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K -tier Heterogeneous Cellular Networks with Wireless Power Transfer

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Abstract—In this paper, we model and analyze the downlink (DL) wireless power transfer and uplink (UL) information transmission of K -tier heterogeneous cellular networks (HCNs). Due to the densely located BSs and universal frequency reuse between all tiers in HCNs, the typical mobile terminal (MT) is allowed to harvest energy from the serving BS by direct beamforming, as well as from the other interfering BSs. Equipped with large storage battery, the typical MT utilizes the harvested energy to provide constant transmit power for the UL information transmission. Stochastic geometry is used to model and evaluate the intrinsic relationship between the energy harvested from the BSs in the DL and the information transmission performance in the UL. To well evaluate the system performance, we derive exact expressions for the maximum transmit power at MT and the UL average ergodic rate. Our results show that the UL average ergodic rate per random MT is not significantly improved by increasing the energy conversion efficiency.

I. INTRODUCTION

The high depletion of energy due to the upsurge growth of smart phones, netbooks, tablets, and machine-to-machine (M2M) communication devices, evokes the rise of various energy harvesting technologies. Radio frequency (RF) wireless power transfer is one of those cost-effective energy harvesting technologies, which can prolong the lifetime of mobile devices and provide continuous and stable energy for wireless energy constrained networks [1]. Meanwhile, traditional cellular systems are moving towards heterogeneous cellular networks (HCNs) for higher network capacity and increased spatial spectrum efficiency [2].

The advent of HCNs marks a new era of viability for RF wireless power transfer in multi-tier cellular networks. In practice, wireless power transfer functions well in short ranges. For this reason HCNs are a good candidate for wireless power transfer where the densely deployed BSs in HCNs can fulfill the short range power transfer requirement and provide energy for nearby mobile terminals (MTs). In downlink (DL) HCNs, each MT is associated with the BS which provides the maximum received power. As such, the serving BS acts as a dedicated RF energy source, similar to “power beacon” in [3]. Meanwhile, due to the universal frequency reuse, the typical MT also endures high levels of interference from the nearby interfering BSs. These densely deployed interfering BSs are typically located close to the typical MT, that act as another energy source for the MT [4]. Inspired by the aforementioned potential benefits brought by RF wireless power transfer, we propose a novel multi-cell wireless powered HCNs in

which all the multi-antenna BSs provide RF energy to the single antenna MTs in the DL. The MTs will then utilize the harvested energy to deliver information to the BSs in the uplink (UL).

RF wireless power transfer was proposed for cellular networks in [3, 5]. In [3], the energy sources, namely power beacons, were implemented in the existing cellular network to power the UL information transmission of mobile devices. In [5], a cognitive underlay device-to-device (D2D) communication in multi-channel cellular networks was proposed, where the D2D transmitters harvest RF energy from the DL and the UL transmissions from both the macrocell BSs and the cellular users. Unfortunately, the impact of RF wireless power transfer on HCNs is less well understood.

In this work, we formulate a tractable model for the DL wireless power transfer and the UL information transmission in HCNs with biased-received-power cell association. In this model, the BS providing the maximum biased received power for the typical MT in both DL and UL, is selected as the serving BS. Stochastic geometry has been widely applied in modeling sensor networks [6], cognitive radio networks [7], cellular networks [8] and HCNs [9]. This powerful tool is used to evaluate the intrinsic relationship between the energy harvested from the BSs in the DL and the information transmission performance in the UL in our model. Assuming large storage battery in each MT, we derive an exact expression for the DL maximum transmit power of a typical MT associated with the k th tier. Based on the maximum transmit power, we derive exact expressions for the UL average ergodic rate to characterize the link reliability and spectral efficiency. Our results show that the UL performance is not significantly improved by increasing the energy conversion efficiency.

II. SYSTEM DESCRIPTION

We consider a K -tier HCN in which the BSs of each tier are spatially distributed in \mathbb{R}^2 according to an independent and homogenous Poisson point process (PPP), Φ_k , with density λ_k . Each tier is assumed to have a different transmit power, $P_{t,k}$, a different path-loss exponent, η_k , and a different bias factor, B_k , for $k \in \{1, \dots, K\}$, where $B_k \geq 1$ [10, 11]. In addition, the MTs are assumed to be spatially distributed in \mathbb{R}^2 according to a homogenous PPP, Φ_U with density λ_u . The density of MTs is assumed to be high enough, and each MT has data ready for transmission, such that saturated traffic conditions hold. It

is further assumed that each BS is active and serves one MT per channel. The MTs in each cell of each tier are assumed to transmit in orthogonal channels [12]. Full-frequency reuse is also assumed to improve the spectrum efficiency [11].

The MTs and the BSs in the k th tier are assumed to be equipped with 1 and N_k antennas, respectively. Each MT has a rechargeable battery with large storage and is capable of converting the RF received power into a direct current (DC) signal to recharge the available battery. A harvest-then-transmit communication protocol for wireless power transfer is considered, where the MTs harvest energy from the BS in the DL, and then utilize the stored energy to power the operating circuits and transmit information to the BS [13]. More specifically, let T denote the duration of a communication block, which consists of DL and UL transmissions. The first and second sub-blocks of duration αT and $(1 - \alpha)T$ are allocated to DL and UL transmissions, respectively, where α ($0 \leq \alpha \leq 1$) is the time allocation factor. In the DL, the typical MT harvests energy from all the active BSs of the K -tier HCN. In the UL, the typical MT transmits information to its serving BS. The fading channels in DL and UL transmissions are assumed to be quasi-static, i.e., the fading channels are fixed within each sub-block and independently change from one block to another.

A. Cell Association

Without loss of generality, using the Slivnyak-Mecke's theorem [14], we assume that the typical MT is located at the origin. The biased-received-power (BRP), $P_{r,k}$, from a BS x at the location $\mathbf{x} \in \mathbb{R}^2$ can be formulated as follows

$$P_{r,k} = P_{t,k} \|\mathbf{x}\|^{-\eta_k} B_k, \quad (1)$$

where $\|\mathbf{x}\|$ denotes the BS-to-MT distance.

We consider a cell association criterion based on DL maximum BRP. According to this criterion, the BS offers the maximum received power for the typical MT is selected [10]. Accordingly, the selected serving BS for the typical MT x^* can be formulated as follows

$$\begin{aligned} x^* &= \arg \max_{x \in \{x_k^*\}} P_{t,k} \|\mathbf{x}\|^{-\eta_k} B_k, \\ \text{with } x_k^* &= \arg \max_{x \in \Phi_k} P_{t,k} \|\mathbf{x}\|^{-\eta_k} \end{aligned} \quad (2)$$

where x_k^* for $k \in \{1, \dots, K\}$ denotes the BS of tier k that is closest to the MT, and Φ_k denotes the position sets of BSs in the k th tier.

It is worth mentioning that: i) if $B_k = 1$ for $k \in \{1, \dots, K\}$, the MT is served by the BS providing the best received power, and ii) if $B_k = 1/P_{t,k}$, the MT is served by the BS providing the smallest path-loss.

B. Downlink Power Transfer

In the DL power transfer phase, each N -antenna BS uses maximal-ratio transmission (MRT) beamforming to transfer the power towards its own intended MT. For mathematical tractability, we consider a short range propagation model to

avoid the singularity caused by proximity between BSs and MTs [3, 15].

For a typical MT u_0 located at the origin that is associated with its serving BS S_{k,x^*} in the k th tier, its received power is presented as

$$P_{r_{u_0,k}} = \underbrace{P_{t,S_{k,x^*}} \|\mathbf{h}_{S_{k,x^*}}\|^2 L_0 \left(\max \left\{ \|\mathbf{x}_{S_{k,x^*}}\|, d \right\} \right)^{-\eta_k}}_{I_{S_{k,x^*}}} + \underbrace{\sum_{j=1}^K \sum_{S_x \in \Phi_j \setminus S_{k,x^*}} P_{t,S_j} \left| \mathbf{h}_{S_x u_0} \frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|} \right|^2 L_0 \left(\max \left\{ \|\mathbf{x}_{S_x u_0}\|, d \right\} \right)^{-\eta_j}}_{I_{S_x}}, \quad (3)$$

where $d \geq 1$ is a constant value, L_0 is the path loss for a reference distance $r_0 = 1$, which is typically $(4\pi/v)^{-2}$, and v is the wavelength. In (3), $\mathbf{h}_{S_{k,x^*}} \in \mathcal{C}^{1 \times N}$ is the small-scale fading channel vector from the serving BS in the k th tier to the typical MT, $\mathbf{h}_{S_x u_0} \in \mathcal{C}^{1 \times N}$ is the small-scale fading interfering channel vector from the interfering BS at a location \mathbf{x} to the typical MT, and $\frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|}$ is the MRT beamforming vector of interfering BS at a location \mathbf{x} , where $\mathbf{g}_{S_x u_j} \in \mathcal{C}^{1 \times N}$ is the small-scale fading channel vector from the interfering BS at a location \mathbf{x} to its associated MT. All the channels are assumed to experience Rayleigh fading such that $\|\mathbf{h}_{S_{k,x^*}}\|^2 \sim \text{Gamma}(N, 1)$. According to [16], $\mathbf{h}_{S_x u_0} \frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|}$ is a zero-mean complex Gaussian variable, which is independent of $\mathbf{g}_{S_x u_j}^H$, such that $\left| \mathbf{h}_{S_x u_0} \frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|} \right|^2 \sim \exp(1)$.

C. Uplink Information Transmission

In the UL information transmission phase, the MTs remain associated with their serving BSs that powered them in the DL power transfer phase¹, and consume the stored energy to transmit information signals to their serving BSs. Note that the saturated traffic conditions holds, and each BS serves one MT per channel [12]. As such, the density of interfering MTs served per channel is $\lambda_u = \sum_{k=1}^K \lambda_k$.

The MTs are assumed to have large storage battery to enable reliable transmission power. As suggested by Lemma 2 in [3], for MTs with large storage battery, the randomness of the instantaneous received power is lost and the active MTs in the k th tier can transmit with fixed power up to $\phi \mathbb{E}\{P_{r_{u_0,k}}\}$, where $\phi = \mu \frac{\alpha}{1-\alpha}^2$. Note that the energy consumed for the UL information transmission should not exceed the harvested energy, thus the transmit power $\phi \mathbb{E}\{P_{r_{u_0,k}}\}$ is assumed to fulfill the UL requirement. Here, μ is the the RF-to-DC conversion efficiency with $0 < \mu < 1$ [18].

In the UL information transmission phase, each N -antenna BS employs maximal-ratio combiner (MRC) to combine the

¹In the current cellular networks, the UL cell association is based on DL cell association criterion [17].

²The processing power in the UL is ignored [13].

received signals. The signal-to-interference-plus-noise ratio (SINR) at the serving BS with MRC in the k th tier is given by

$$SINR_k = \frac{\phi \mathbb{E} \{P_{r_u,k}\} \left\| \mathbf{h}_{u_0, S_{k_x^*}} \right\|^2 L_0 \left\| \mathbf{x}_{u_0, S_{k_x^*}} \right\|^{-\eta_k}}{I_U + \delta^2}, \quad (4)$$

where $I_U = \sum_{j=1}^K I_{U,j}$, $I_{U,j} = \sum_{\mathbf{u}_{x,j} \in \tilde{\Phi}_j \setminus \mathbf{u}} \phi \mathbb{E} \{P_{r_u,j}\}$

$\left\| \frac{\mathbf{h}_{u_0, S_{k_x^*}}^H}{\left\| \mathbf{h}_{u_0, S_{k_x^*}}^H \right\|} \mathbf{h}_{u_{x,j}, S_{k_x^*}} \right\|^2 L_0 \left\| \mathbf{x}_{u_{x,j}, S_{k_x^*}} \right\|^{-\eta_j}$, $\mathbf{h}_{u_0, S_{k_x^*}} \in \mathcal{C}^{N \times 1}$ is the small-scale fading channel vector from the typical MT \mathbf{u}_0 to the serving BS in the k th tier, $\mathbf{h}_{u_{x,j}, S_{k_x^*}} \in \mathcal{C}^{N \times 1}$ is the small-scale fading channel vector from the interfering MT $\mathbf{u}_{x,j}$ in the j th tier to the serving BS in the k th tier, $\frac{\mathbf{h}_{u_0, S_{k_x^*}}^H}{\left\| \mathbf{h}_{u_0, S_{k_x^*}}^H \right\|}$ is the MRC vector of the serving BS in the k th tier, $\tilde{\Phi}_j$ is the point process corresponding to the interfering MTs in the j th tier, and δ^2 is the noise power.

III. SYSTEM-LEVEL PERFORMANCE EVALUATION OF DOWNLINK POWER TRANSFER

To determine the efficiency of DL wireless power transfer, we derive the maximum transmit power at the typical MT in the k th tier based on (3) in the following theorem.

A. Maximum Transmit Power

Theorem 1. The maximum transmit power at the typical MT associated with the k th tier BS using BRP cell association is given by

$$P_{t_u,k} = \phi \mathbb{E} \{P_{r_{u_0,k}}\} = \phi \frac{P_{t,B_{k_x^*}} N_k L_0}{\Pi_k} (d^{-\eta_k} \Psi_1 + \Psi_2) + \phi \frac{L_0}{\Pi_k} \sum_{j=1}^K 2\pi P_{t,j} \lambda_j \left(\Psi_3 + (\eta_j - 2)^{-1} \Psi_4 \right), \quad (5)$$

where

$$\Psi_1 = \int_0^d x \exp \left\{ - \sum_{j_1=1}^K \zeta_{k,j_1} x^{2\eta_k/\eta_{j_1}} \right\} dx, \quad (6)$$

$$\Psi_2 = \int_d^\infty x^{-(\eta_k-1)} \exp \left\{ - \sum_{j_2=1}^K \zeta_{k,j_2} x^{2\eta_k/\eta_{j_2}} \right\} dx, \quad (7)$$

$$\Psi_3 = \int_0^{\chi_{j,k}} \frac{x}{2d^{\eta_j}} \left(\frac{\eta_j d^2}{(\eta_j - 2)} - \rho_{j,k}^{2/\eta_j} x^{2\eta_j/\eta_k} \right) \exp \left\{ - \sum_{j_3=1}^K \zeta_{k,j_3} x^{2\eta_k/\eta_{j_3}} \right\} dx, \quad (8)$$

$$\Psi_4 = \int_{\chi_{j,k}}^\infty \frac{x \exp \left\{ - \sum_{j_4=1}^K \zeta_{k,j_4} x^{2\eta_k/\eta_{j_4}} \right\}}{(\rho_{j,k}^{1/\eta_j} x^{\eta_k/\eta_j})^{\eta_j-2}} dx, \quad (9)$$

$$\chi_{j,k} = d^{\eta_j/\eta_k} \rho_{j,k}^{-1/\eta_k}, \quad (10)$$

$$\Pi_k = \int_0^\infty r \exp \left\{ - \sum_{j_0=1}^K \zeta_{k,j_0} r^{2\eta_k/\eta_{j_0}} \right\} dr, \quad (11)$$

$$\zeta_{k,j} = \pi \lambda_j \rho_{j,k}^{2/\eta_j}, \quad (12)$$

and

$$\rho_{j,k} = P_{t,j} B_j / P_{t,k} B_k. \quad (13)$$

Proof. See Appendix A. \square

In the following, we present the maximum transmit power at the typical MT in HCNs for the special case of single antenna BSs and equal path loss exponent in each tier.

Corollary 1. With $N_k = 1$ and $\{\eta_k\} = \eta$, the maximum transmit power at the typical MT associated with the k th tier reduces to

$$P_{t_u,k} = \phi P_{t,k} N_k L_0 [d^{-\eta} \Lambda_1 + \Lambda_2] + \phi L_0 \sum_{j=1}^K \pi P_{t,j} \lambda_j [\Lambda_3 + (\eta-2)^{-1} \Lambda_4], \quad (14)$$

where

$$\Lambda_1 = 1 - \exp \{ -d^2 \varpi_k \}, \quad (15)$$

$$\Lambda_2 = (\varpi_k)^{\eta/4} d^{-\eta/2} \exp \{ -d^2 \varpi_k / 2 \} W_{-\eta/4, 1/2(1-\eta/2)}(d^2 \varpi_k), \quad (16)$$

$$\Lambda_3 = \frac{\eta}{\eta-2} d^{(2-\eta)} (1 - \exp \{ -\varpi_k d^2 \rho_{j,k}^{-2} \}) - \frac{\rho_{j,k}^{2/\eta}}{d^\eta \varpi_k} \Upsilon \left(2, \varpi_k d^2 \rho_{j,k}^{-2/\eta} \right), \quad (17)$$

$$\Lambda_4 = 2\rho_{j,k}^{-1/\eta-1/2} \varpi_k^{7/2-3\eta/4} d^{-\eta/2+1} \exp \{ -d^2 \rho_{j,k}^{-2/\eta} \varpi_k / 2 \}, \quad (18)$$

$$\varpi_k = \sum_{j_1=1}^K \zeta_{k,j_1} = \sum_{j_1=1}^K \pi \lambda_{j_1} (P_{t,j_1} B_{j_1} / P_{t,k} B_k)^{2/\eta}, \quad (19)$$

and $W_{\lambda,\mu}(z)$ is Whittaker function [19].

IV. SYSTEM-LEVEL PERFORMANCE EVALUATION OF UPLINK INFORMATION TRANSMISSION

In this section, we characterize the UL average ergodic rate as an important performance indicator for the spectral efficiency of the UL transmission in HCNs.

A. UL Average Ergodic Rate

The UL average ergodic rate per random MT of a K -tier HCN is given by [10]

$$R = \sum_{k=1}^K J_k R_k, \quad (20)$$

where J_k is given in (21). In (20), R_k is the UL average ergodic rate of a typical MT associated with the k th tier, which represents the average data rate of a random MT associated with the k th tier. In each tier, one MT per cell is active.

The probability that the typical MT is associated with a BS in the k th tier is given as

$$J_k = 2\pi\lambda_k \int_0^\infty r \exp\left\{-\sum_{j_0=1}^K \zeta_{k,j_0} r^{2\eta_{k,j_0}/\eta_{j_0}}\right\} dr, \quad (21)$$

where $\zeta_{k,j}$ is given in (12).

For a typical MT at a distance $\|\mathbf{x}_{u_0, B_{k,x^*}}\|$ from the serving BS, the UL average ergodic rate of a typical MT associated with the k th tier is defined as

$$R_k = \mathbb{E}_{\|\mathbf{x}_{u_0, S_{k,x^*}}\|} \left[\mathbb{E}_{SINR_k} \left[\frac{(1-\alpha)T}{T} \ln \left(1 + SINR_k \left(\|\mathbf{x}_{u_0, S_{k,x^*}}\| \right) \right) \right] \right]. \quad (22)$$

1) *General Case:* We derive the UL average ergodic rate of a typical MT associated with the k th tier in the following theorem.

Theorem 2. The UL average ergodic rate of a typical MT associated with the k th tier is derived as

$$R_k = \frac{(1-\alpha)}{\Pi_k} \left[T_1 - \sum_{m=1}^{N-1} \frac{\phi^{-m}}{(-1)^m} \sum_{i=1}^m \frac{1}{m_i! i! m_i} T_2 \right], \quad (23)$$

where

$$T_1 = \int_0^\infty \int_0^\infty \frac{r}{1+x} \exp\left\{-\sigma^2 \tau_k x r^{\eta_k} - \sum_{j=1}^K \left(\vartheta_{k,j} x^{\frac{2}{\eta_j}} + \zeta_{k,j} \right) r^{\frac{2\eta_k}{\eta_j}}\right\} dx dr \quad (24)$$

and

$$T_2 = \int_0^\infty \int_0^\infty \exp\left\{-\sigma^2 \tau_k x r^{\eta_k} - \sum_{j=1}^K \left(\vartheta_{k,j} x^{\frac{2}{\eta_j}} + \zeta_{k,j} \right) r^{\frac{2\eta_k}{\eta_j}}\right\} \frac{r}{1+x} \left(-\sigma^2 \tau_k \phi x r^{\eta_k} - \sum_{j=1}^K \vartheta_{k,j} \frac{2}{\eta_j} \phi x^{\frac{2}{\eta_j}} r^{\frac{2\eta_k}{\eta_j}} \right)^{m_1} \prod_{l=2}^m \left(-\sum_{j=1}^K \vartheta_{k,j} \prod_{i=0}^{l-1} \left(\frac{2}{\eta_j} - i \right) \phi^l x^{\frac{2}{\eta_j}} r^{\frac{2\eta_k}{\eta_j}} \right)^{m_l} dx dr, \quad (25)$$

respectively.

In (24) and (25), τ_k and $\vartheta_{k,j}$ are given by

$$\tau_k = (\phi \mathbb{E}\{P_{r_u, k}\} L_0)^{-1} \quad (26)$$

and

$$\vartheta_{k,j} = \pi \lambda_j (\mathbb{E}\{P_{r_u, j}\} / \mathbb{E}\{P_{r_u, k}\})^{\frac{2}{\eta_j}} \Gamma\left(1 + \frac{2}{\eta_j}\right) \Gamma\left(1 - \frac{2}{\eta_j}\right), \quad (27)$$

respectively. Note that $\zeta_{k,j}$ and $\mathbb{E}\{P_{r_u, k}\}$ in (24) and (25) are given in (12) and (5), respectively.

Proof. See Appendix B. \square

For the UL transmission of HCNs with high density of MTs, the interference power from the interfering MTs in each tier

dominates the performance, and as such the thermal noise is ignored. In the sequel we then present the UL average ergodic rate of a typical MT associated with the k th tier in HCNs with single antenna BSs, equal path loss exponent in each tier, and no thermal noise.

2) *Special Case 1:* Single antenna BSs, equal path loss exponent in each tier, and no thermal noise.

Corollary 2. When $N_k = 1$, $\{\eta_k\} = \eta$, and $\sigma^2 = 0$, the UL average ergodic rate of a typical MT associated with the k th tier is derived as

$$R_k = (1-\alpha) \int_0^\infty \left[(1+x) \left(1 + x^{\frac{2}{\eta}} \frac{\sum_{j=1}^K \vartheta_{k,j}}{\varpi_k} \right) \right]^{-1} dx. \quad (28)$$

3) *Special Case 2:* Single antenna BSs, equal path loss exponent $\eta = 4$ in each tier, and no thermal noise.

Corollary 3. When $N_k = 1$, $\{\eta_k\} = 4$, and $\sigma^2 = 0$, the UL average ergodic rate of a typical MT associated with the k th tier reduces to

$$R_k = (1-\alpha) \left[\pi \left(\frac{\sum_{j=1}^K \vartheta_{k,j}}{\varpi_k} \right) - 2 \log \left(\frac{\sum_{j=1}^K \vartheta_{k,j}}{\varpi_k} \right) \right] \left[1 + \left(\frac{\sum_{j=1}^K \vartheta_{k,j}}{\varpi_k} \right)^2 \right], \quad (29)$$

where

$$\frac{\sum_{j=1}^K \vartheta_{k,j}}{\varpi_k} = \sum_{j=1}^K \left[\frac{(P_{t,j} B_j / P_{t,k} B_k)}{(\mathbb{E}\{P_{r_u, j}\} / \mathbb{E}\{P_{r_u, k}\})} \right]^{-1/2} \Gamma(3/2) \Gamma(1/2).$$

We find that the UL average ergodic rate in the interference-limited scenario is independent of the RF-to-DC conversion efficiency. This can be seen from (4) where μ in the $SINR_k$ disappear in the interference-limited regime with $\sigma^2 = 0$.

V. NUMERICAL RESULTS

In this section, we plot the exact maximum transmit power and the UL average ergodic rate using (5) and (23), respectively. The analytical results are validated by Monte Carlo simulations as shown in each figure. In all the figures, we set the path loss at 1 meters of $L_0 = -38.5$ dB, and the thermal noise $\sigma^2 = -104$ dB for 10 MHz bandwidth. In Figs. 1 and 2, we set $\alpha_1 = 2.8$, $\alpha_2 = 2.5$, $\lambda_1 = 10^{-3}$, $B_1 = B_2 = 1$, $P_{t, S_1} = 46$ dBm, $P_{t, S_2} = 30$ dBm, $d = 1$, $\mu = 0.5$, and $\alpha = 0.45$.

Fig. 1 plots the maximum transmit power at the typical MT versus the density of picocell BSs λ_2 . The following insights are observed: 1) The maximum transmit power at the MT in macrocell or picocell increases with increasing λ_2 . This can be explained by (A.2) that increasing λ_2 decreases the distance $\|\mathbf{x}_{S_{k,x^*}}\|$ between the typical MT and the serving BS. 2) We see that the maximum transmit power at the MT in the macrocell is higher and grows sharper than that at the MT in

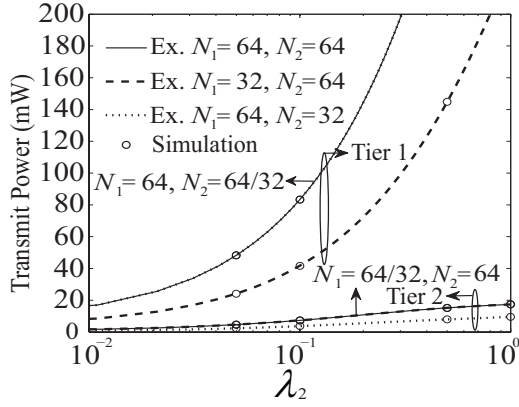


Fig. 1. Impact of picocell BS density and number of BS antennas on the maximum transmit power.

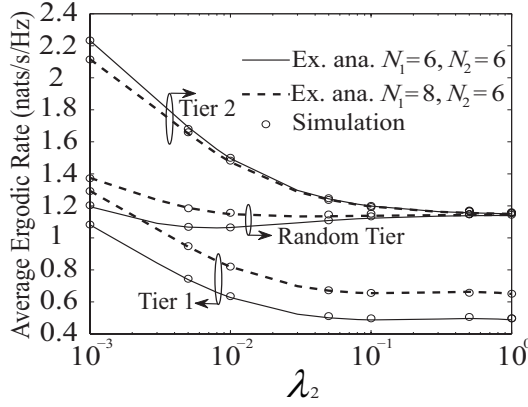


Fig. 2. Impact of picocell BS density and number of BS antennas on the UL average ergodic rate.

the picocell. This is due to the fact that the transmit power of the macrocell BS is much higher than that of the picocell BS; and 3) We find that for $N_1 = 64$, increasing the number of antennas at the picocell BS N_2 has no effect on the maximum transmit power at the MT in the macrocell. Similarly, for $N_2 = 64$, increasing the number of antennas at the macrocell BS N_1 has no effect on the maximum transmit power at the MT in the picocell. These observations are attributed to the fact that changing the number of antennas at the interfering BS does not change the distribution of $\left| \mathbf{h}_{S_x u_0} \frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|} \right|^2$ in (3).

Fig. 2 plot the UL average ergodic rate per MT versus the density of picocell BSs λ_2 . The key observations are as follows: We observe that the UL average ergodic rate per MT is not significantly improved with increasing λ_2 , which is due to the tradeoff between the benefits from the increased transmit power of MT and the detrimental effect brought by the increased interference from the other MTs in the picocell. We also see that increasing the number of antennas at the macrocell BS improves the UL average ergodic

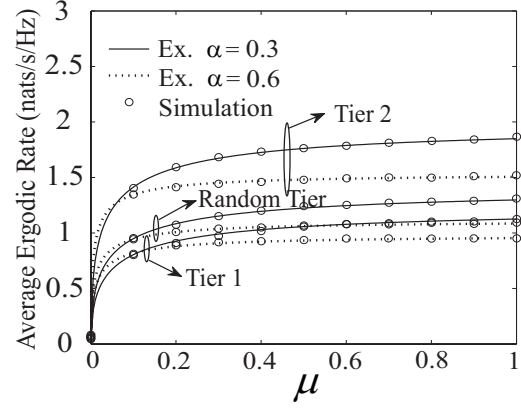


Fig. 3. Impact of energy conversion efficiency on the UL average ergodic rate.

rate per random MT.

Fig. 3 plots the UL average ergodic rate per MT versus the energy conversion efficiency μ . We see that the UL average ergodic rate per MT improves with increasing μ . This is because for small μ , the noise plays a dominant role in the received SINR as shown in (4), and thus the SINR increases with increasing μ in the noise-limited scenario. However, for large μ , the aggregate interference overtakes the effect of noise, and the SIR dominates the performance in the interference-limited scenario. In this case, the SINR remains almost unchanged with increasing μ , and thus the UL average ergodic rate is almost saturated.

VI. CONCLUSION

We have developed a tractable model for the DL wireless power transfer along with UL information transmission in HCNs. We derived useful expressions for the exact maximum transmit power and UL average ergodic rate. The intrinsic relationship between the energy harvested from the BSs in the DL and the information transmission in the UL are well demonstrated using the derived results and simulation results. It is shown that while the maximum transmit power at the MT can be improved by adding more picocell BS, the UL average ergodic rate per random MT is not significantly improved. More importantly, the UL average ergodic rate improves with increasing the energy conversion efficiency.

APPENDIX A

A PROOF OF THEOREM 1

Based on (3), using $\mathbb{E} \left\{ \left\| \mathbf{h}_{S_{k_x^*}} \right\|^2 \right\} = N_k$ and the polar-coordinate system, we first calculate $\mathbb{E} \{ I_{S_{k_x^*}} \}$ as

$$\begin{aligned} \mathbb{E} \{ I_{S_{k_x^*}} \} &= P_{t, S_{k_x^*}} N_k L_0 \mathbb{E} \left\{ \left(\max \left\{ \left\| \mathbf{x}_{S_{k_x^*}} \right\|, d \right\} \right)^{-\eta_k} \right\} \\ &= P_{t, S_{k_x^*}} N_k L_0 \left[\int_0^d f_{\|\mathbf{x}_{S_{k_x^*}}\|}(x) d^{-\eta_k} dx \right. \\ &\quad \left. + \int_d^\infty f_{\|\mathbf{x}_{S_{k_x^*}}\|}(x) x^{-\eta_k} dx \right]. \end{aligned} \quad (\text{A.1})$$

In (A.1), the PDF of $\|\mathbf{x}_{S_{k,x^*}}\|$ is given by [10]

$$f_{\|\mathbf{x}_{B_{k,x^*}}\|}(x) = \frac{x}{\Pi_k} \exp \left\{ - \sum_{j=1}^K \zeta_{k,j} x^{2\eta_k/\eta_j} \right\}, \quad (\text{A.2})$$

where Π_k is given in (11), $\zeta_{k,j}$ and $\rho_{j,k}$ are given in (12) and (13), respectively.

We then turn our attention to the expectation of aggregate interference, which is derived as

$$\mathbb{E}\{I_{S_x}\} = \mathbb{E}_{S_{x^*}} \left\{ \mathbb{E}_{\Phi_j} \left\{ \sum_{S_x \in \Phi_j \setminus S_{x^*}} (\max\{\|\mathbf{x}_{S_x u_0}\|, d\})^{-\eta_j} \right\} \right\}. \quad (\text{A.3})$$

Given the distance between the typical MT and the serving BS as $\|\mathbf{x}_{S_{k,x^*}}\| = x$, the interfering BSs need to be located outside a disc of radius $\rho_{j,k}^{1/\eta_j} x^{\eta_j/\eta_k}$ to satisfy the BRP cell association. According to this, the radius between the interfering BSs and the typical MT should be larger than $r_{I,\min} = \rho_{j,k}^{1/\eta_j} x^{\eta_j/\eta_k}$ [10]. We proceed by applying the Campbell's theorem to (A.3) to derive

$$\begin{aligned} \mathbb{E}\{I_{S_x}\} &= \sum_{j=1}^K \mathbb{E} \left\{ P_{t,j} L_0 \left| \mathbf{h}_{S_x u_0} \frac{\mathbf{g}_{S_x u_j}^H}{\|\mathbf{g}_{S_x u_j}\|} \right|^2 \right\} \\ &\quad \mathbb{E}_{S_{x^*}} \left\{ \lambda_j \int_{R^2/r_{I,\min}^2} (\max\{x, d\})^{-\eta_j} dx \right\} \\ &= \sum_{j=1}^K 2\pi P_{t,j} L_0 \lambda_j \int_0^\infty \int_{r_{I,\min}}^\infty (\max\{r, d\})^{-\eta_j} r dr \\ &\quad f_{\|\mathbf{x}_{S_{k,x^*}}\|}(x) dx. \end{aligned} \quad (\text{A.4})$$

By inserting $r_{I,\min} = \rho_{j,k}^{1/\eta_j} x^{\eta_j/\eta_k}$ into $\mathbb{E}\{I_{S_x}\}$, we have

$$\begin{aligned} \mathbb{E}\{I_{S_x}\} &= \sum_{j=1}^K 2\pi P_{t,j} L_0 \lambda_j \left[\int_0^{x_d} \left(d^{-\eta_j} \int_{\rho_{j,k}^{1/\eta_j} x^{\eta_j/\eta_k}}^d r dr \right. \right. \\ &\quad \left. \left. + \int_d^\infty r^{-(\eta_j-1)} dr \right) f_{\|\mathbf{x}_{S_{k,x^*}}\|}(x) dx + \right. \\ &\quad \left. \int_{x_d}^\infty \int_{\rho_{j,k}^{1/\eta_j} x^{\eta_j/\eta_k}}^\infty r^{-(\eta_j-1)} r dr f_{\|\mathbf{x}_{S_{k,x^*}}\|}(x) dx \right] \end{aligned} \quad (\text{A.5})$$

where $x_d = d^{\eta_k/\eta_j} \rho_{j,k}^{-\eta_k/\eta_j^2}$.

Substituting the PDF of $\|\mathbf{x}_{S_{k,x^*}}\|$ in (A.2) into (A.5), we obtain $\mathbb{E}\{I_{S_x}\}$.

Combining $\mathbb{E}\{I_{S_{k,x^*}}\}$ in (A.1) and $\mathbb{E}\{I_{S_x}\}$ in (A.5), we derive the maximum transmit power at the typical MT associated with the k th tier as Theorem 1.

APPENDIX B A PROOF OF THEOREM 2

Based on (22), the UL average ergodic rate of a typical MT associated with the k th tier is derived as

$$\begin{aligned} R_k &= (1 - \alpha) \int_0^\infty \int_0^\infty \Pr[SINR_k(x) > e^t - 1] dt \\ &\quad f_{\|\mathbf{x}_{u_0, B_{k,x^*}}\|}(x) dx. \end{aligned} \quad (\text{B.1})$$

Substituting (4) into (B.1), we obtain Theorem 2.

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