

Supplementary Material C

Evaluation of the algorithm sensitivities to the bone placements in the CT Coordinate System.

To evaluate the sensibility of our algorithm to the initial position and orientation relative to the CT Scan coordinate frame, each bone model was moved in 3D space by applying a randomly generated position matrix (roto-translation matrix) to the model triangulated surface (Information on the distribution of basis vectors can be seen in Figure 1). Subsequently, the ACS constructions were performed, and the obtained ACSs were transformed back to original coordinate system for comparison with the original ACS. This procedure was repeated 5 times to get a large sample of bone model placements.

Random translation vectors were generated as follow:

$$\vec{T} = 500. \begin{pmatrix} randn \\ randn \\ randn \end{pmatrix}$$

with *randn* the MATLAB function giving a normally distributed random number from $\mathcal{N}(0, 1)$.

Random rotation matrices were derived from unit quaternions $\mathbf{q}_{unit}^{rotation}$, generated as follow:

$$\mathbf{q}^{random} = \begin{pmatrix} randn \\ randn \\ randn \\ randn \end{pmatrix} \quad \text{then} \quad \mathbf{q}_{unit}^{rotation} = \frac{\mathbf{q}^{random}}{\|\mathbf{q}^{random}\|}$$

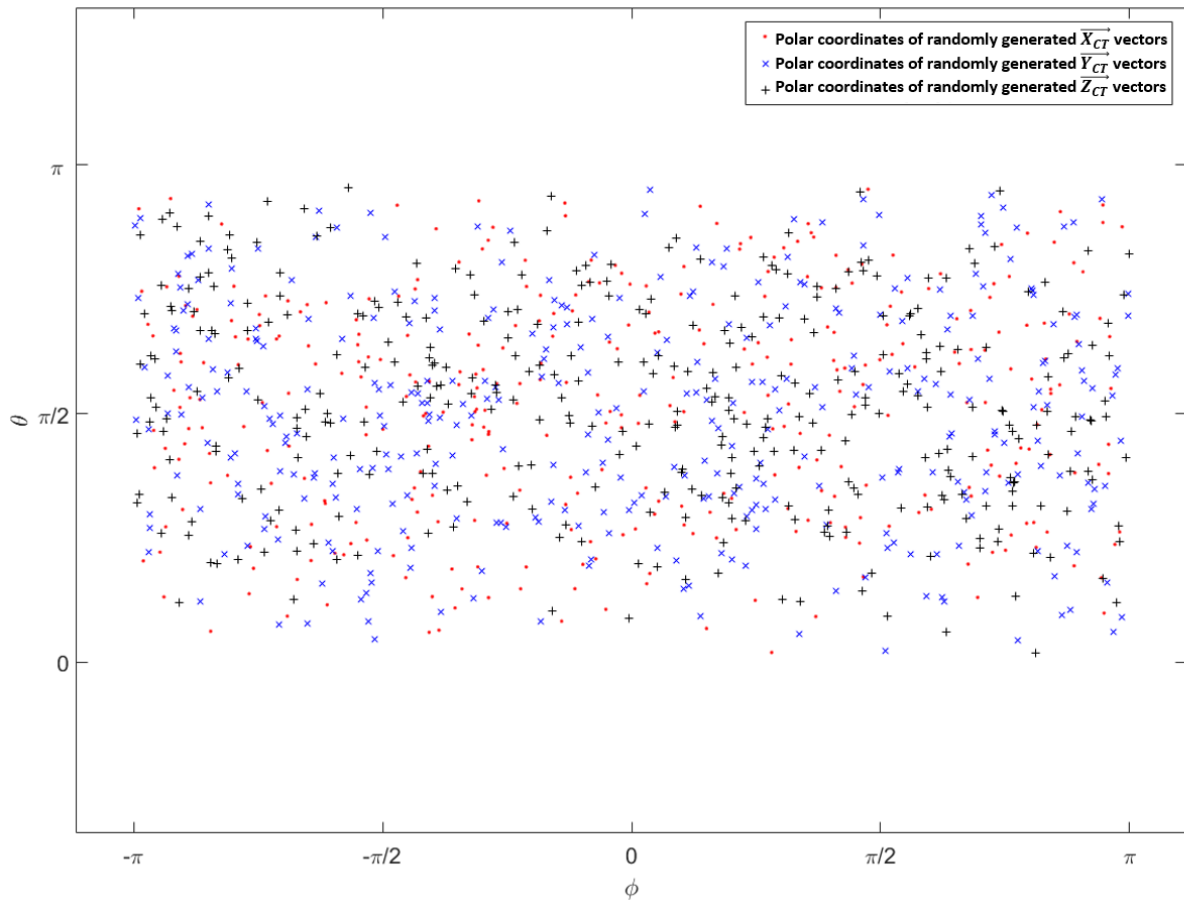


Figure 1: Representation of the basis vectors of all the generated random position matrices. As basis vectors are unit vectors they are represented by their spherical coordinates for clarity. We can see the wide range of the 360 generated random position matrices.

The results show that for the methods displaying the least variability (PCC for the femur, PIAAS for the patella and PIAASL for the tibia) the influence of the initial position of the bones relative to the CT Scan coordinate frame is virtually null, axis displaying lower than 10^{-9} degrees of deviations and the distance between centers being inferior to 10^{-9} mm (cf: Table 1). Influence of initial placement for other methods is also low, with axis deviations under the millidegree and position differences inferior to the μm . The differences can originate from the optimization tolerance during the surface fitting or the accumulation of round off errors that generate small variations in the selected AS sets.

Methods		Maximal Between Axis Angle [°]			Between Center distances [mm]		
		Operator 1	Operator 2	Operator 3	Operator 1	Operator 2	Operator 3
		Mean	Mean	Mean	Mean	Mean	Mean
		Max	Max	Max	Max	Max	Max
FEMUR	PCC	5,0E-12	4,2E-12	4,5E-12	5,1E-11	4,6E-11	4,7E-11
		3,0E-11	2,0E-11	1,4E-11	1,2E-10	1,3E-10	1,5E-10
	PCS	1,6E-11	2,2E-11	1,9E-11	4,7E-11	4,7E-11	4,5E-11
		1,0E-10	1,2E-10	9,5E-11	9,3E-11	1,2E-10	1,0E-10
	CE	4,7E-07	7,2E-07	5,4E-07	4,2E-07	8,0E-06	4,4E-07
		6,0E-06	2,0E-05	7,0E-06	3,0E-06	9,0E-04	4,9E-06
PATELLA	PIAAS	4,8E-12	5,4E-12	5,2E-12	4,4E-11	4,3E-11	4,2E-11
		1,2E-11	1,2E-11	1,1E-11	9,2E-11	7,9E-11	8,7E-11
	RL	1,9E-08	4,6E-12	6,5E-10	1,3E-08	4,3E-11	4,0E-10
		2,2E-06	3,0E-11	2,6E-08	1,6E-06	8,3E-11	1,4E-08
	VR	1,9E-08	4,7E-12	6,5E-10	4,5E-11	4,3E-11	4,2E-11
		2,2E-06	1,5E-11	2,6E-08	9,2E-11	8,3E-11	9,4E-11
TIBIA	PIAASL	4,6E-12	4,8E-12	4,5E-12	4,5E-11	4,5E-11	4,5E-11
		9,6E-12	1,1E-11	1,1E-11	8,5E-11	8,6E-11	9,4E-11
	ECASE	3,9E-12	3,9E-12	3,7E-12	4,5E-11	4,5E-11	4,4E-11
		8,0E-12	9,1E-12	8,4E-12	8,3E-11	8,7E-11	9,3E-11
	CASB	4,3E-12	4,6E-12	4,3E-12	4,5E-11	4,5E-11	4,5E-11
		1,1E-11	1,1E-11	1,0E-11	8,3E-11	8,7E-11	9,3E-11

Table 1: Maximal differences between original ACSs and randomly moved ones, in terms of orientations (between axis angle) and origins (between center distance). For each method the maximal deviation is highlighted by a red box.