



Memory Bandits: Towards the Switching Bandit Problem Best Resolution

Réda Alami, Odalric-Ambrym Maillard, Raphaël Féraud

► To cite this version:

Réda Alami, Odalric-Ambrym Maillard, Raphaël Féraud. Memory Bandits: Towards the Switching Bandit Problem Best Resolution. MLSS 2018 - Machine Learning Summer School, Aug 2018, Madrid, Spain. hal-01879251

HAL Id: hal-01879251

<https://hal.science/hal-01879251>

Submitted on 22 Sep 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

MULTI-ARMED BANDIT



For each step $t = 1, \dots, T$

- The player chooses an arm $k_t \in \mathcal{K}$
- The reward k_t is revealed $x_{k_t} \in [0, 1]$
- Bernoulli rewards: $x_{k_t} \sim \mathcal{B}(\mu_{k_t, t})$

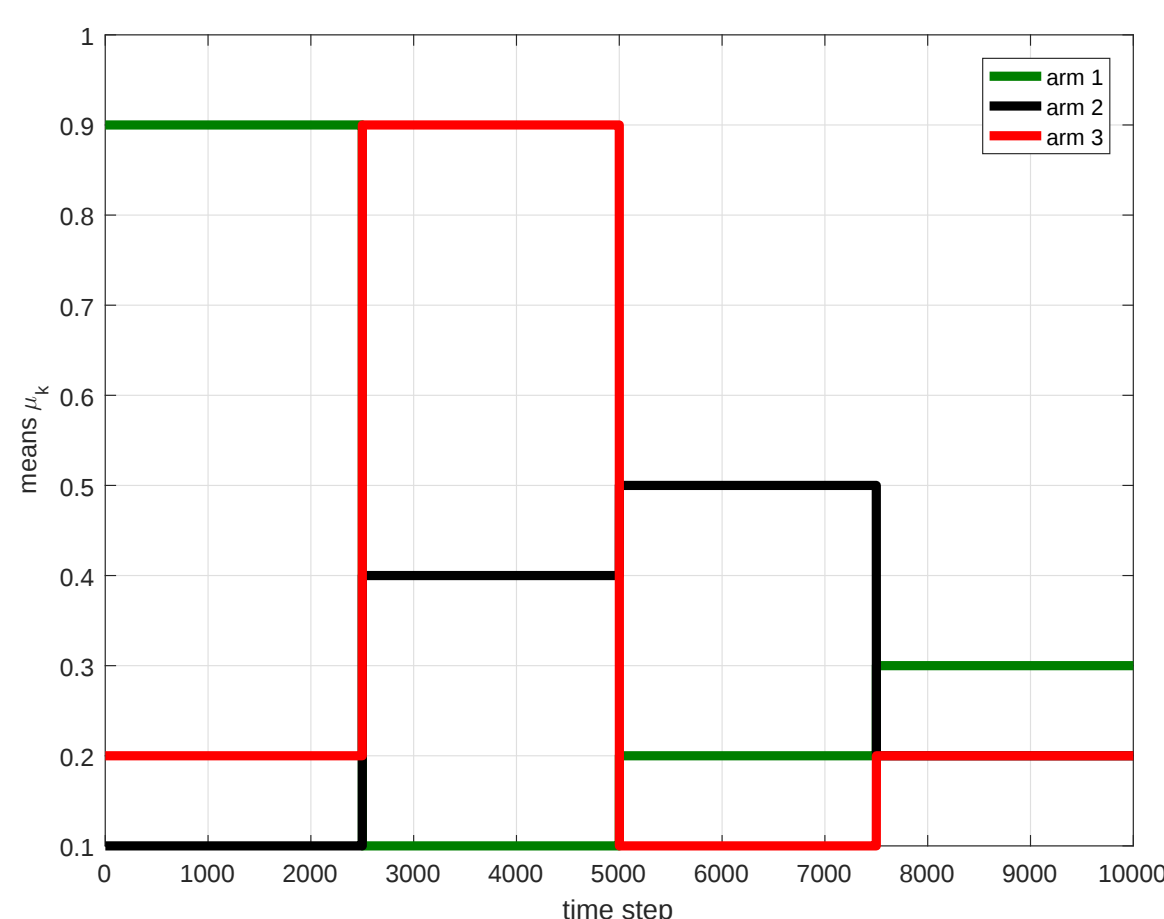
Objective: Minimize the pseudo regret $\mathcal{R}_T$:

$$\mathcal{R}_T = \underbrace{\sum_{t=1}^T \mu_t^*}_{\text{Best policy}} - \underbrace{\mathbb{E} \left[\sum_{t=1}^T x_{k_t} \right]}_{\text{Your policy}} \quad \mu_t^* = \max_k \mu_{k, t}$$

SWITCHING ENVIRONMENT

$$\mu_{k, t} = \begin{cases} \mu_{k, t-1} & \text{probability } 1 - \rho \\ \mu_{\text{new}} \sim \mathcal{U}(0, 1) & \text{probability } \rho \end{cases}$$

where ρ is the switching rate.



THOMPSON SAMPLING (TS)

$$\begin{cases} \text{success counter : } \alpha_k = \#(x_k = 1) + \alpha_0 \\ \text{failure counter : } \beta_k = \#(x_k = 0) + \beta_0 \end{cases}$$

At each step $t = 1, \dots, T$:

1. *Characterization:* $\theta_k \sim \text{Beta}(\alpha_k, \beta_k)$
2. *Decision:* $k_t = \arg \max_k \theta_k$
3. *Play:* $x_{k_t} \sim \mathcal{B}(\mu_{k_t, t})$
4. *Update:* $\begin{cases} \alpha_{k_t} = \alpha_{k_t} + 1 & \text{if } x_{k_t} = 1 \\ \beta_{k_t} = \beta_{k_t} + 1 & \text{if } x_{k_t} = 0 \end{cases}$

$$\mathcal{R}_T \leq (1 + \epsilon) \sum_k \frac{\mu_k^* - \mu_k}{KL(\mu_k, \mu_k^*)} (\log T + \log \log T)$$

(Lai and Robbins (1985) lower bound)
 $KL(\bullet, \bullet)$ = Kullback-Leibler divergence

REFERENCES

R. P. Adams and D.J.C MacKay, *Bayesian online changepoint detection*, arXiv, 2007.

J. Mellor and J. Shapiro, *Thompson Sampling in switching environments with Bayesian online changepoint detection*, AISTATS, 2013.

GLOBAL SWITCHING TS WITH BAYESIAN AGGREGATION

Learning with a growing number of Thompson Sampling $f_{i, t}$: i denotes the starting time and t the current time. $\mathbb{P}(f_{i, t})$: weight at time t of the Thompson sampling starting at time i .

Initialization: $\mathbb{P}(f_{1, 1}) = 1, t = 1,$
 $\forall k \in \mathcal{K} \alpha_{k, f_{1, 1}} = \alpha_0, \beta_{k, f_{1, 1}} = \beta_0$

-1- Decision process: at each time t :

- $\forall i \leq t, \forall k: \theta_{k, f_{i, t}} \sim \text{Beta}(\alpha_{k, f_{i, t}}, \beta_{k, f_{i, t}})$
- Play (**Bayesian Aggregation**):

$$k_t = \arg \max_k \sum_{i < t} \mathbb{P}(f_{i, t}) \theta_{k, f_{i, t}}$$

-4- Distribution of experts update:

- Update previous experts: $\mathbb{P}(f_{i, t+1}) \propto (1 - \rho) \cdot \mathbb{P}(\mathbf{x}_t | \mathbf{f}_{i, t}) \cdot \mathbb{P}(f_{i, t}) \quad \forall i \leq t$
- Create new expert $f_{t+1, t+1}$: $\mathbb{P}(f_{t+1, t+1}) \propto \rho \sum_{i=1}^t \mathbb{P}(f_{i, t})$
- Prior: $\alpha_{k, f_{i, t}} = \alpha_0, \beta_{k, f_{i, t}} = \beta_0$

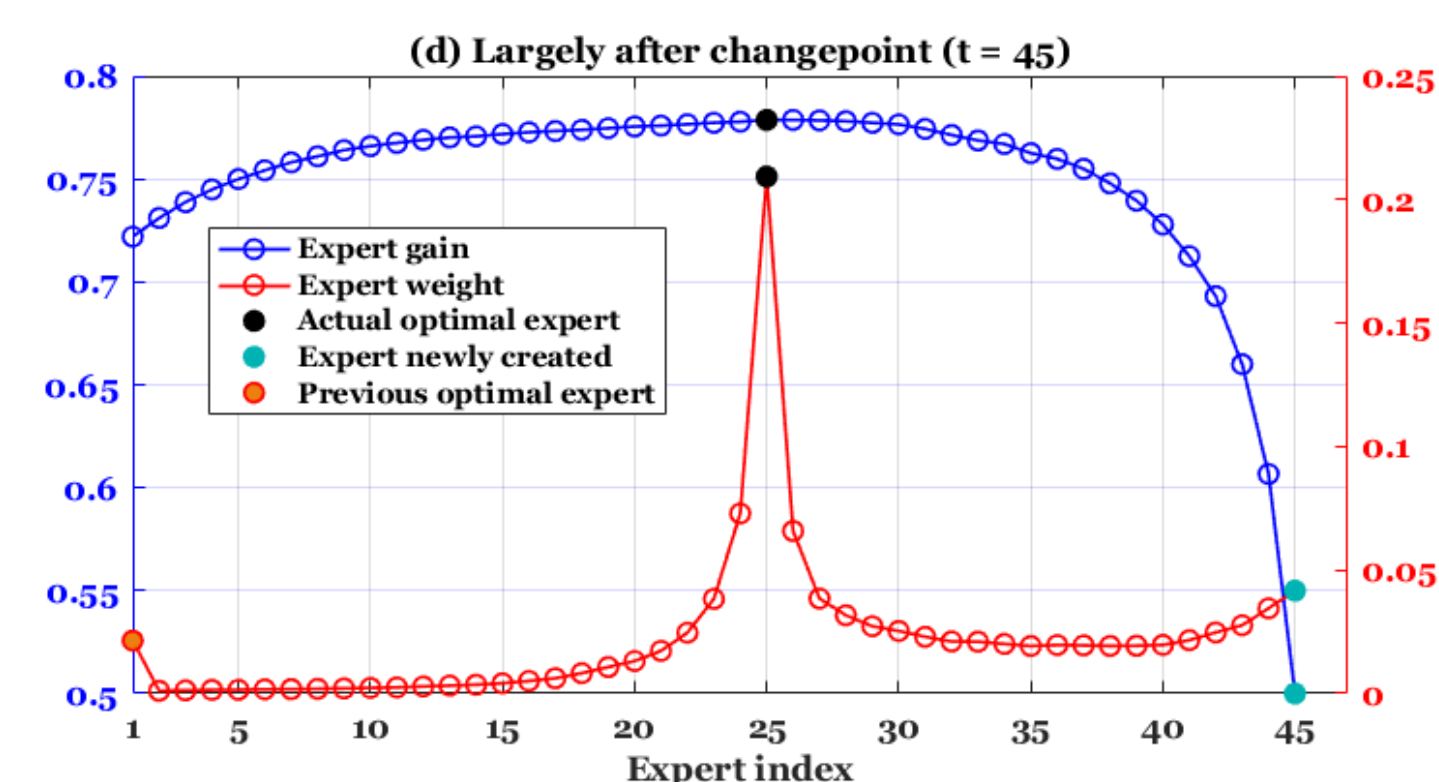
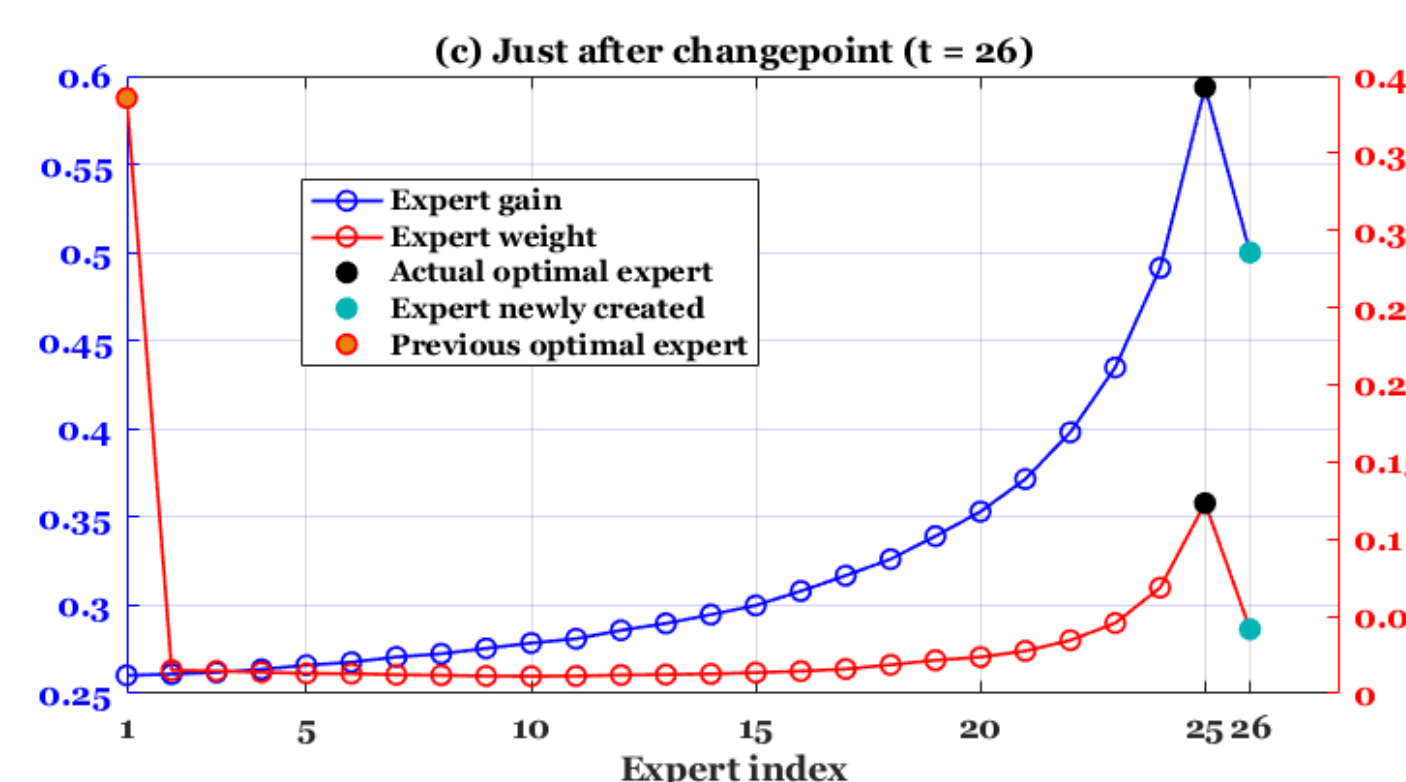
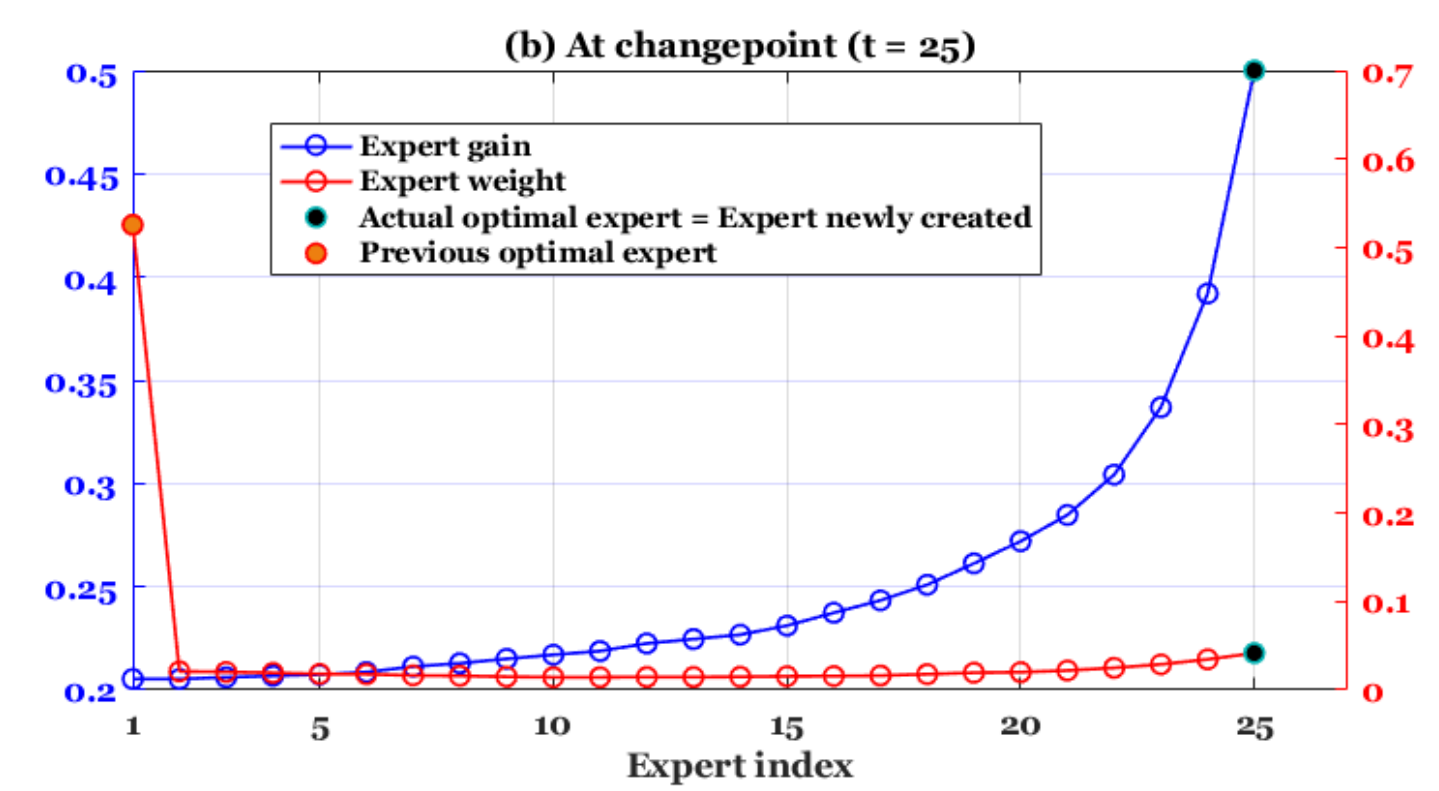
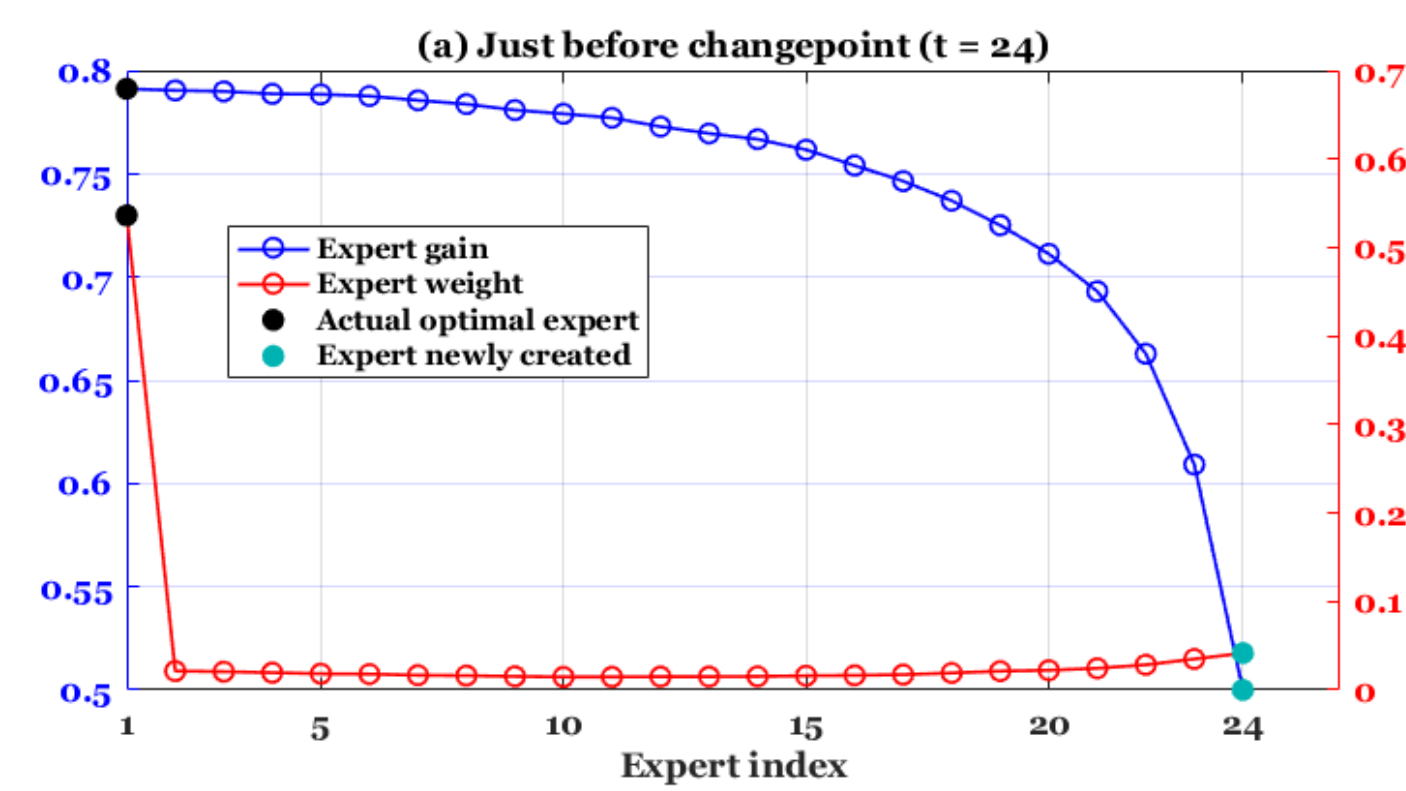
-2- Instantaneous gain update:

$$\forall i \leq t \mathbb{P}(x_t | f_{i, t}) = \begin{cases} \frac{\alpha_{k_t, f_{i, t}}}{\beta_{k_t, f_{i, t}} + \alpha_{k_t, f_{i, t}}} & \text{if } x_{k_t} = 1 \\ \frac{\beta_{k_t, f_{i, t}}}{\beta_{k_t, f_{i, t}} + \alpha_{k_t, f_{i, t}}} & \text{if } x_{k_t} = 0 \end{cases}$$

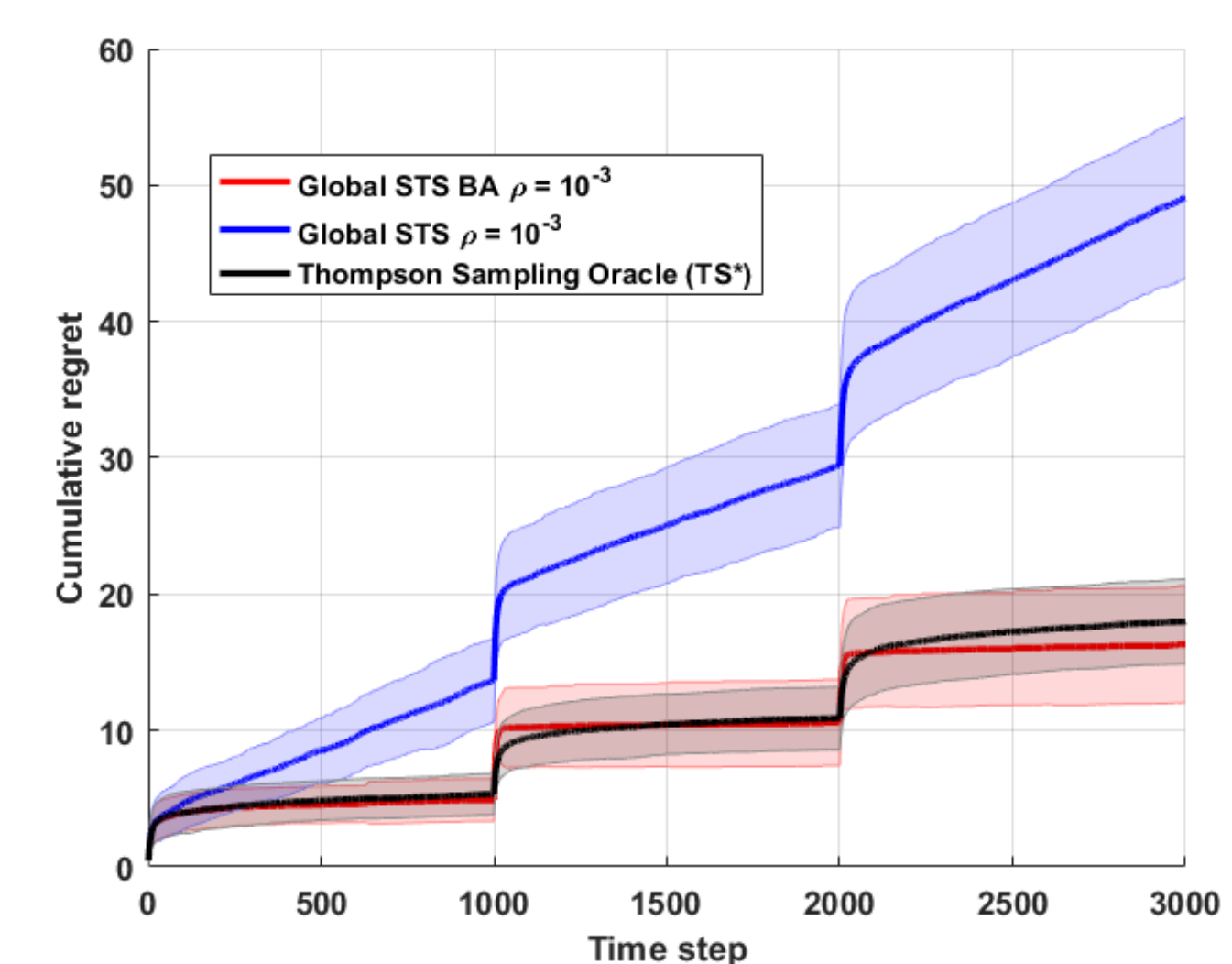
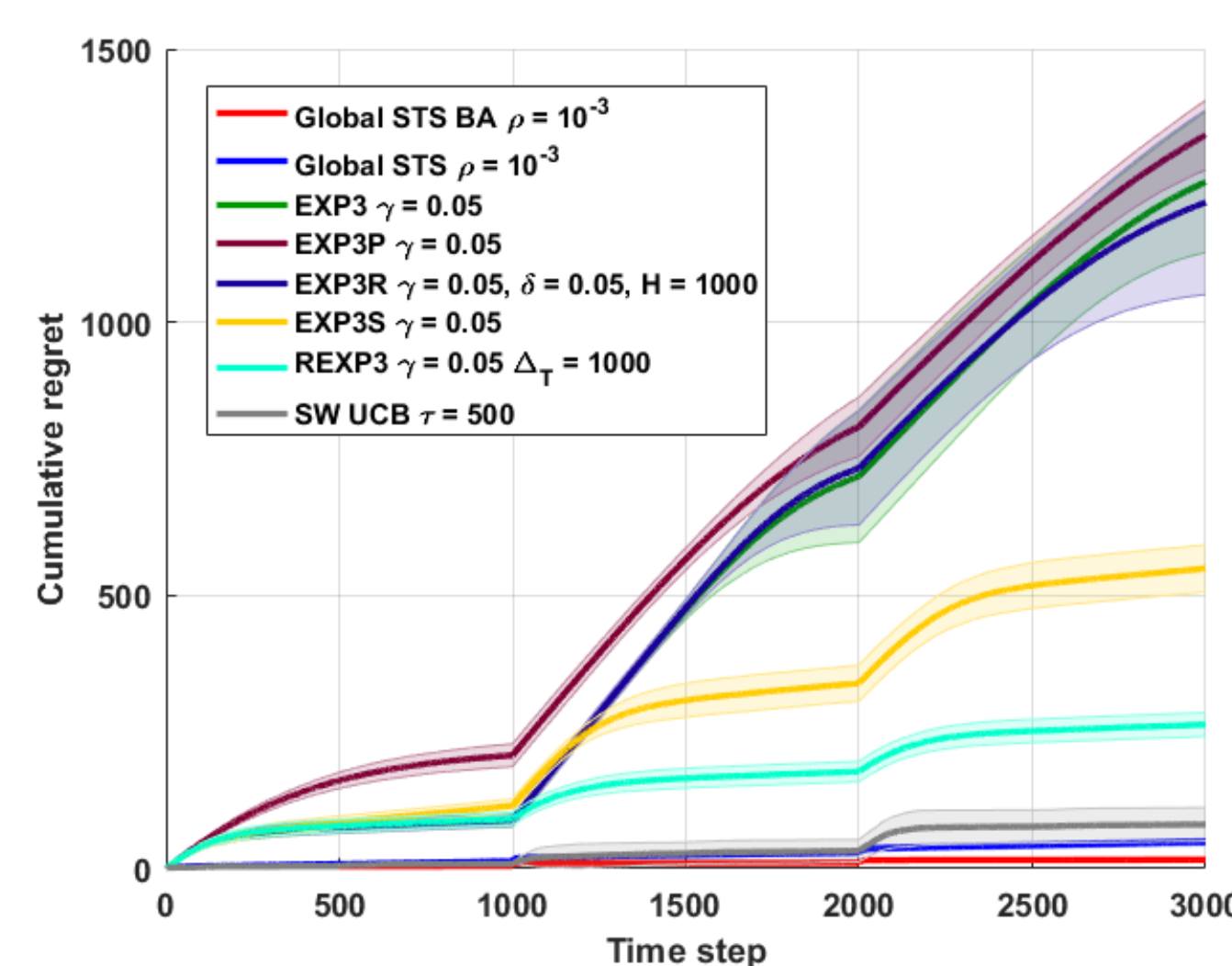
-3- Arm hyperparameters update:

$$\forall i \leq t \begin{cases} \alpha_{k_t, f_{i, t}} = \alpha_{k_t, f_{i, t}} + 1 & \text{if } x_{k_t} = 1 \\ \beta_{k_t, f_{i, t}} = \beta_{k_t, f_{i, t}} + 1 & \text{if } x_{k_t} = 0 \end{cases}$$

TRACKING THE OPTIMAL EXPERT



COMPARISON WITH STATE-OF-THE-ART



SENSITIVITY ANALYSIS OF PARAMETERS (ρ AND M)

