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Memory Bandits: Towards the Switching Bandit Problem Best Resolution

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► **To cite this version:**

Réda Alami, Odalric-Ambrym Maillard, Raphaël Féraud. Memory Bandits: Towards the Switching Bandit Problem Best Resolution. MLSS 2018 - Machine Learning Summer School, Aug 2018, Madrid, Spain. hal-01879251

HAL Id: hal-01879251

<https://hal.science/hal-01879251v1>

Submitted on 22 Sep 2018

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MULTI-ARMED BANDIT



For each step $t = 1, \dots, T$

- The player chooses an arm $k_t \in \mathcal{K}$
- The reward k_t is revealed $x_{k_t} \in [0, 1]$
- Bernoulli rewards: $x_{k_t} \sim \mathcal{B}(\mu_{k_t, t})$

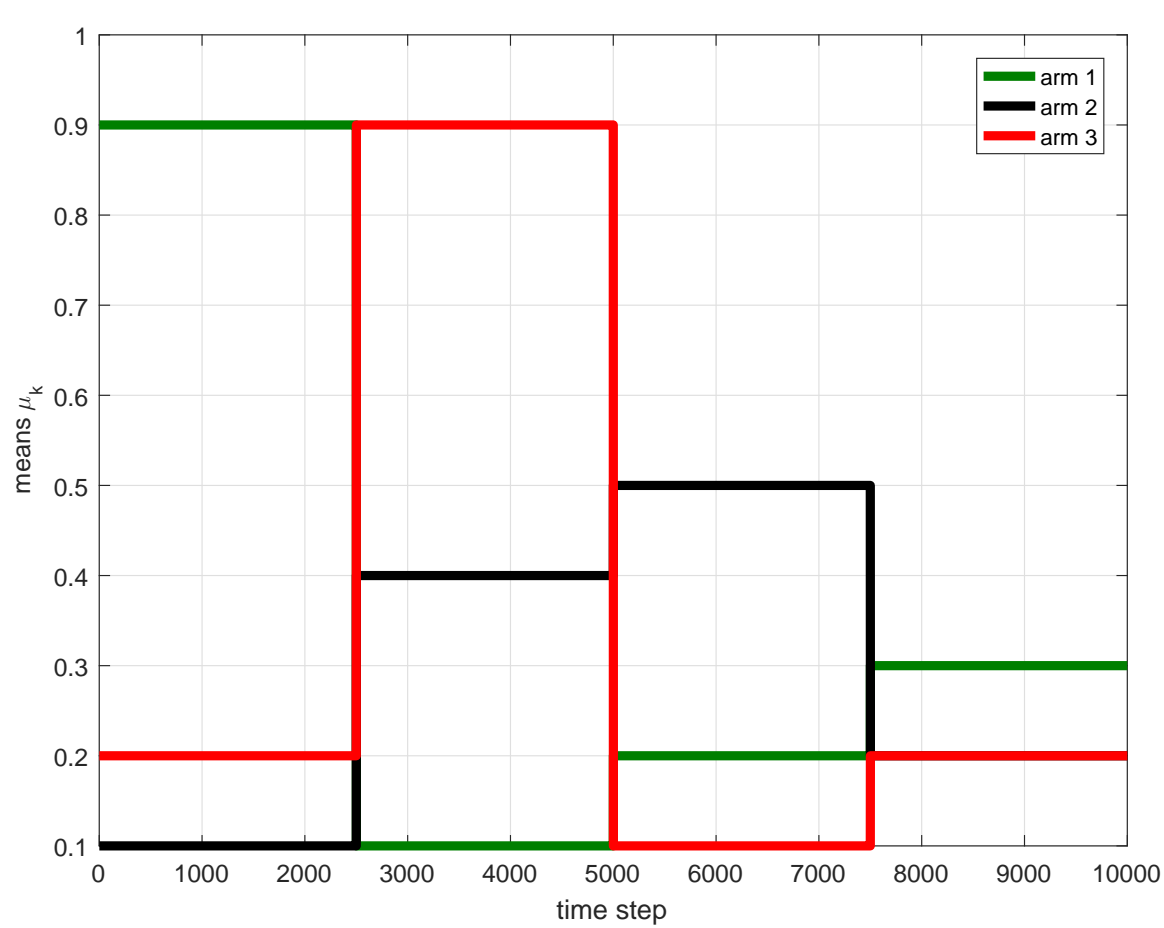
Objective: Minimize the pseudo regret \mathcal{R}_T :

$$\mathcal{R}_T = \underbrace{\sum_{t=1}^T \mu_t^*}_{\text{Best policy}} - \underbrace{\mathbb{E} \left[\sum_{t=1}^T x_{k_t} \right]}_{\text{Your policy}} \quad \mu_t^* = \max_k \mu_{k, t}$$

SWITCHING ENVIRONMENT

$$\mu_{k, t} = \begin{cases} \mu_{k, t-1} & \text{probability } 1 - \rho \\ \mu_{new} \sim \mathcal{U}(0, 1) & \text{probability } \rho \end{cases}$$

where ρ is the switching rate.



THOMPSON SAMPLING (TS)

$$\begin{cases} \text{success counter : } \alpha_k = \#(x_k = 1) + \alpha_0 \\ \text{failure counter : } \beta_k = \#(x_k = 0) + \beta_0 \end{cases}$$

At each step $t = 1, \dots, T$:

1. **Characterization:** $\theta_k \sim \text{Beta}(\alpha_k, \beta_k)$
2. **Decision:** $k_t = \arg \max_k \theta_k$
3. **Play:** $x_{k_t} \sim \mathcal{B}(\mu_{k_t, t})$
4. **Update:** $\begin{cases} \alpha_{k_t} = \alpha_{k_t} + 1 & \text{if } x_{k_t} = 1 \\ \beta_{k_t} = \beta_{k_t} + 1 & \text{if } x_{k_t} = 0 \end{cases}$

$$\mathcal{R}_T \leq (1 + \epsilon) \sum_k \frac{\mu_k^* - \mu_k}{KL(\mu_k, \mu_k^*)} (\log T + \log \log T)$$

(Lai and Robbins (1985) lower bound)
 $KL(\bullet, \bullet)$ = Kullback-Leibler divergence

REFERENCES

R. P. Adams and D.J.C MacKay, *Bayesian online changepoint detection*, arXiv, 2007.

J. Mellor and J. Shapiro, *Thompson Sampling in switching environments with Bayesian online changepoint detection*, AISTATS, 2013.

GLOBAL SWITCHING TS WITH BAYESIAN AGGREGATION

Learning with a growing number of Thompson Sampling $f_{i, t}$: i denotes the starting time and t the current time. $\mathbb{P}(f_{i, t})$: weight at time t of the Thompson sampling starting at time i .

Initialization: $\mathbb{P}(f_{1, 1}) = 1, t = 1,$
 $\forall k \in \mathcal{K} \alpha_{k, f_{1, 1}} = \alpha_0, \beta_{k, f_{1, 1}} = \beta_0$

-1- Decision process: at each time t :

- $\forall i \leq t, \forall k: \theta_{k, f_{i, t}} \sim \text{Beta}(\alpha_{k, f_{i, t}}, \beta_{k, f_{i, t}})$
- Play (**Bayesian Aggregation**):

$$k_t = \arg \max_k \sum_{i < t} \mathbb{P}(f_{i, t}) \theta_{k, f_{i, t}}$$

-4- Distribution of experts update:

- Update previous experts: $\mathbb{P}(f_{i, t+1}) \propto (1 - \rho) \cdot \mathbb{P}(\mathbf{x}_t | \mathbf{f}_{i, t}) \cdot \mathbb{P}(f_{i, t}) \quad \forall i \leq t$
- Create new expert $f_{t+1, t+1}$: $\mathbb{P}(f_{t+1, t+1}) \propto \rho \sum_{i=1}^t \mathbb{P}(f_{i, t})$
- Prior: $\alpha_{k, f_{i, t}} = \alpha_0, \beta_{k, f_{i, t}} = \beta_0$

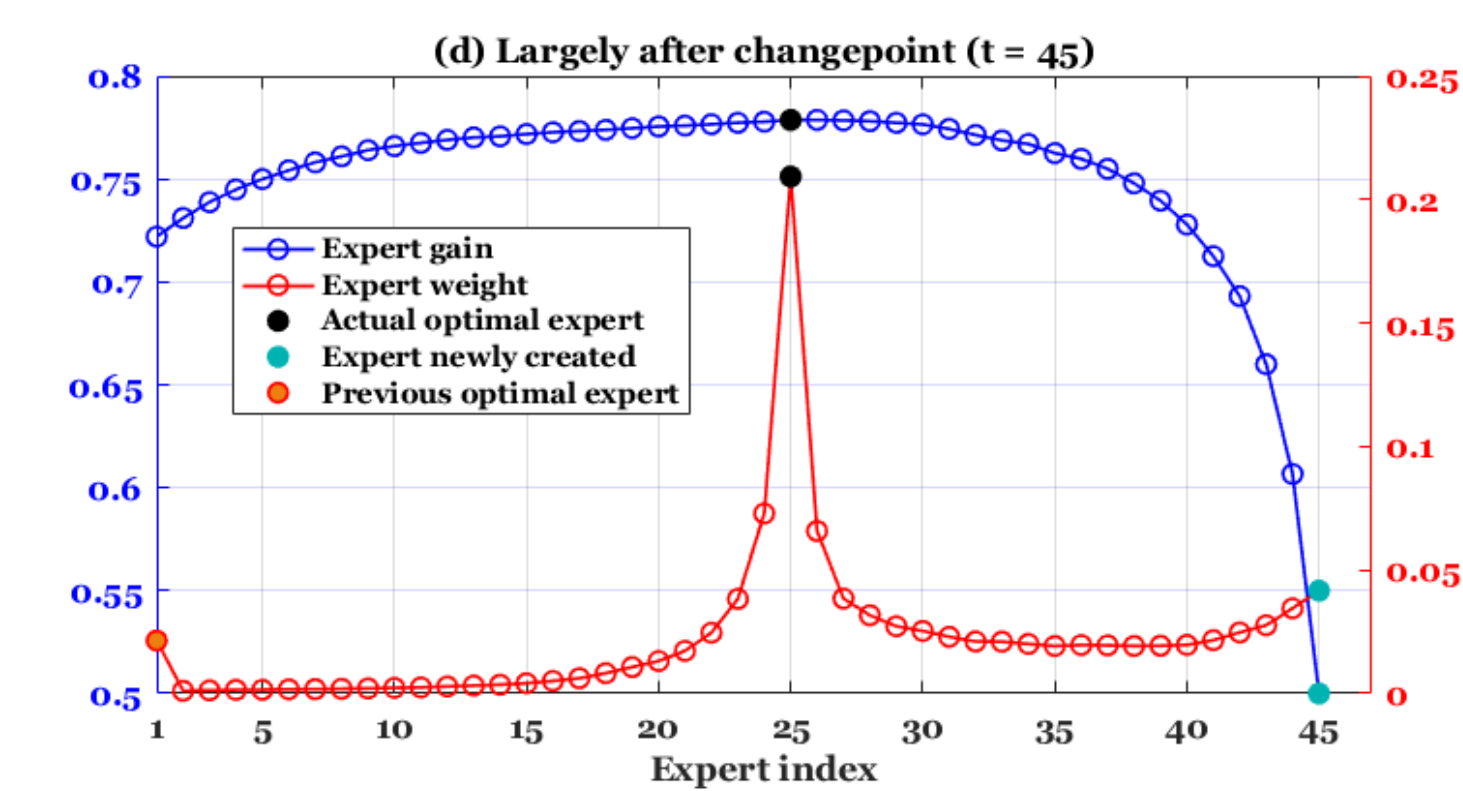
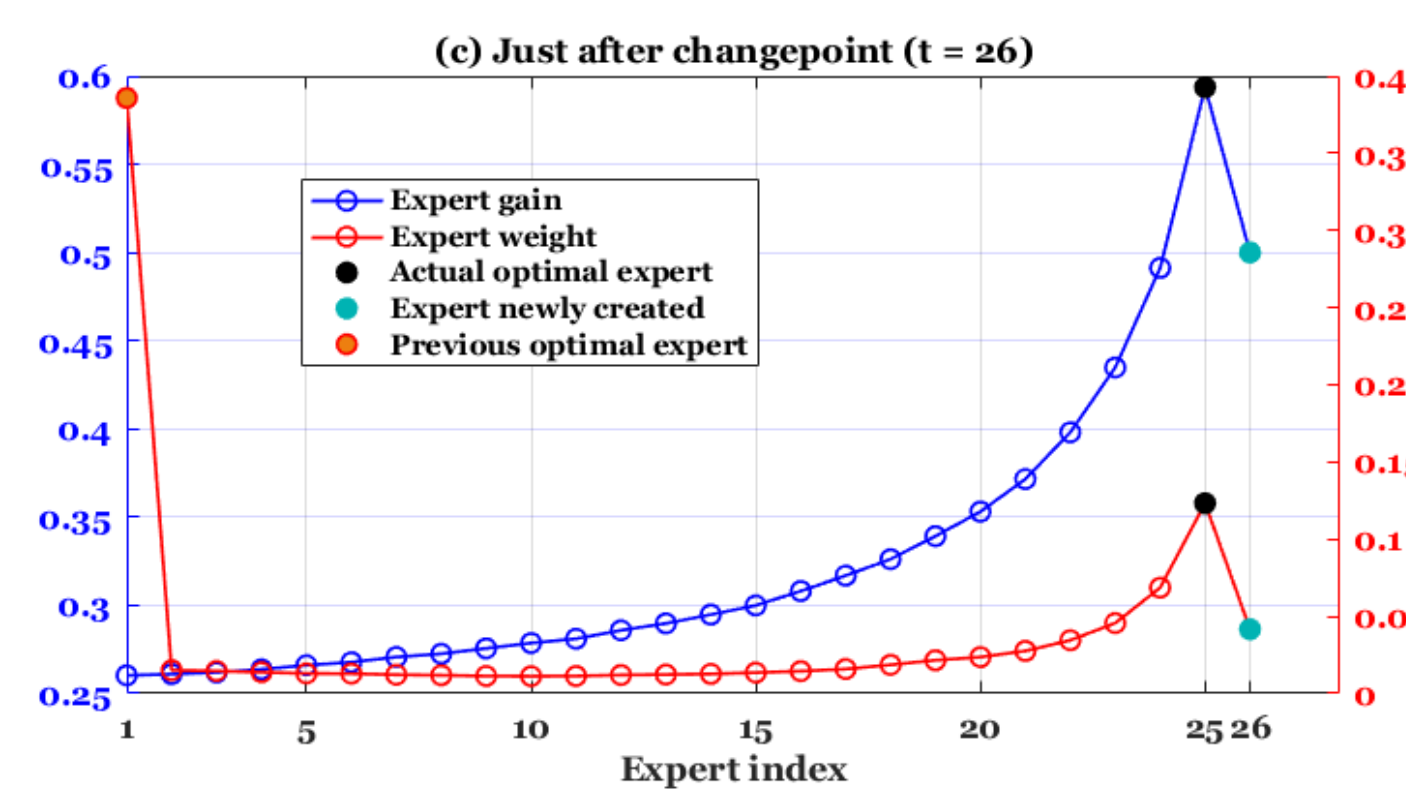
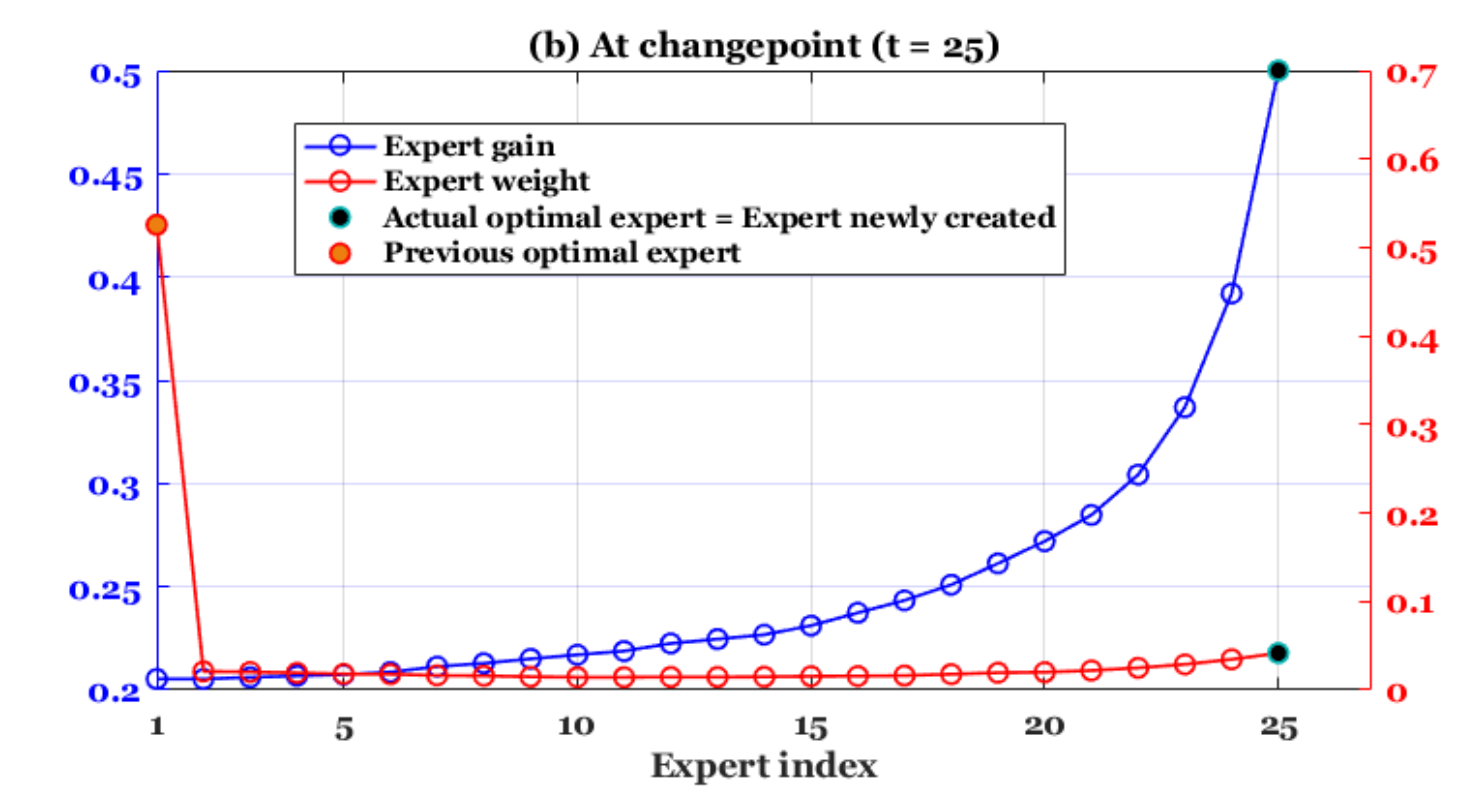
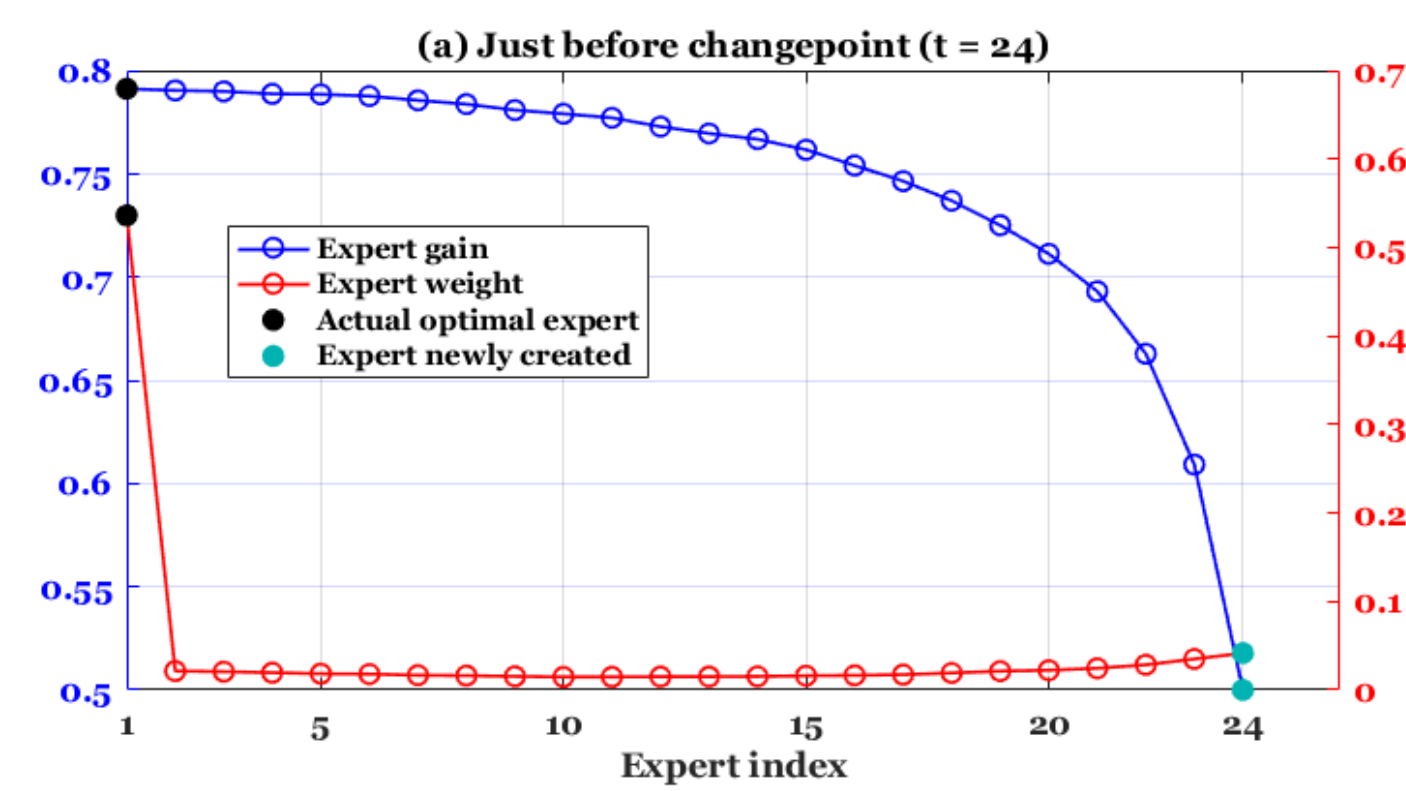
-2- Instantaneous gain update:

$$\forall i \leq t \mathbb{P}(x_t | f_{i, t}) = \begin{cases} \frac{\alpha_{k_t, f_{i, t}}}{\beta_{k_t, f_{i, t}} + \alpha_{k_t, f_{i, t}}} & \text{if } x_{k_t} = 1 \\ \frac{\beta_{k_t, f_{i, t}}}{\beta_{k_t, f_{i, t}} + \alpha_{k_t, f_{i, t}}} & \text{if } x_{k_t} = 0 \end{cases}$$

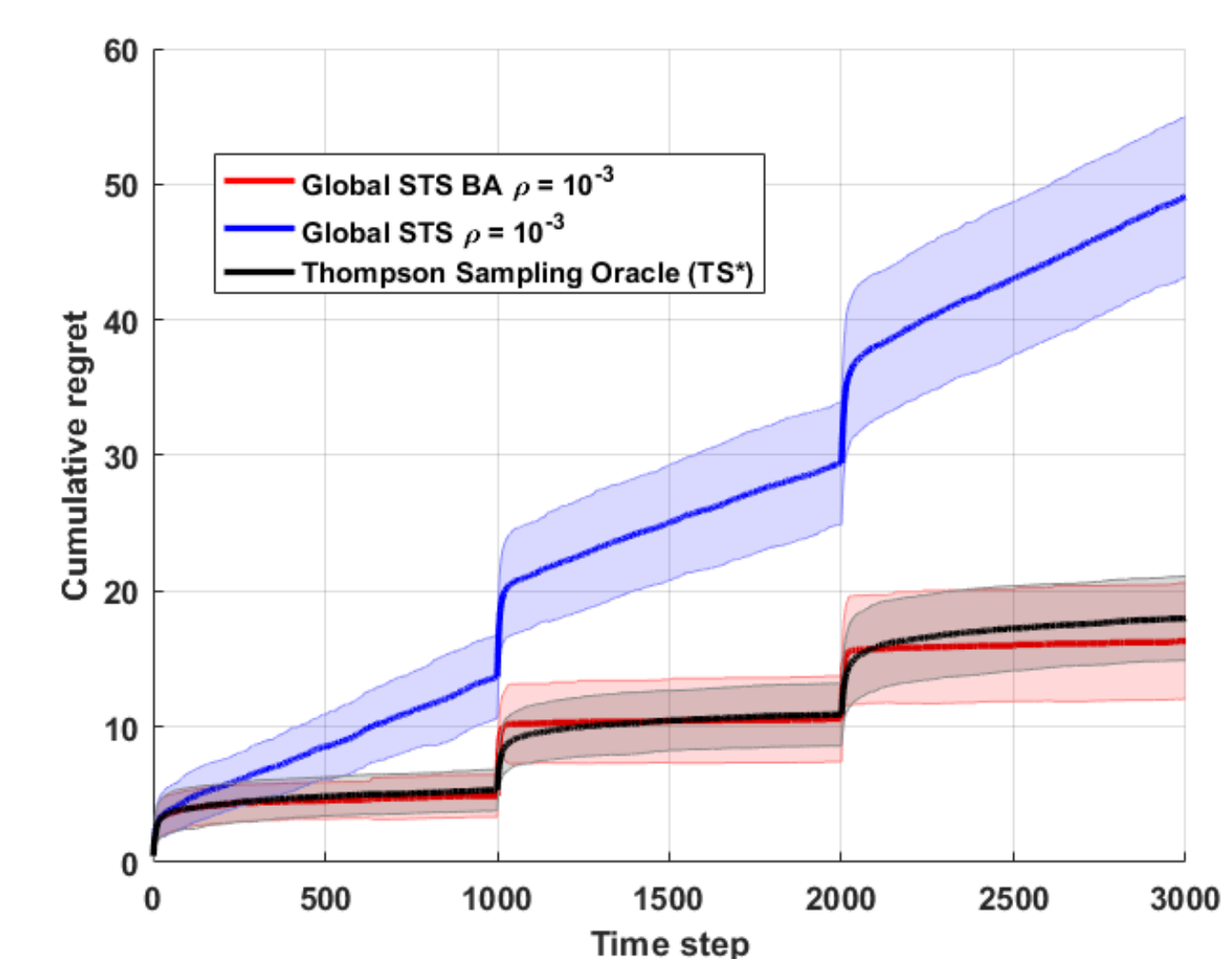
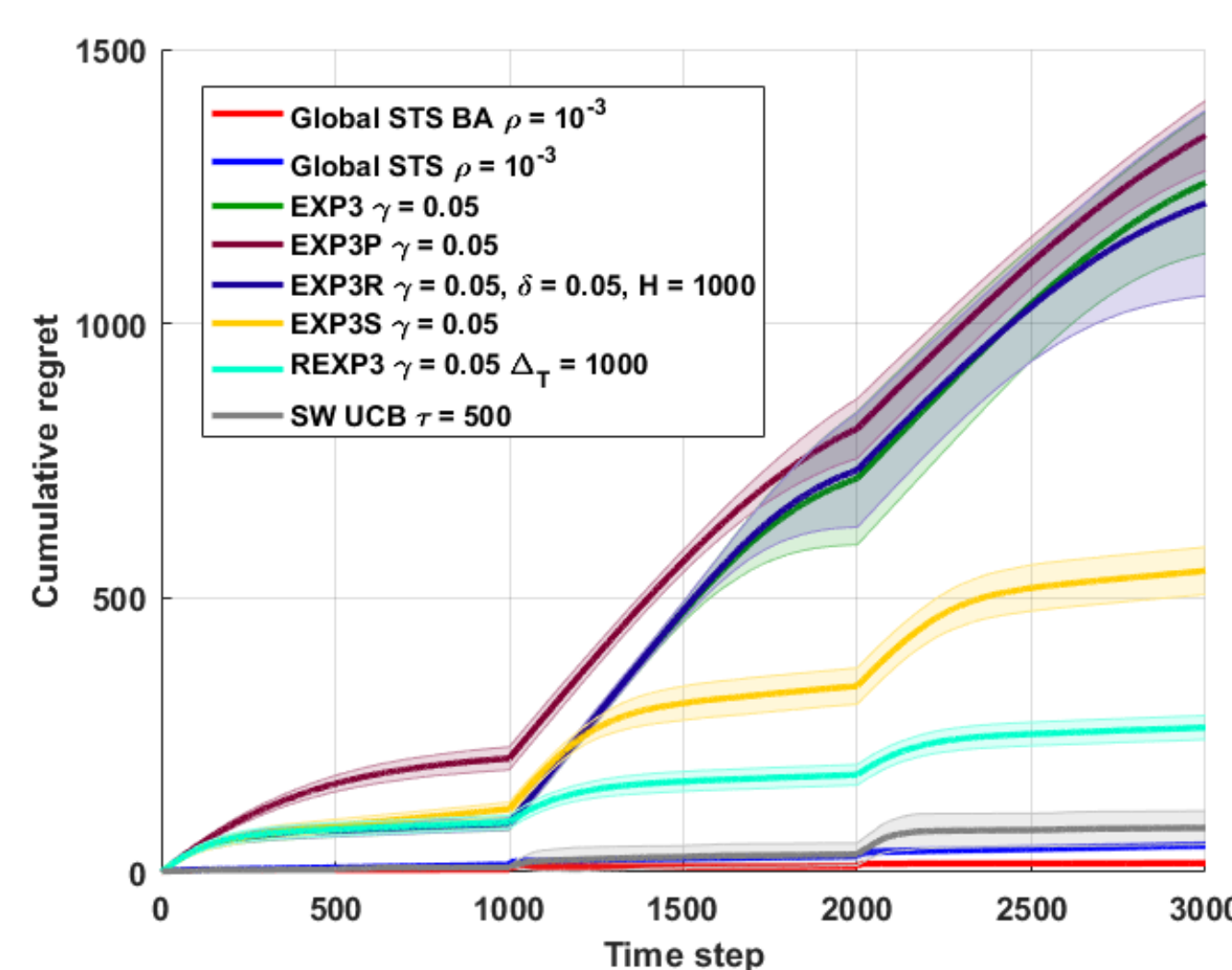
-3- Arm hyperparameters update:

$$\forall i \leq t \begin{cases} \alpha_{k_t, f_{i, t}} = \alpha_{k_t, f_{i, t}} + 1 & \text{if } x_{k_t} = 1 \\ \beta_{k_t, f_{i, t}} = \beta_{k_t, f_{i, t}} + 1 & \text{if } x_{k_t} = 0 \end{cases}$$

TRACKING THE OPTIMAL EXPERT



COMPARISON WITH STATE-OF-THE-ART



SENSITIVITY ANALYSIS OF PARAMETERS (ρ AND M)

