

Taking a three-dimensional and viscous flow in the centrifugal pump into consideration, 3D Reynolds-averaged Navier-Stokes equations are solved by commercial code ANSYS CFX 14.5.

In the steady simulation, the shear stress transport (SST)  $k-\omega$  turbulence model is chosen and the multireference frame (MRF) technique is applied. The inlet and outlet boundary conditions are respectively set as total pressure and flow rate. The interface between the rotor and stator is defined as the Frozen rotor. The physical time scale is set to  $0.1\omega$ , where  $\omega$  is the angular velocity of the impeller. Root mean square (RMS) residual values of the momentum and mass are set to be less than  $10^{-5}$ . The performance results are obtained after about 600 iterations.

In the transient simulation, the boundary conditions is the same with that of the steady simulation. The interface between the rotor and stator is changed to "transient rotor stator". The accuracy of discretization in space is set as second order accuracy and second-order backward Euler scheme is used for the time discretization. The total calculating time for the pump is 12 revolutions and the every step time is  $1.149 \times 10^{-4}$ s, corresponding to the rotating angle of  $1^\circ$ . Therefore, the results of final 3 revolutions are used to analyze the pressure fluctuation distributions in the pump. The steady simulation is used to calculate the performance of pump and the transient simulation is conducted to compare with the inner flow characteristics of the original pump and optimized one. The numerical simulations were carried out on a Dell Workstation with an Intel Xeon CPU E5-1650 (12 CPUs) and a clock speed of 3.2 GHz.

## 1.4 Experimental validation

The performance predicted by numerical simulation is necessarily validated by the experiments before conducting the optimization process. The performance of the low specific speed pump was test in

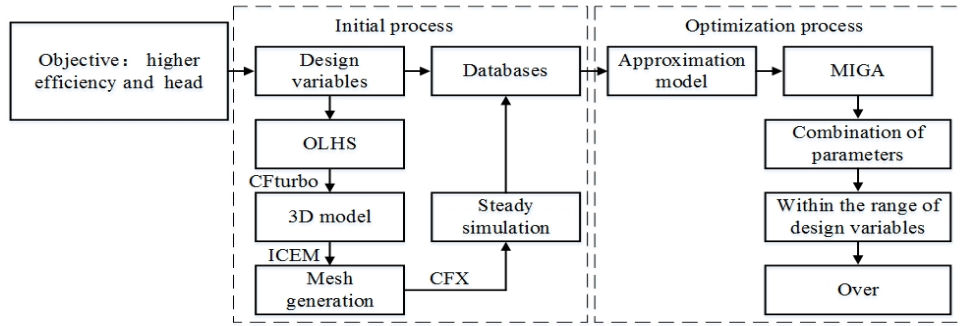
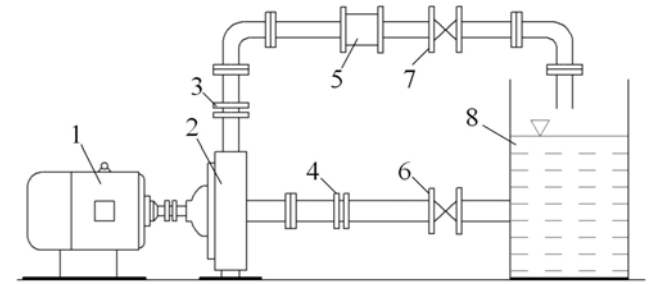


Fig.6 Flow chart of the optimization procedure

the open test rig, as Fig 4. showed. The pressure in the inlet and outlet is measured by the WT200 pressure transducers, whose precisions are 0.1%. The flow rates are measured by the IFC 300 electromagnetic flowmeter, whose precision is 0.3%. The comparison between the simulated head and experimental one is showed in Fig.5. The trend of the head predicted is similar to the experimental performance. The prediction deviations of head is 3.3% under the design condition, and that under  $0.7Q_d$  is 3.9%. Therefore, numerical simulation used in the optimization process is reliable.



1.Motor 2.Pump 3.Pressure sensor 4.Pressure sensor 5.Electromagnetic flowmeter 6.Valve 7.Valve 8.Tank

Fig 4. Diagram of the open test rig

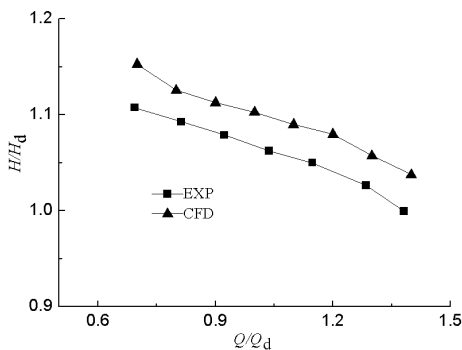


Fig 5. Validation of head coefficient

## 2. Optimization process

The optimization procedure is illustrated in Fig.6. At first, the design variables that have great influence on the pump's performance are determined. Secondly, Optimal Latin Hypercube Sampling (OLHS) is used to generate different schemes of impeller. The impellers are designed within the design space, and then the mesh generation and steady simulation are executed.

The databases are formed by the design variables and the objectives composed of the hydraulic efficiency and head. The approximation model is constructed and the best combination of parameters is obtained based on multi-island genetic algorithm through solving the approximation model. The advantage of the optimization technique is that it can reduce the time of finding the best combination precisely and reduce the computational resource.

### 2.1 Two objectives

The optimization objectives are not only to improve the hydraulic efficiency under design condition, but also to ensure the head to meet the requirements.

$$\eta_{1.0Q} = \max(f(b_2, \varphi, \beta_2)) \quad (1)$$

### 2.2 Design variables

According to the design experience and parameters of impeller that have great influence on the efficiency and head<sup>[14]</sup>, the design parameters that blade outlet width  $b_2$ 、 blade outlet angle  $\beta_2$  and wrap angle  $\varphi$  are selected as the input variables, as showed in table 2.

Table 2 Ranges of design parameters

Design variables	Lower	Upper	origin
$b_2/\text{mm}$	5	8	6
$\beta_2/^\circ$	27	33	30
$\varphi/^\circ$	135	165	150

### 2.3 Optimal Latin Hypercube Sampling method

It is important to make sure that the database is enough to construct the approximation model, and the design space formed by design variables is feasible. Optimal Latin hypercube sampling method, which is improved from the Latin hypercube sampling, is one of the most effective way to build the design space. And the advantage of the sampling method is that it improves the uniformity of the Latin hypercube sampling and the great ability of space filling can make better fitting functions between the objectives and design variables. In the optimization process, 20 design points was generated by optimal Latin hypercube sampling method and the corresponding numerical results are shown in table 3. It can be found that the efficiency of the ninth scheme is highest, whose head

also meet the requirements.

**Table 3** Designed schemes and objective values

Design schemes	$b_2$ /mm	$\beta_2/^\circ$	$\varphi/^\circ$	$\eta_n/\%$	H/m
1	7.84	28.26	146.05	71.40	9.20
2	6.89	33.00	139.74	70.78	9.02
3	6.42	29.84	157.11	75.27	8.82
4	6.11	32.37	150.79	75.30	8.71
5	7.37	32.68	152.37	71.93	9.30
6	5.16	28.89	160.26	79.57	8.59
7	5.47	29.53	147.63	78.20	8.71
8	7.21	31.11	165.00	73.12	9.21
9	5.00	31.42	155.53	79.69	8.56
10	5.79	29.21	135.00	76.95	8.77
11	7.68	30.16	153.95	71.81	9.23
12	7.05	28.58	136.58	73.47	9.10
13	6.58	27.95	149.21	74.72	8.97
14	5.32	27.00	142.89	78.93	8.63
15	8.00	30.79	141.32	70.68	9.36
16	7.53	27.63	158.68	72.82	9.22
17	5.63	31.74	138.16	77.26	8.77
18	5.95	32.05	163.42	75.96	8.80
19	6.26	27.32	161.84	76.40	8.85
20	6.74	30.47	144.47	72.64	8.89

## 2.4 Kriging model and Multi-island genetic algorithm

In this paper, the approximation model, called Kriging model, is used to obtain the best combination of design variables of impeller. The function of the Kriging model is to build the approximate mathematic expressions between the objectives and design variables. By solving the mathematic expressions with optimization algorithm, the best combination of parameters can be obtained within the design space quickly.

Kriging model consists of regression function and relative function. The regression function can be polynomial fitting of zero order, first order and second order. The relative function is Gaussian function, linear function, exponent function and so on. Supposed that design variables  $x$  of  $n$  dimensions is  $[x_1, x_2 \dots x_n]^T$ , the response values  $y(x)$  is  $[y_1, y_2 \dots y_n]^T$ , so the  $y(x)$  can be written as followed:

$$y(x)=f(x)+\varepsilon \quad (2)$$

$$f(x)=ag(x)+h(x) \quad (3)$$

where  $f(x)$  is Kriging model.  $\varepsilon$  is random error between the value of Kriging model and response and it

follows normal distribution.  $g(x)$  is the regression function of Kriging model,  $\alpha$  is the coefficient of regression function,  $h(x)$  is correlation function and Gaussian function is selected.

Each design point in the Multi-Island Genetic Algorithm (MIGA), like other genetic algorithms, is perceived as an individual with a certain value of fitness based on the value of the objective function and constraint penalty. The main feature of Multi-Island Genetic Algorithm is that each population of individuals is divided into several sub-populations called "islands". Some individuals are then selected from each island and migrated to different islands periodically. This operation is called "migration". The advantage of the multi-island genetic algorithm is to avoid the phenomenon of "premature", which exists in the general genetic algorithms. The main parameters of multi-island genetic algorithm is showed in table 4.

**Table 4** Detailed parameters of multi-island GA

Parameters of setup	values
Interval of Migration	5
Number of Generations	50
Number of Islands	10
Rate of Crossover	0.9
Rate of Migration	0.01
Rate of Mutation	0.01
Sub-Population Size	10

## 3. Results and discussions

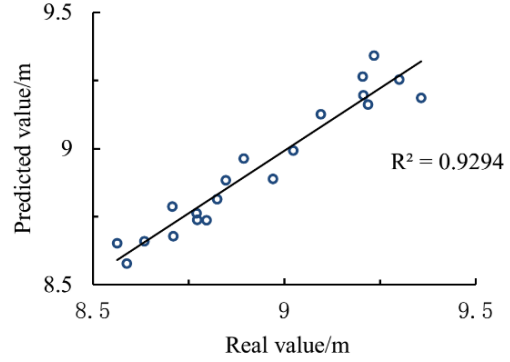
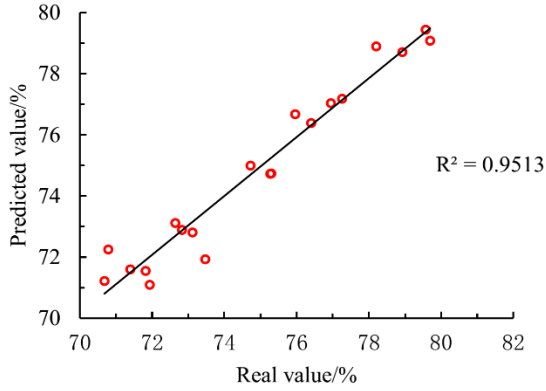
### 3.1 Accuracy of approximation model

It is very necessary to assess the accuracy of Kriging model before the optimization, thus the R-square error is used to analyze the difference between the efficiency and head calculated by numerical simulation and those predicted by the Kriging model. The value of R-square is over than 0.9, as Fig 7. showed, it means that the Kriging model can have a satisfactory accuracy of prediction. Therefore, the Kriging model can be applied obtain fine mathematic relationship between the objectives and design parameters combined with genetic algorithm.

### 3.2 Comparison of optimal parameters

The best combination of impeller parameters is figured out by solving the approximation model with multi-island genetic algorithm. The optimized design parameters for impeller of the centrifugal pump are predicted by the Kriging model showed in table 5. It can be seen that the blade outlet width is optimized to smaller, the wrap angle is a little lager, whereas the blade outlet angle is almost the same with the original one. The hydraulic efficiency of the optimized impeller increases by 3.2% under the design condition, whose head also meet the requirements. Besides, the efficiency and head obtained predicted by Kriging model have a good agreement with those calculated by CFD, with a relative deviation of 0.06% and 3.4%, respectively.





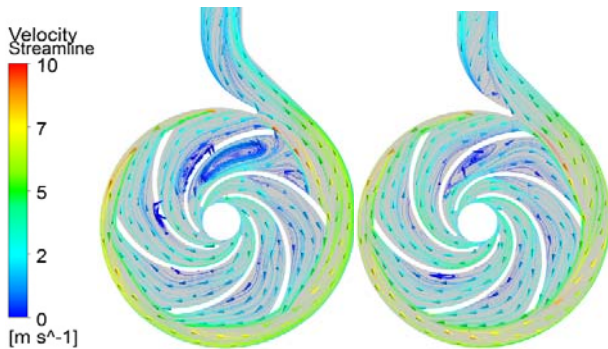
(a) Error analysis of approximation model of efficiency (b) Error analysis of approximation model of head  
**Fig 7.** Error analysis of R square

**Table 5** Comparisons of performance between the origin and optimization

	$b_2/\text{mm}$	$\beta_2/^\circ$	$\varphi/^\circ$	$\eta/\%$ (Predicted)	$H/\text{m}$ (Predicted)	$\eta/\%$ (CFD)	$H/\text{m}$ (CFD)
Original design	6	30	150	—	—	76.642	8.823
Optimal design	5	29.64	156.1	79.89	8.54	79.845	8.838

### 3.3 Comparison of steady simulations

To find reasonable factors for the improvement of performance in design flow rate, the inner flow characteristics of the original pump and optimized one have been compared, as showed in Fig.8. The flow separation phenomena occur on the leading surface of the blades in the original pump, but those in the optimized one is much better, because the original one based on increasing-capacity design method operates under the partload condition. The vortex region in optimized impeller is smaller than that in original one and the flow is improved, which mainly results in the improvement of the efficiency of the optimized one. But some impact loss arise in the volute of the optimized pump, thus the matching between the impeller and volute still need to improve.



(a) Original scheme (b) Optimized scheme  
**Fig 8.** Comparison of velocity distribution in the impeller under design condition

### 3.4 Comparison of unsteady simulations

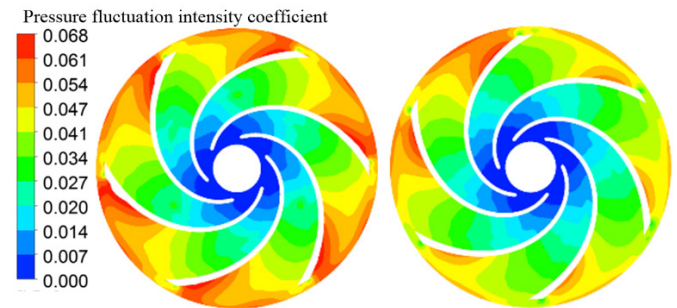
In order to assess the unsteady pressure fluctuation distribution intensity directly and

comprehensively, a statistical method is applied to analyze the pressure fluctuation intensity on each grid node in the fluid domains. Pressure standard deviation is calculated to indicate the pressure fluctuation intensity<sup>[11]</sup>. A lower pressure standard deviation value indicates smaller pressure fluctuation intensity. The pressure on each grid node can be given as  $p_i=p(x, y, z, t)$ , the pressure fluctuation intensity  $p_{sdv}$  is defined as follows:

$$\bar{p} = \frac{1}{N} \sum_{n=1}^N p_i(x, y, z, \frac{n}{360}T) \quad (4)$$

$$p_{sdv} = \sqrt{\frac{1}{N-1} \sum_{n=1}^N \left( p(x, y, z, \frac{n}{360}T) - \bar{p} \right)^2} \quad (5)$$

where  $T$  is the period of one impeller revolution,  $N = 1080$ .



(a) Original impeller (b) Optimized impeller  
**Fig 9.** Pressure fluctuation intensity distribution in the impeller

The comparisons between the pressure fluctuation intensity distribution in the original impeller and that in optimized one are showed in Fig.9. The pressure fluctuation intensity in these two schemes increases

along the flow passage. The minimum pressure fluctuation intensity occurs in the inlet and the maximum exists near to the trailing edge at the pressure side, and that on the pressure side is larger than that on the suction side. It is clear that the area of large pressure fluctuation intensity decreases in the optimized impeller under design condition.

The pressure fluctuation intensity distribution in the volutes are showed in Fig.10. The pressure fluctuation intensity is relatively large from the tongue to cross section II and becomes weaker from cross section VII to VIII, and the largest one is around the tongue because of the interactions of the impeller and volute. Comparing the original scheme, the pressure fluctuation in the optimized one is obviously weakened, which is beneficial to improve the operating reliability.

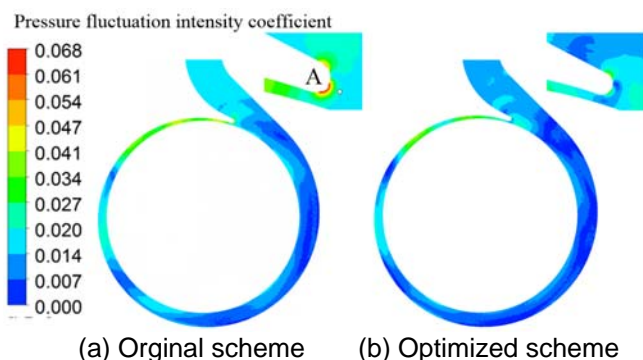
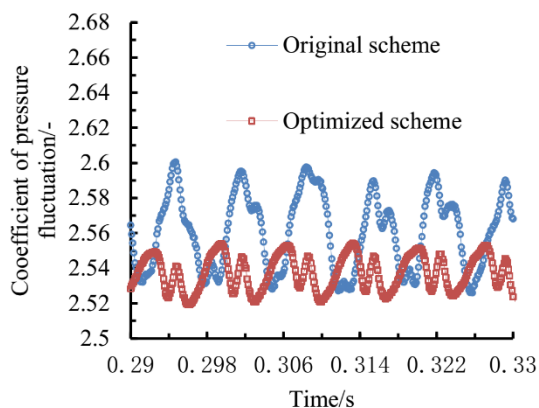
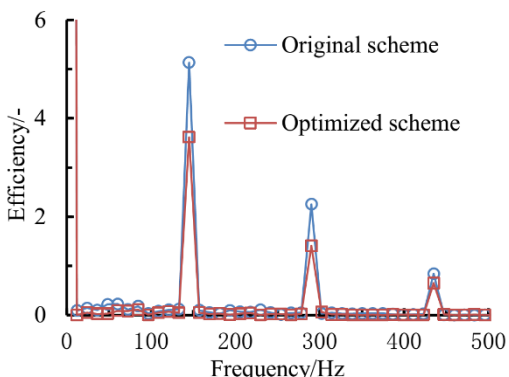


Fig 10. pressure fluctuation intensity distribution in the volute



(a) Time domain of pressure fluctuation



(b) Frequency domain of pressure fluctuation  
Fig 11. Characteristics of pressure fluctuation of the monitor in the volute tongue

The monitor A near the tongue is set to record the pressure fluctuation of the last revolution showed in Fig.10. The Fig.11 (a) shows that the pressure fluctuation is regular and composed of 6 peaks and 6 troughs in a rotating cycle. The amplitude of pressure fluctuation coefficient in the original scheme is larger than that in the optimized one. The frequency of pressure fluctuation of these two schemes are determined by the blade passing frequency, and the amplitude of the dominant frequency in the original scheme is larger than that in the optimized one, but the amplitude of the second frequency in the original scheme is smaller.

#### 4 Conclusions

An optimization approach based on numerical simulation, design of experiment, approximation model and multi-island genetic algorithm is proposed to improve the performance of the low-specific-speed centrifugal pump and the inner flow characteristics are analyzed to illustrate the improvement of performance.

(1) Kriging model is effective to obtain the fine relationship between the objectives and design parameters and predict the efficiency and head with a high accuracy.

(2) The hydraulic efficiency of optimized impeller increases by 3.2% comparing to the original one under design flow rate, and the head also meets the requirements.

(3) The velocity flow fields through this optimization are improved. The unsteady pressure fluctuation intensity in the optimized scheme decreases under design flow rate.

#### ACKNOWLEDGMENTS

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