Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, Harold Vincent Poor

Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, and H. Vincent Poor

***Abstract*—In this paper, an achievable η-Nash equilibrium (η-NE) region for the two-user Gaussian interference channel with noisy channel-output feedback is presented for all $\eta \geqslant 1$. This result is obtained in the scenario in which each transmitter-receiver pair chooses its own transmit-receive configuration in order to maximize its own individual information transmission rate. At an η-NE, any unilateral deviation by either of the pairs does not increase the corresponding individual rate by more than η bits per channel use.**

***Index Terms*—Gaussian Interference Channel, Noisy channel-output feedback, η-Nash equilibrium region.**

I. Introduction

The interference channel (IC) is one of the simplest yet insightful multi-user channels in network information theory. An important class of ICs is the two-user Gaussian interference channel (GIC) in which there exist two point-to-point links subject to mutual interference and independent Gaussian noise sources. In this model, each output signal is a noisy version of the sum of the two transmitted signals affected by the corresponding channel gains. The analysis of this channel can be made considering two general scenarios: (1) a centralized scenario in which the entire network is controlled by a central entity that configures both transmitter-receiver pairs; and (2) a decentralized scenario in which each transmitter-receiver pair autonomously configures its transmission-reception parameters. In the former, the fundamental limits are characterized by the capacity region, which is approximated to within a fixed number of bits in [1] for the case without feedback; in [2] for the case with perfect channel-output feedback; and in [3] and [4] for the case with noisy channel-output feedback. In the latter, the fundamental limits are characterized by the η-Nash equilibrium (η-NE) region. The η-NE of the GIC is approximated in the cases without feedback and with perfect channel-output feedback in [5] and [6], respectively.

Victor Quintero is with the Department of Telecommunications, Universidad del Cauca, 19003, Popayán, Cauca, Colombia. (vflorez@unicauca.edu.co).

Samir M. Perlaza and Jean-Marie Gorce are with the laboratoire CITI (a joint laboratory between the Université de Lyon, INRIA, and INSA de Lyon). 6 Avenue des Arts, F-69621, Villeurbanne, France. ({samir.perlaza, jean-marie.gorce}@inria.fr).

H. Vincent Poor and Samir M. Perlaza are with the Department of Electrical Engineering at Princeton University, Princeton, NJ 08544 USA. (poor@princeton.edu).

This research was supported in part by the European Commission under Marie Skłodowska-Curie Individual Fellowship No. 659316; in part by the INSA Lyon - SPIE ICS chair on the Internet of Things; in part by the Administrative Department of Science, Technology, and Innovation of Colombia (Colciencias), fellowship No. 617-2013; and in part by the U. S. National Science Foundation under Grants CNS-1702808 and ECCS-1647198.

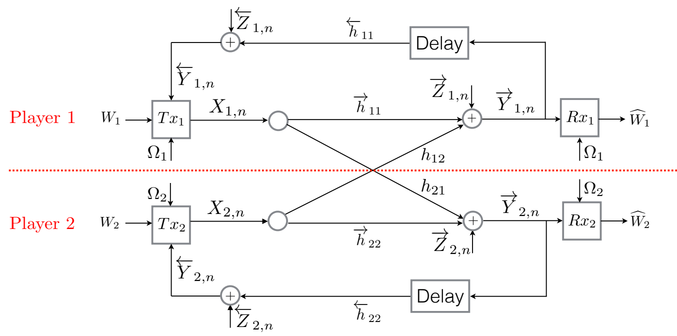

Fig. 1. Two-User Decentralized Gaussian interference channel with noisy channel-output feedback at channel use n.

In this paper the η-NE region of the GIC is studied assuming that there exists a noisy feedback link from each receiver to its corresponding transmitter. The η-NE region is approximated by two regions for all $\eta \geqslant 1$: a region for which an equilibrium transmit-receive configuration is presented for each of the information rate pairs (an achievable region); and a region for which any information rate pair that is outside of this region cannot be an η-NE (impossibility region). The focus of this paper is on the achievable region.

The results presented in this paper are a generalization of the results presented in [5] and [6], and they are obtained thanks to the analysis of linear deterministic approximations in [7] and [8].

II. Decentralized Gaussian Interference Channels with Noisy Channel-Output Feedback

Consider the two-user decentralized Gaussian interference channel with noisy channel-output feedback (D-GIC-NOF) depicted in Figure 1. Transmitter i, with $i \in \{1,2\}$, communicates with receiver i subject to the interference produced by transmitter j, with $j \in \{1,2\}\backslash\{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \ldots, \lfloor 2^{N_i R_i} \rfloor\}$, where N_i denotes the fixed block-length in channel uses and R_i the information transmission rate in bits per channel use. At each block, transmitter i sends the codeword $\boldsymbol{X}_i = (X_{i,1}, X_{i,2}, \ldots, X_{i,N_i})^{\mathsf{T}} \in \mathcal{C}_i \subseteq \mathbb{R}^{N_i}$, where $\mathcal{C}_i$ is the codebook of transmitter i. The channel coefficient from transmitter j to receiver i is denoted by h_{ij}; the channel coefficient from transmitter i to receiver i is denoted by $\overrightarrow{h}_{ii}$; and the channel coefficient from channel-output i to transmitter i is denoted by $\overleftarrow{h}_{ii}$. All channel coefficients are assumed to be non-negative real numbers. At a given channel use $n \in \{1,2,\ldots,N\}$, with

$$N = \max(N_1, N_2), \tag{1}$$

the channel output at receiver i is denoted by $\overrightarrow{Y}_{i,n}$. During channel use n, the input-output relation of the channel model is given by

$$\overrightarrow{Y}_{i,n}=\overrightarrow{h}_{ii}X_{i,n} + h_{ij}X_{j,n} + \overrightarrow{Z}_{i,n}, \tag{2}$$

where $X_{i,n} = 0$ for all n such that $N \geqslant n > N_i$ and $\overrightarrow{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise at the input of receiver i. Let $d > 0$ be the finite feedback delay measured in channel uses. At the end of channel use n, transmitter i observes $\overleftarrow{Y}_{i,n}$, which consists of a scaled and noisy version of $\overrightarrow{Y}_{i,n-d}$. More specifically,

$$\overleftarrow{Y}_{i,n}=\begin{cases}\overleftarrow{Z}_{i,n} & \text{for } n\in \{1,2,\ldots,d\}\\ \overleftarrow{h}_{ii}\overrightarrow{Y}_{i,n-d}+\overleftarrow{Z}_{i,n}, & \text{for } n\in \{d+1,d+2,\ldots,N\},\end{cases} \tag{3}$$

where $\overleftarrow{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise in the feedback link of transmitter-receiver pair i. The random variables $\overrightarrow{Z}_{i,n}$ and $\overleftarrow{Z}_{i,n}$ are assumed to be independent. In the following, without loss of generality, the feedback delay is assumed to be one channel use, i.e., $d = 1$. The encoder of transmitter i is defined by a set of deterministic functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \ldots, f_{i,N_i}^{(N)}$, with $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathcal{X}_i$ and for all $n \in \{2,3,\ldots,N_i\}$, $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \mathbb{R}^{n-1} \rightarrow \mathcal{X}_i$, such that

$$X_{i,1}=f_{i,1}^{(N)}\left(W_i, \Omega_i\right), \text{ and} \tag{4a}$$

$$X_{i,n}=f_{i,n}^{(N)}\left(W_i, \Omega_i, \overleftarrow{Y}_{i,1}, \overleftarrow{Y}_{i,2}, \ldots, \overleftarrow{Y}_{i,n-1}\right), \tag{4b}$$

where Ω_i is an additional index randomly generated. The index Ω_i is assumed to be known by both transmitter i and receiver i, while unknown by transmitter j and receiver j.

The components of the input vector $\boldsymbol{X}_i$ are real numbers subject to an average power constraint

$$\frac{1}{N_i}\sum_{n=1}^{N_i}\mathbb{E}_{X_{i,n}}\left[X_{i,n}^2\right] \leq 1. \tag{5}$$

The decoder of receiver i is defined by a deterministic function $\psi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathcal{W}_i$. At the end of the communication, receiver i uses the vector $\left(\overrightarrow{Y}_{i,1},\ \overrightarrow{Y}_{i,2},\ \ldots,\ \overrightarrow{Y}_{i,N}\right)$ and the index Ω_i to obtain an estimate

$$\widehat{W}_i=\psi_i^{(N)}\left(\Omega_i, \overrightarrow{Y}_{i,1}, \overrightarrow{Y}_{i,2}, \ldots, \overrightarrow{Y}_{i,N}\right), \tag{6}$$

of the message index W_i. A *transmit-receive configuration* for transmitter-receiver pair i, denoted by s_i, can be described in terms of the block-length N_i, the rate R_i, the codebook $\mathcal{C}_i$, the encoding functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \ldots, f_{i,N_i}^{(N)}$, and the decoding function $\psi_i^{(N)}$, etc. The average error probability at decoder i given the configurations s_1 and s_2, denoted by $p_i(s_1,s_2)$, is given by

$$p_i(s_1,s_2)=\Pr\left[W_i \neq \widehat{W}_i\right]. \tag{7}$$

Within this context, a rate pair $(R_1,R_2) \in \mathbb{R}_+^2$ is said to be achievable if it complies with the following definition.

Definition 1 (Achievable Rate Pairs): A rate pair $(R_1,R_2) \in \mathbb{R}_+^2$ is achievable if there exists at least one pair of configurations (s_1,s_2) such that the decoding bit error probabilities $p_1(s_1,s_2)$ and $p_2(s_1,s_2)$ can be made arbitrarily small by letting the block-lengths N_1 and N_2 grow to infinity.

The aim of transmitter i is to autonomously choose its transmit-receive configuration s_i in order to maximize its achievable rate R_i. Note that the rate achieved by transmitter-receiver i depends on both configurations s_1 and s_2 due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel. The fundamental limits of the two-user D-GIC-NOF in Figure 1 can be described by six parameters: $\overrightarrow{\mathrm{SNR}}_i$, $\overleftarrow{\mathrm{SNR}}_i$, and INR_{ij}, with $i \in \{1,2\}$ and $j \in \{1,2\}\setminus\{i\}$, which are defined as follows:

$$\overrightarrow{\mathrm{SNR}}_i\triangleq\overrightarrow{h}_{ii}^{2}, \tag{8}$$

$$\mathrm{INR}_{ij}\triangleq h_{ij}^2 \text{ and} \tag{9}$$

$$\overleftarrow{\mathrm{SNR}}_i\triangleq\overleftarrow{h}_{ii}^{2}\left(\overrightarrow{h}_{ii}^{2} + 2\overrightarrow{h}_{ii}h_{ij} + h_{ij}^2 + 1\right). \tag{10}$$

The analysis presented in this paper focuses exclusively on the case in which $\mathrm{INR}_{ij} > 1$ for all $i \in \{1,2\}$ and $j \in \{1,2\} \setminus \{i\}$. The reason for exclusively considering this case follows from the fact that when $\mathrm{INR}_{ij} \leqslant 1$, the transmitter-receiver pair i is impaired mainly by noise instead of interference. In this case, feedback does not bring a significant rate improvement. Denote by $\mathcal{C}$ the capacity region of the two-user GIC-NOF with fixed parameters $\overrightarrow{\mathrm{SNR}}_1$, $\overrightarrow{\mathrm{SNR}}_2$, INR_{12}, INR_{21}, $\overleftarrow{\mathrm{SNR}}_1$, and $\overleftarrow{\mathrm{SNR}}_2$. The achievable region $\underline{\mathcal{C}}$ in [4, Theorem 2] and the converse region $\overline{\mathcal{C}}$ in [4, Theorem3] approximate the capacity region $\mathcal{C}$ to within 4.4 bits [4].

III. Game Formulation

The competitive interaction between the two transmitter-receiver pairs in the interference channel can be modeled by the following game in normal-form:

$$\mathcal{G} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right). \tag{11}$$

The set $\mathcal{K} = \{1,2\}$ is the set of players, that is, the set of transmitter-receiver pairs. The sets $\mathcal{A}_1$ and $\mathcal{A}_2$ are the sets of actions of players 1 and 2, respectively. An action of a player $i \in \mathcal{K}$, which is denoted by $s_i \in \mathcal{A}_i$, is basically its transmit-receive configuration as described above. The utility function of player i is $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$ and it is defined as the information rate of transmitter i,

$$u_i(s_1,s_2) = \begin{cases} R_i, & \text{if } \quad p_i(s_1,s_2) < \epsilon \\ 0, & \quad\quad\ \text{otherwise,}\end{cases} \tag{12}$$

where $\epsilon > 0$ is an arbitrarily small number. This game formulation for the case without feedback was first proposed in [9] and [10].

A class of transmit-receive configurations that are particularly important in the analysis of this game is referred to as

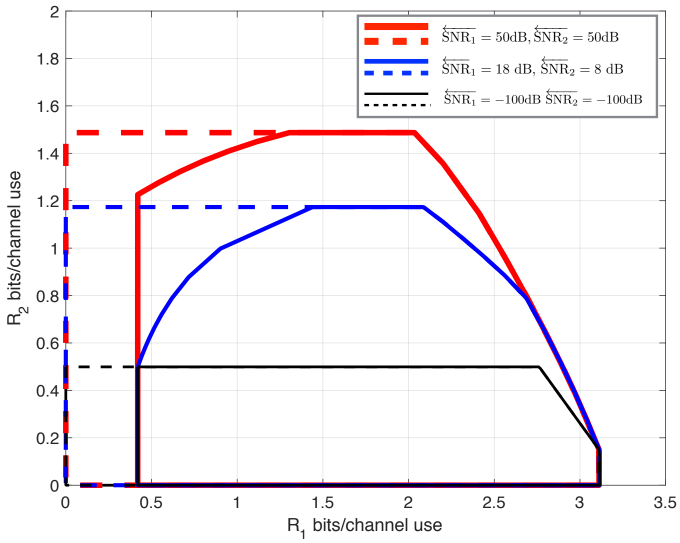

Fig. 2. Achievable capacity regions $\mathcal{C}$ (dashed-lines) in [4, Theorem 2] and achievable η-NE regions $\underline{\mathcal{N}}_\eta$ (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 3$ dB, $\mathrm{INR}_{12} = 16$ dB, $\mathrm{INR}_{21} = 9$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$.

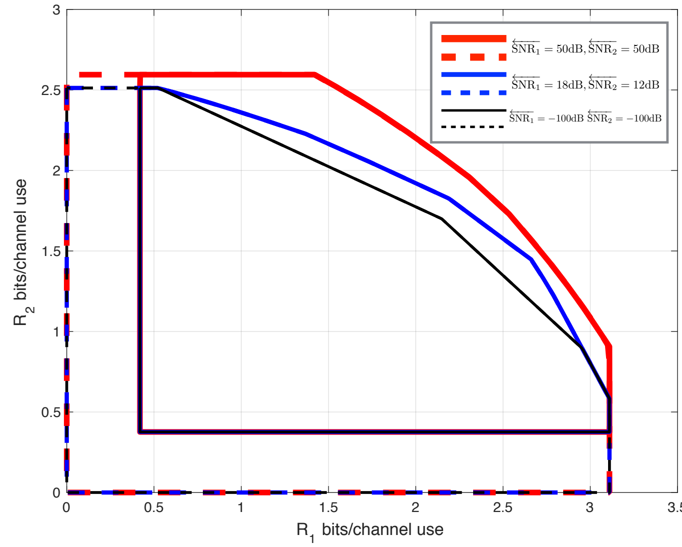

Fig. 3. Achievable capacity regions $\mathcal{C}$ (dashed-lines) in [4, Theorem 2] and achievable η-NE regions $\underline{\mathcal{N}}_\eta$ (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 18$ dB, $\mathrm{INR}_{12} = 16$ dB, $\mathrm{INR}_{21} = 10$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

the set of η-Nash equilibria (η-NE), with $\eta > 0$. This type of configurations satisfy the following definition.

Definition 2 (η-Nash equilibrium): Given a positive real η, an action profile (s_1^*, s_2^*) is an η-Nash equilibrium (NE) in the game $\mathcal{G} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right)$, if for all $i \in \mathcal{K}$ and for all $s_i \in \mathcal{A}_i$, it follows that

$$u_i(s_i, s_j^*) \leqslant u_i(s_i^*, s_j^*) + \eta. \tag{13}$$

Let (s_1^*, s_2^*) be an η-Nash equilibrium action profile. Then, none of the transmitters can increase its own transmission rate more than η bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for η sufficiently large, from Definition 2, any pair of configurations can be an η-NE. Alternatively, for $\eta = 0$, the definition of Nash equilibrium is obtained [11]. In this case, if a pair of configurations is a Nash equilibrium ($\eta = 0$), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible η-NE rate pairs (R_1, R_2) of the game in (11) with the smallest η for which there exists at least one equilibrium configuration pair.

The set of rate pairs that can be achieved at an η-NE is known as the η-Nash equilibrium (η-NE) region.

Definition 3 (η-NE Region): Let $\eta > 0$ be fixed. An achievable rate pair (R_1, R_2) is said to be in the η-NE region of the game $\mathcal{G} = \left(\mathcal{K}, \{\mathcal{A}_k\}_{k\in\mathcal{K}}, \{u_k\}_{k\in\mathcal{K}}\right)$ if there exists a pair $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that is an η-NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \textit{and} \quad u_2(s_1^*, s_2^*) = R_2. \tag{14}$$

The η-NE regions of the two-user GIC with and without perfect channel-output feedback have been approximated to within a constant number of bits in [5] and [6], respectively. The next section introduces a generalization of these results.

IV. Main Results

A. Achievable η-Nash Equilibrium Region

Let the η-NE region (Definition 3) of the D-GIC-NOF be denoted by $\mathcal{N}_\eta$. This section introduces a region $\underline{\mathcal{N}}_\eta \subseteq \mathcal{N}_\eta$ that is achievable using a coding scheme that combines rate splitting [12], common randomness [5], [6], block Markov superposition coding [13] and backward decoding [14]. In the following, this coding scheme is referred to as randomized Han-Kobayashi scheme with noisy channel-output feedback (RHK-NOF). This coding scheme is presented in [8] and uses the same techniques of the schemes in [5] and [6]. Therefore, the focus of this section is on the results rather than the description of the scheme. A motivated reader is referred to [15]. The RHK-NOF is proved to be an η-NE action profile with $\eta \geqslant 1$. That is, any unilateral deviation from the RHK-NOF by any of the transmitter-receiver pairs might lead to an individual rate improvement which is at most one bit per channel use. The description of the achievable η-Nash region $\underline{\mathcal{N}}_\eta$ is presented using the constants $a_{1,i}$; the functions $a_{2,i} : [0,1] \to \mathbb{R}_+$, $a_{l,i} : [0,1]^2 \to \mathbb{R}_+$, with $l \in \{3, \ldots, 6\}$; and $a_{7,i} : [0,1]^3 \to \mathbb{R}_+$, which are defined as follows, for all $i \in \{1,2\}$, with $j \in \{1,2\} \setminus \{i\}$:

$$a_{1,i} = \frac{1}{2}\log\left(2 + \frac{\overrightarrow{\mathrm{SNR}}_i}{\mathrm{INR}_{ji}}\right) - \frac{1}{2}, \tag{15a}$$

$$a_{2,i}(\rho) = \frac{1}{2}\log\Big(b_{1,i}(\rho) + 1\Big) - \frac{1}{2}, \tag{15b}$$

$$a_{3,i}(\rho,\mu) = \frac{1}{2}\log\left(\frac{\overleftarrow{\mathrm{SNR}}_i\Big(b_{2,i}(\rho) + 2\Big) + b_{1,i}(1) + 1}{\overleftarrow{\mathrm{SNR}}_i\Big((1-\mu)b_{2,i}(\rho) + 2\Big) + b_{1,i}(1) + 1}\right), \tag{15c}$$

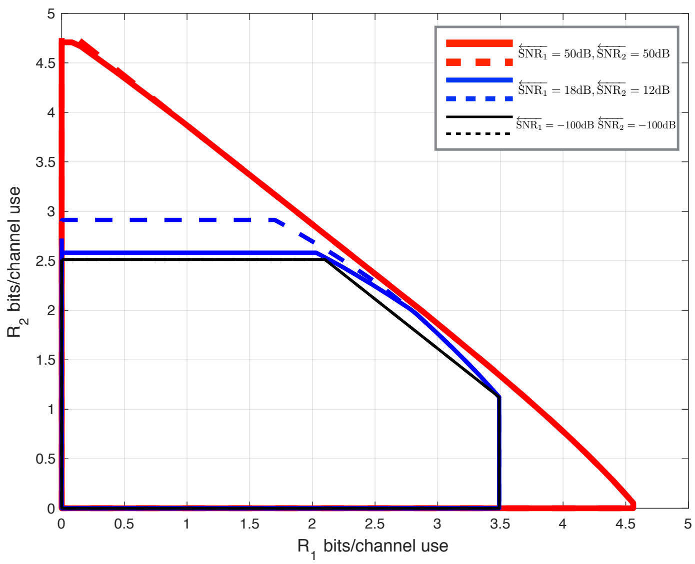

Fig. 4. Achievable capacity regions $\underline{\mathcal{C}}$ (dashed-lines) in [4, Theorem 2] and achievable η-NE regions $\underline{\mathcal{N}}_\eta$ (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 18$ dB, $\mathrm{INR}_{12} = 48$ dB, $\mathrm{INR}_{21} = 30$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

$$a_{4,i}(\rho,\mu)=\frac{1}{2}\log\left(\Big(1-\mu\Big)b_{2,i}(\rho)+2\right)-\frac{1}{2}, \qquad (15\text{d})$$

$$a_{5,i}(\rho,\mu)=\frac{1}{2}\log\left(2+\frac{\overrightarrow{\mathrm{SNR}}_i}{\mathrm{INR}_{ji}}+\Big(1-\mu\Big)b_{2,i}(\rho)\right)-\frac{1}{2}, \qquad (15\text{e})$$

$$a_{6,i}(\rho,\mu)=\frac{1}{2}\log\left(\frac{\overrightarrow{\mathrm{SNR}}_i}{\mathrm{INR}_{ji}}\left(\Big(1-\mu\Big)b_{2,j}(\rho)+1\right)+2\right)-\frac{1}{2},$$

$$a_{7,i}(\rho,\mu_1,\mu_2)=\frac{1}{2}\log\left(\frac{\overrightarrow{\mathrm{SNR}}_i}{\mathrm{INR}_{ji}}\left(\Big(1-\mu_i\Big)b_{2,j}(\rho)+1\right)+\Big(1-\mu_j\Big)b_{2,i}(\rho)+2\right)-\frac{1}{2}, \qquad (15\text{f})$$

where the functions $b_{l,i} : [0,1] \rightarrow \mathbb{R}_+$, with $l \in \{1,2\}$ are defined as follows:

$$b_{1,i}(\rho)=\overrightarrow{\mathrm{SNR}}_i + 2\rho\sqrt{\overrightarrow{\mathrm{SNR}}_i\mathrm{INR}_{ij}} + \mathrm{INR}_{ij} \text{ and} \qquad (16\text{a})$$

$$b_{2,i}(\rho)=\Big(1-\rho\Big)\mathrm{INR}_{ij}-1. \qquad (16\text{b})$$

Note that the functions in (15) and (16) depend on $\overrightarrow{\mathrm{SNR}}_1$, $\overrightarrow{\mathrm{SNR}}_2$, INR_{12}, INR_{21}, $\overleftarrow{\mathrm{SNR}}_1$, and $\overleftarrow{\mathrm{SNR}}_2$, however as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, the achievable η-NE region is presented by Theorem 1 on the next page. The proof of Theorem 1 is presented in [8]. The inequalities in (17) are additional conditions to those defining the region $\underline{\mathcal{C}}$ in [4, Theorem 2]. More specifically, the η-NE region is described by the intersection of the achievable region $\underline{\mathcal{C}}$ and the set of rate pairs (R_1, R_2) satisfying (17).

Figure 2 shows the achievable region $\underline{\mathcal{C}}$ in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η-NE region $\underline{\mathcal{N}}_\eta$ in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 3$ dB, $\mathrm{INR}_{12} = 16$ dB, $\mathrm{INR}_{21} = 9$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$. Note that in this case, the feedback parameter $\overleftarrow{\mathrm{SNR}}_2$ does not have an effect on the achievable η-NE region $\underline{\mathcal{N}}_\eta$ and the achievable capacity region $\underline{\mathcal{C}}$ ([4, Theorem 2]). This is due to the fact that when one transmitter-receiver pair is in low interference regime (LIR) and the other transmitter-receiver pair is in high interference regime (HIR), feedback is useless on the transmitter-receiver pair in HIR [15], [16].

Figure 3 shows the achievable region $\underline{\mathcal{C}}$ in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η-NE region $\underline{\mathcal{N}}_\eta$ in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 18$ dB, $\mathrm{INR}_{12} = 16$ dB, $\mathrm{INR}_{21} = 10$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. Figure 4 shows the achievable region $\underline{\mathcal{C}}$ in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η-NE region $\underline{\mathcal{N}}_\eta$ in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\mathrm{SNR}}_1 = 24$ dB, $\overrightarrow{\mathrm{SNR}}_2 = 18$ dB, $\mathrm{INR}_{12} = 48$ dB, $\mathrm{INR}_{21} = 30$ dB, $\overleftarrow{\mathrm{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overleftarrow{\mathrm{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. In this case, the achievable η-NE region $\underline{\mathcal{N}}_\eta$ in Theorem 1 and achievable region $\underline{\mathcal{C}}$ on the capacity region [4, Theorem 2] are almost identical, which implies that in the cases in which $\overrightarrow{\mathrm{SNR}}_i < \mathrm{INR}_{ij}$, for both $i \in \{1,2\}$, with $j \in \{1,2\} \setminus \{i\}$, the achievable η-NE region is almost the same as the achievable capacity region in the centralized case studied in [4]. At low values of $\overleftarrow{\mathrm{SNR}}_1$ and $\overleftarrow{\mathrm{SNR}}_2$, the achievable η-NE region approaches the rectangular region reported in [5] for the case of the two-user decentralized GIC (D-GIC). Alternatively, for high values of $\overleftarrow{\mathrm{SNR}}_1$ and $\overleftarrow{\mathrm{SNR}}_2$, the achievable η-NE region approaches the region reported in [6] for the case of the two-user decentralized GIC with perfect channel-output feedback (D-GIC-POF). These observations are formalized by the following corollaries.

Denote by $\underline{\mathcal{N}}_{\eta\mathrm{PF}}$ the achievable η-NE region of the two-user D-GIC-POF presented in [6]. The region $\underline{\mathcal{N}}_{\eta\mathrm{PF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 1 (η-NE Region with Perfect Output Feedback): Let $\underline{\mathcal{N}}_{\eta\mathrm{PF}}$ denote the achievable η-NE region of the two-user D-GIC-POF with fixed parameters $\overrightarrow{\mathrm{SNR}}_i$ and INR_{ij}, with $i \in \{1,2\}$ and $j \in \{1,2\} \setminus \{i\}$. Then, the following holds:

$$\underline{\mathcal{N}}_{\eta\mathrm{PF}} = \lim_{\substack{\overleftarrow{\mathrm{SNR}}_1 \rightarrow \infty \\ \overleftarrow{\mathrm{SNR}}_2 \rightarrow \infty}} \underline{\mathcal{N}}_\eta. \qquad (18)$$

Denote by $\underline{\mathcal{N}}_{\eta\mathrm{WF}}$ the achievable η-NE region of the two-user D-GIC presented in [5]. The region $\underline{\mathcal{N}}_{\eta\mathrm{WF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 2 (η-NE Region without Output Feedback): Let $\underline{\mathcal{N}}_{\eta\mathrm{WF}}$ denote the achievable η-NE region of the two-user D-GIC, with fixed parameters $\overrightarrow{\mathrm{SNR}}_i$ and INR_{ij}, with $i \in \{1,2\}$

Theorem 1: Let $\eta \geqslant 1$ be fixed. The achievable η-NE region $\underline{\mathcal{N}}_\eta$ is given by the closure of all possible achievable rate pairs $(R_1, R_2) \in \underline{\mathcal{C}}$ in [4, Theorem 2] that satisfy, for all $i \in \{1,2\}$ and $j \in \{1,2\} \setminus \{i\}$, the following conditions:

$$R_i \geqslant \Big(a_{2,i}(\rho) - a_{3,i}(\rho,\mu_j) - a_{4,i}(\rho,\mu_j) - \eta\Big)^+, \tag{17a}$$

$$\begin{aligned} R_i \leqslant \min\Big(& a_{2,i}(\rho) + a_{3,j}(\rho,\mu_i) + a_{5,j}(\rho,\mu_i) - a_{2,j}(\rho) + \eta, \\ & a_{3,i}(\rho,\mu_j) + a_{7,i}(\rho,\mu_1,\mu_2) + 2a_{3,j}(\rho,\mu_i) + a_{5,j}(\rho,\mu_i) - a_{2,j}(\rho) + \eta, \\ & a_{2,i}(\rho) + a_{3,i}(\rho,\mu_j) + 2a_{3,j}(\rho,\mu_i) + a_{5,j}(\rho,\mu_i) + a_{7,j}(\rho,\mu_1,\mu_2) - 2a_{2,j}(\rho) + 2\eta\Big), \end{aligned} \tag{17b}$$

$$R_1 + R_2 \leqslant a_{1,i} + a_{3,i}(\rho,\mu_j) + a_{7,i}(\rho,\mu_1,\mu_2) + a_{2,j}(\rho) + a_{3,j}(\rho,\mu_1) - a_{2,i}(\rho) + \eta, \tag{17c}$$

for all $(\rho,\mu_1,\mu_2) \in \Big[0, \Big(1 - \max\Big(\frac{1}{\mathrm{INR}_{12}}, \frac{1}{\mathrm{INR}_{21}}\Big)\Big)^+\Big] \times [0,1] \times [0,1]$.

---

and $j \in \{1,2\} \backslash \{i\}$. Then, the following holds:

$$\underline{\mathcal{N}}_{\eta \mathrm{WF}} = \lim_{\substack{\overleftarrow{\mathrm{SNR}}_1 \to 0 \\ \overleftarrow{\mathrm{SNR}}_2 \to 0 \\ \rho = 0.}} \underline{\mathcal{N}}_\eta. \tag{19}$$

B. Imposibility Region

This section introduces an imposibility region, denoted by $\overline{\mathcal{N}}_\eta$. That is, $\overline{\mathcal{N}}_\eta \supseteq \mathcal{N}_\eta$. More specifically, any rate pair $(R_1, R_2) \in \overline{\mathcal{N}}_\eta^{\mathsf{c}}$ is not an η-NE. This region is described in terms of the convex region $\overline{\mathcal{B}}_{\mathrm{G-IC-NOF}}$. Here, for the case of the two-user D-GIC-NOF, the region $\overline{\mathcal{B}}_{\mathrm{G-IC-NOF}}$ is given by the closure of the rate pairs $(R_1, R_2) \in \mathbb{R}_+^2$ that satisfy for all $i \in \{1,2\}$, with $j \in \{1,2\} \setminus \{i\}$:

$$\overline{\mathcal{B}}_{\mathrm{G-IC-NOF}} = \Big\{(R_1, R_2) \in \mathbb{R}_+^2 : R_i \geqslant L_i, \text{ for all } i \in \mathcal{K} = \{1,2\}\Big\}, \tag{20}$$

where,

$$L_i \triangleq \left(\frac{1}{2}\log\left(1 + \frac{\overrightarrow{\mathrm{SNR}}_i}{1 + \mathrm{INR}_{ij}}\right) - \eta\right)^+. \tag{21}$$

Note that L_i is the rate achieved by the transmitter-receiver pair i when it saturates the power constraint in (5) and treats interference as noise. Following this notation, the imposibility region of the two-user GIC-NOF, i.e., $\overline{\mathcal{N}}_\eta$, can be described as follows.

Theorem 2: Let $\eta \geqslant 1$ be fixed. The imposibility region $\overline{\mathcal{N}}_\eta$ of the two-user D-GIC-NOF is given by the closure of all possible non-negative rate pairs $(R_1, R_2) \in \overline{\mathcal{C}} \cap \overline{\mathcal{B}}_{\mathrm{G-IC-NOF}}$ for all $\rho \in [0,1]$.

The impossibility region in Theorem 2 has been first presented in [6] and it is very loose in this case. A better impossibility region is presented in [15].

V. Conclusions

In this paper, an achievable η-Nash equilibrium (η-NE) region for the two-user Gaussian interference channel with noisy channel-output feedback has been presented for all $\eta \geqslant 1$. This result generalizes the existing achievable regions of the η-NE for the the cases without feedback and with perfect channel-output feedback.

References

[1] R. H. Etkin, D. N. C. Tse, and W. Hua, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[2] C. Suh and D. N. C. Tse, "Feedback capacity of the Gaussian interference channel to within 2 bits," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2667–2685, May. 2011.

[3] S.-Q. Le, R. Tandon, M. Motani, and H. V. Poor, "Approximate capacity region for the symmetric Gaussian interference channel with noisy feedback," *IEEE Trans. Inf. Theory*, vol. 61, no. 7, pp. 3737–3762, Jul. 2015.

[4] V. Quintero, S. M. Perlaza, I. Esnaola, and J.-M. Gorce, "Approximate capacity region of the two-user Gaussian interference channel with noisy channel-output feedback," *IEEE Trans. Inf. Theory*, vol. 64, no. 7, pp. 5326–5358, Jul 2018.

[5] R. A. Berry and D. N. C. Tse, "Shannon meets Nash on the interference channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2821–2836, May. 2011.

[6] S. M. Perlaza, R. Tandon, H. V. Poor, and Z. Han, "Perfect output feedback in the two-user decentralized interference channel," *IEEE Trans. Inf. Theory*, vol. 61, no. 10, pp. 5441–5462, Oct. 2015.

[7] V. Quintero, S. M. Perlaza, J.-M. Gorce, and H. V. Poor, "Nash region of the linear deterministic interference channel with noisy output feedback," in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT)*, Aachen, Germany, Jun. 2017.

[8] ——, "Decentralized interference channels with noisy output feedback," INRIA, Lyon, France, Tech. Rep. 9011, Jan. 2017.

[9] R. D. Yates, D. Tse, and Z. Li, "Secret communication on interference channels," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Toronto, Canada, Jul. 2008.

[10] R. Berry and D. N. C. Tse, "Information theoretic games on interference channels," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Toronto,Canada, Jul. 2008.

[11] J. F. Nash, "Equilibrium points in n-person games," *Proc. National Academy of Sciences of the United States of America*, vol. 36, no. 1, pp. 48–49, Jan. 1950.

[12] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.

[13] T. M. Cover and C. S. K. Leung, "An achievable rate region for the multiple-access channel with feedback," *IEEE Trans. Inf. Theory*, vol. 27, no. 3, pp. 292–298, May. 1981.

[14] F. M. J. Willems, "Information theoretical results for multiple access channels," Ph.D. dissertation, Katholieke Universiteit, Leuven, Belgium, Oct. 1982.

[15] V. Quintero, "Noisy channel-output feedback in the interference channel," Ph.D. dissertation, Université de Lyon, Lyon, France., Dec. 2017.

[16] V. Quintero, S. M. Perlaza, I. Esnaola, and J.-M. Gorce, "When does output feedback enlarge the capacity of the interference channel?" *IEEE Trans. Commun.*, vol. 66, no. 2, pp. 615–628, Feb. 2018.