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# Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, and H. Vincent Poor

**Abstract**—In this paper, an achievable  $\eta$ -Nash equilibrium ( $\eta$ -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback is presented for all  $\eta \geq 1$ . This result is obtained in the scenario in which each transmitter-receiver pair chooses its own transmit-receive configuration in order to maximize its own individual information transmission rate. At an  $\eta$ -NE, any unilateral deviation by either of the pairs does not increase the corresponding individual rate by more than  $\eta$  bits per channel use.

**Index Terms**—Gaussian Interference Channel, Noisy channel-output feedback,  $\eta$ -Nash equilibrium region.

## I. INTRODUCTION

The interference channel (IC) is one of the simplest yet insightful multi-user channels in network information theory. An important class of ICs is the two-user Gaussian interference channel (GIC) in which there exist two point-to-point links subject to mutual interference and independent Gaussian noise sources. In this model, each output signal is a noisy version of the sum of the two transmitted signals affected by the corresponding channel gains. The analysis of this channel can be made considering two general scenarios: (1) a centralized scenario in which the entire network is controlled by a central entity that configures both transmitter-receiver pairs; and (2) a decentralized scenario in which each transmitter-receiver pair autonomously configures its transmission-reception parameters. In the former, the fundamental limits are characterized by the capacity region, which is approximated to within a fixed number of bits in [1] for the case without feedback; in [2] for the case with perfect channel-output feedback; and in [3] and [4] for the case with noisy channel-output feedback. In the latter, the fundamental limits are characterized by the  $\eta$ -Nash equilibrium ( $\eta$ -NE) region. The  $\eta$ -NE of the GIC is approximated in the cases without feedback and with perfect channel-output feedback in [5] and [6], respectively.

Victor Quintero is with the Department of Telecommunications, Universidad del Cauca, 19003, Popayán, Cauca, Colombia. (vflorez@unicauca.edu.co).

Samir M. Perlaza and Jean-Marie Gorce are with the laboratoire CITI (a joint laboratory between the Université de Lyon, INRIA, and INSA de Lyon), 6 Avenue des Arts, F-69621, Villeurbanne, France. ({samir.perlaza, jean-marie.gorce}@inria.fr).

H. Vincent Poor and Samir M. Perlaza are with the Department of Electrical Engineering at Princeton University, Princeton, NJ 08544 USA. (poor@princeton.edu).

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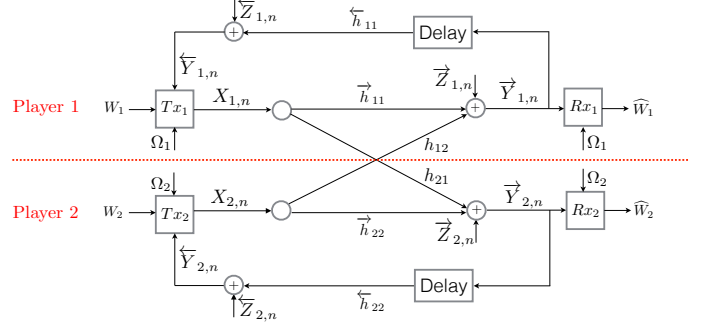


Fig. 1. Two-User Decentralized Gaussian interference channel with noisy channel-output feedback at channel use  $n$ .

In this paper the  $\eta$ -NE region of the GIC is studied assuming that there exists a noisy feedback link from each receiver to its corresponding transmitter. The  $\eta$ -NE region is approximated by two regions for all  $\eta \geq 1$ : a region for which an equilibrium transmit-receive configuration is presented for each of the information rate pairs (an achievable region); and a region for which any information rate pair that is outside of this region cannot be an  $\eta$ -NE (impossibility region). The focus of this paper is on the achievable region.

The results presented in this paper are a generalization of the results presented in [5] and [6], and they are obtained thanks to the analysis of linear deterministic approximations in [7] and [8].

## II. DECENTRALIZED GAUSSIAN INTERFERENCE CHANNELS WITH NOISY CHANNEL-OUTPUT FEEDBACK

Consider the two-user decentralized Gaussian interference channel with noisy channel-output feedback (D-GIC-NOF) depicted in Figure 1. Transmitter  $i$ , with  $i \in \{1, 2\}$ , communicates with receiver  $i$  subject to the interference produced by transmitter  $j$ , with  $j \in \{1, 2\} \setminus \{i\}$ . There are two independent and uniformly distributed messages,  $W_i \in \mathcal{W}_i$ , with  $\mathcal{W}_i = \{1, 2, \dots, \lfloor 2^{N_i R_i} \rfloor\}$ , where  $N_i$  denotes the fixed block-length in channel uses and  $R_i$  the information transmission rate in bits per channel use. At each block, transmitter  $i$  sends the codeword  $\mathbf{X}_i = (X_{i,1}, X_{i,2}, \dots, X_{i,N_i})^T \in \mathcal{C}_i \subseteq \mathcal{R}^{N_i}$ , where  $\mathcal{C}_i$  is the codebook of transmitter  $i$ . The channel coefficient from transmitter  $j$  to receiver  $i$  is denoted by  $h_{ij}$ ; the channel coefficient from transmitter  $i$  to receiver  $i$  is denoted by  $\bar{h}_{ii}$ ; and the channel coefficient from channel-output  $i$  to transmitter  $i$  is denoted by  $\bar{h}_{ii}$ . All channel coefficients are assumed to be non-negative real numbers. At a given channel

use  $n \in \{1, 2, \dots, N\}$ , with

$$N = \max(N_1, N_2), \quad (1)$$

the channel output at receiver  $i$  is denoted by  $\vec{Y}_{i,n}$ . During channel use  $n$ , the input-output relation of the channel model is given by

$$\vec{Y}_{i,n} = \vec{h}_{ii} X_{i,n} + h_{ij} X_{j,n} + \vec{Z}_{i,n}, \quad (2)$$

where  $X_{i,n} = 0$  for all  $n$  such that  $N \geq n > N_i$  and  $\vec{Z}_{i,n}$  is a real Gaussian random variable with zero mean and unit variance that represents the noise at the input of receiver  $i$ . Let  $d > 0$  be the finite feedback delay measured in channel uses. At the end of channel use  $n$ , transmitter  $i$  observes  $\vec{Y}_{i,n}$ , which consists of a scaled and noisy version of  $\vec{Y}_{i,n-d}$ . More specifically,

$$\overleftarrow{Y}_{i,n} = \begin{cases} \overleftarrow{Z}_{i,n} & \text{for } n \in \{1, 2, \dots, d\} \\ \overleftarrow{h}_{ii} \vec{Y}_{i,n-d} + \overleftarrow{Z}_{i,n} & \text{for } n \in \{d+1, d+2, \dots, N\}, \end{cases} \quad (3)$$

where  $\overleftarrow{Z}_{i,n}$  is a real Gaussian random variable with zero mean and unit variance that represents the noise in the feedback link of transmitter-receiver pair  $i$ . The random variables  $\vec{Z}_{i,n}$  and  $\overleftarrow{Z}_{i,n}$  are assumed to be independent. In the following, without loss of generality, the feedback delay is assumed to be one channel use, i.e.,  $d = 1$ . The encoder of transmitter  $i$  is defined by a set of deterministic functions  $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$ , with  $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathcal{X}_i$  and for all  $n \in \{2, 3, \dots, N_i\}$ ,  $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \mathbb{R}^{n-1} \rightarrow \mathcal{X}_i$ , such that

$$X_{i,1} = f_{i,1}^{(N)}(W_i, \Omega_i), \text{ and} \quad (4a)$$

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega_i, \overleftarrow{Y}_{i,1}, \overleftarrow{Y}_{i,2}, \dots, \overleftarrow{Y}_{i,n-1}), \quad (4b)$$

where  $\Omega_i$  is an additional index randomly generated. The index  $\Omega_i$  is assumed to be known by both transmitter  $i$  and receiver  $i$ , while unknown by transmitter  $j$  and receiver  $j$ .

The components of the input vector  $\mathbf{X}_i$  are real numbers subject to an average power constraint

$$\frac{1}{N_i} \sum_{n=1}^{N_i} \mathbb{E}_{X_{i,n}} [X_{i,n}^2] \leq 1. \quad (5)$$

The decoder of receiver  $i$  is defined by a deterministic function  $\psi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathcal{W}_i$ . At the end of the communication, receiver  $i$  uses the vector  $(\vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N})$  and the index  $\Omega_i$  to obtain an estimate

$$\widehat{W}_i = \psi_i^{(N)}(\Omega_i, \vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N}), \quad (6)$$

of the message index  $W_i$ . A *transmit-receive configuration* for transmitter-receiver pair  $i$ , denoted by  $s_i$ , can be described in terms of the block-length  $N_i$ , the rate  $R_i$ , the codebook  $\mathcal{C}_i$ , the encoding functions  $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$ , and the decoding function  $\psi_i^{(N)}$ , etc. The average error probability at decoder  $i$  given the configurations  $s_1$  and  $s_2$ , denoted by  $p_i(s_1, s_2)$ , is given by

$$p_i(s_1, s_2) = \Pr[W_i \neq \widehat{W}_i]. \quad (7)$$

Within this context, a rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is said to be achievable if it complies with the following definition.

**Definition 1 (Achievable Rate Pairs):** A rate pair  $(R_1, R_2) \in \mathbb{R}_+^2$  is achievable if there exists at least one pair of configurations  $(s_1, s_2)$  such that the decoding bit error probabilities  $p_1(s_1, s_2)$  and  $p_2(s_1, s_2)$  can be made arbitrarily small by letting the block-lengths  $N_1$  and  $N_2$  grow to infinity.

The aim of transmitter  $i$  is to autonomously choose its transmit-receive configuration  $s_i$  in order to maximize its achievable rate  $R_i$ . Note that the rate achieved by transmitter-receiver  $i$  depends on both configurations  $s_1$  and  $s_2$  due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel. The fundamental limits of the two-user D-GIC-NOF in Figure 1 can be described by six parameters:  $\overrightarrow{\text{SNR}}_i$ ,  $\overleftarrow{\text{SNR}}_i$ , and  $\text{INR}_{ij}$ , with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , which are defined as follows:

$$\overrightarrow{\text{SNR}}_i \triangleq \vec{h}_{ii}^2, \quad (8)$$

$$\text{INR}_{ij} \triangleq h_{ij}^2 \text{ and} \quad (9)$$

$$\overleftarrow{\text{SNR}}_i \triangleq \overleftarrow{h}_{ii}^2 (\vec{h}_{ii}^2 + 2\vec{h}_{ii}h_{ij} + h_{ij}^2 + 1). \quad (10)$$

The analysis presented in this paper focuses exclusively on the case in which  $\text{INR}_{ij} > 1$  for all  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ . The reason for exclusively considering this case follows from the fact that when  $\text{INR}_{ij} \leq 1$ , the transmitter-receiver pair  $i$  is impaired mainly by noise instead of interference. In this case, feedback does not bring a significant rate improvement. Denote by  $\mathcal{C}$  the capacity region of the two-user GIC-NOF with fixed parameters  $\overrightarrow{\text{SNR}}_1, \overrightarrow{\text{SNR}}_2, \text{INR}_{12}, \text{INR}_{21}, \overleftarrow{\text{SNR}}_1$ , and  $\overleftarrow{\text{SNR}}_2$ . The achievable region  $\underline{\mathcal{C}}$  in [4, Theorem 2] and the converse region  $\overline{\mathcal{C}}$  in [4, Theorem 3] approximate the capacity region  $\mathcal{C}$  to within 4.4 bits [4].

### III. GAME FORMULATION

The competitive interaction between the two transmitter-receiver pairs in the interference channel can be modeled by the following game in normal-form:

$$\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}). \quad (11)$$

The set  $\mathcal{K} = \{1, 2\}$  is the set of players, that is, the set of transmitter-receiver pairs. The sets  $\mathcal{A}_1$  and  $\mathcal{A}_2$  are the sets of actions of players 1 and 2, respectively. An action of a player  $i \in \mathcal{K}$ , which is denoted by  $s_i \in \mathcal{A}_i$ , is basically its transmit-receive configuration as described above. The utility function of player  $i$  is  $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$  and it is defined as the information rate of transmitter  $i$ ,

$$u_i(s_1, s_2) = \begin{cases} R_i, & \text{if } p_i(s_1, s_2) < \epsilon \\ 0, & \text{otherwise,} \end{cases} \quad (12)$$

where  $\epsilon > 0$  is an arbitrarily small number. This game formulation for the case without feedback was first proposed in [9] and [10].

A class of transmit-receive configurations that are particularly important in the analysis of this game is referred to as

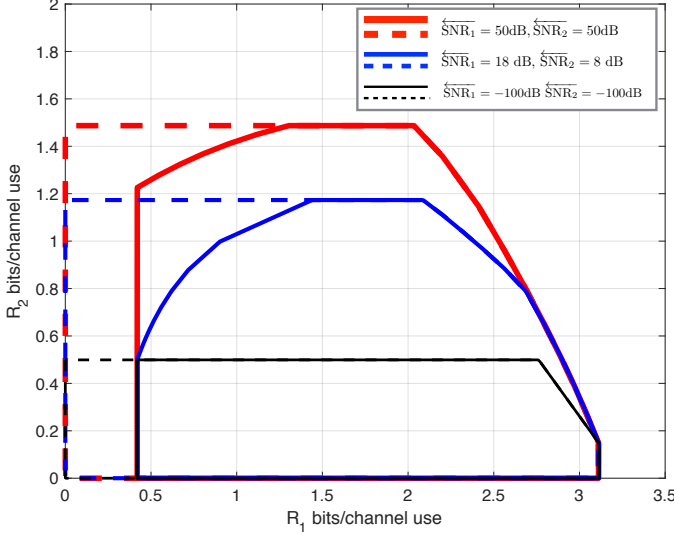


Fig. 2. Achievable capacity regions  $\mathcal{C}$  (dashed-lines) in [4, Theorem 2] and achievable  $\eta$ -NE regions  $\mathcal{N}_\eta$  (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 3$  dB,  $\text{INR}_{12} = 16$  dB,  $\text{INR}_{21} = 9$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 8, 50\}$  dB and  $\eta = 1$ .

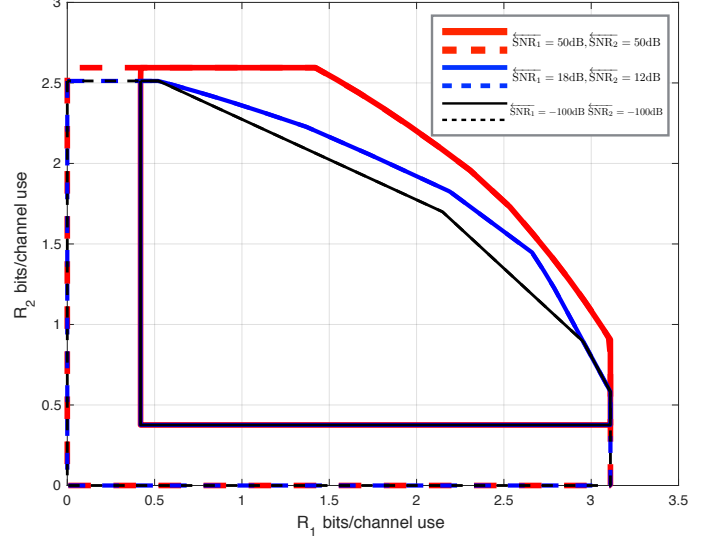


Fig. 3. Achievable capacity regions  $\mathcal{C}$  (dashed-lines) in [4, Theorem 2] and achievable  $\eta$ -NE regions  $\mathcal{N}_\eta$  (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 18$  dB,  $\text{INR}_{12} = 16$  dB,  $\text{INR}_{21} = 10$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$  dB and  $\eta = 1$ .

the set of  $\eta$ -Nash equilibria ( $\eta$ -NE), with  $\eta > 0$ . This type of configurations satisfy the following definition.

**Definition 2 ( $\eta$ -Nash equilibrium):** Given a positive real  $\eta$ , an action profile  $(s_1^*, s_2^*)$  is an  $\eta$ -Nash equilibrium (NE) in the game  $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ , if for all  $i \in \mathcal{K}$  and for all  $s_i \in \mathcal{A}_i$ , it follows that

$$u_i(s_i, s_j^*) \leq u_i(s_i^*, s_j^*) + \eta. \quad (13)$$

Let  $(s_1^*, s_2^*)$  be an  $\eta$ -Nash equilibrium action profile. Then, none of the transmitters can increase its own transmission rate more than  $\eta$  bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for  $\eta$  sufficiently large, from Definition 2, any pair of configurations can be an  $\eta$ -NE. Alternatively, for  $\eta = 0$ , the definition of Nash equilibrium is obtained [11]. In this case, if a pair of configurations is a Nash equilibrium ( $\eta = 0$ ), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible  $\eta$ -NE rate pairs  $(R_1, R_2)$  of the game in (11) with the smallest  $\eta$  for which there exists at least one equilibrium configuration pair.

The set of rate pairs that can be achieved at an  $\eta$ -NE is known as the  $\eta$ -Nash equilibrium ( $\eta$ -NE) region.

**Definition 3 ( $\eta$ -NE Region):** Let  $\eta > 0$  be fixed. An achievable rate pair  $(R_1, R_2)$  is said to be in the  $\eta$ -NE region of the game  $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$  if there exists a pair  $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$  that is an  $\eta$ -NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \text{and} \quad u_2(s_1^*, s_2^*) = R_2. \quad (14)$$

The  $\eta$ -NE regions of the two-user GIC with and without perfect channel-output feedback have been approximated to within a constant number of bits in [5] and [6], respectively. The next section introduces a generalization of these results.

## IV. MAIN RESULTS

### A. Achievable $\eta$ -Nash Equilibrium Region

Let the  $\eta$ -NE region (Definition 3) of the D-GIC-NOF be denoted by  $\mathcal{N}_\eta$ . This section introduces a region  $\mathcal{N}_\eta \subseteq \mathcal{N}_\eta$  that is achievable using a coding scheme that combines rate splitting [12], common randomness [5], [6], block Markov superposition coding [13] and backward decoding [14]. In the following, this coding scheme is referred to as randomized Han-Kobayashi scheme with noisy channel-output feedback (RHK-NOF). This coding scheme is presented in [8] and uses the same techniques of the schemes in [5] and [6]. Therefore, the focus of this section is on the results rather than the description of the scheme. A motivated reader is referred to [15]. The RHK-NOF is proved to be an  $\eta$ -NE action profile with  $\eta \geq 1$ . That is, any unilateral deviation from the RHK-NOF by any of the transmitter-receiver pairs might lead to an individual rate improvement which is at most one bit per channel use. The description of the achievable  $\eta$ -Nash region  $\mathcal{N}_\eta$  is presented using the constants  $a_{1,i}$ ; the functions  $a_{2,i} : [0, 1] \rightarrow \mathbb{R}_+$ ,  $a_{1,i} : [0, 1]^2 \rightarrow \mathbb{R}_+$ , with  $l \in \{3, \dots, 6\}$ ; and  $a_{7,i} : [0, 1]^3 \rightarrow \mathbb{R}_+$ , which are defined as follows, for all  $i \in \{1, 2\}$ , with  $j \in \{1, 2\} \setminus \{i\}$ :

$$a_{1,i} = \frac{1}{2} \log \left( 2 + \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \right) - \frac{1}{2}, \quad (15a)$$

$$a_{2,i}(\rho) = \frac{1}{2} \log \left( b_{1,i}(\rho) + 1 \right) - \frac{1}{2}, \quad (15b)$$

$$a_{3,i}(\rho, \mu) = \frac{1}{2} \log \left( \frac{\overrightarrow{\text{SNR}}_i (b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1}{\overrightarrow{\text{SNR}}_i ((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1} \right), \quad (15c)$$

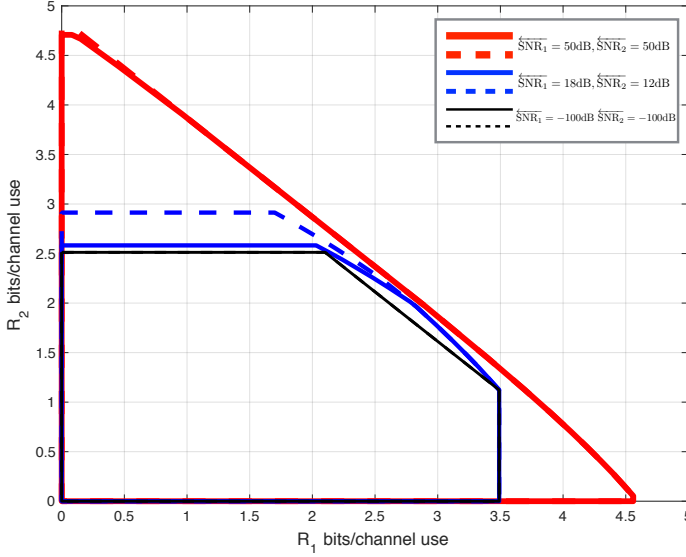


Fig. 4. Achievable capacity regions  $\mathcal{C}$  (dashed-lines) in [4, Theorem 2] and achievable  $\eta$ -NE regions  $\mathcal{N}_\eta$  (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 18$  dB,  $\text{INR}_{12} = 48$  dB,  $\text{INR}_{21} = 30$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$  dB and  $\eta = 1$ .

$$a_{4,i}(\rho, \mu) = \frac{1}{2} \log \left( (1 - \mu) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15d)$$

$$a_{5,i}(\rho, \mu) = \frac{1}{2} \log \left( 2 + \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} + (1 - \mu) b_{2,i}(\rho) \right) - \frac{1}{2}, \quad (15e)$$

$$a_{6,i}(\rho, \mu) = \frac{1}{2} \log \left( \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left( (1 - \mu) b_{2,j}(\rho) + 1 \right) + 2 \right) - \frac{1}{2},$$

$$a_{7,i}(\rho, \mu_1, \mu_2) = \frac{1}{2} \log \left( \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left( (1 - \mu_i) b_{2,j}(\rho) + 1 \right) \right. \\ \left. + (1 - \mu_j) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15f)$$

where the functions  $b_{l,i} : [0, 1] \rightarrow \mathbb{R}_+$ , with  $l \in \{1, 2\}$  are defined as follows:

$$b_{1,i}(\rho) = \overrightarrow{\text{SNR}}_i + 2\rho \sqrt{\overrightarrow{\text{SNR}}_i \text{INR}_{ij}} + \text{INR}_{ij} \text{ and } \quad (16a)$$

$$b_{2,i}(\rho) = (1 - \rho) \text{INR}_{ij} - 1. \quad (16b)$$

Note that the functions in (15) and (16) depend on  $\overrightarrow{\text{SNR}}_1$ ,  $\overrightarrow{\text{SNR}}_2$ ,  $\text{INR}_{12}$ ,  $\text{INR}_{21}$ ,  $\overrightarrow{\text{SNR}}_1$ , and  $\overrightarrow{\text{SNR}}_2$ , however as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, the achievable  $\eta$ -NE region is presented by Theorem 1 on the next page. The proof of Theorem 1 is presented in [8]. The inequalities in (17) are additional conditions to those defining the region  $\mathcal{C}$  in [4, Theorem 2]. More specifically, the  $\eta$ -NE region is described by the intersection of the achievable region  $\mathcal{C}$  and the set of rate pairs  $(R_1, R_2)$  satisfying (17).

Figure 2 shows the achievable region  $\mathcal{C}$  in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  in Theorem 1 of a two-user D-GIC-NOF with

parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 3$  dB,  $\text{INR}_{12} = 16$  dB,  $\text{INR}_{21} = 9$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 8, 50\}$  dB and  $\eta = 1$ . Note that in this case, the feedback parameter  $\overrightarrow{\text{SNR}}_2$  does not have an effect on the achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  and the achievable capacity region  $\mathcal{C}$  ([4, Theorem 2]). This is due to the fact that when one transmitter-receiver pair is in low interference regime (LIR) and the other transmitter-receiver pair is in high interference regime (HIR), feedback is useless on the transmitter-receiver pair in HIR [15], [16].

Figure 3 shows the achievable region  $\mathcal{C}$  in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  in Theorem 1 of a two-user D-GIC-NOF with parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 18$  dB,  $\text{INR}_{12} = 16$  dB,  $\text{INR}_{21} = 10$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$  dB and  $\eta = 1$ . Figure 4 shows the achievable region  $\mathcal{C}$  in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  in Theorem 1 of a two-user D-GIC-NOF with parameters  $\overrightarrow{\text{SNR}}_1 = 24$  dB,  $\overrightarrow{\text{SNR}}_2 = 18$  dB,  $\text{INR}_{12} = 48$  dB,  $\text{INR}_{21} = 30$  dB,  $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$  dB,  $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$  dB and  $\eta = 1$ . In this case, the achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  in Theorem 1 and achievable region  $\mathcal{C}$  on the capacity region [4, Theorem 2] are almost identical, which implies that in the cases in which  $\overrightarrow{\text{SNR}}_i < \text{INR}_{ij}$ , for both  $i \in \{1, 2\}$ , with  $j \in \{1, 2\} \setminus \{i\}$ , the achievable  $\eta$ -NE region is almost the same as the achievable capacity region in the centralized case studied in [4]. At low values of  $\overrightarrow{\text{SNR}}_1$  and  $\overrightarrow{\text{SNR}}_2$ , the achievable  $\eta$ -NE region approaches the rectangular region reported in [5] for the case of the two-user decentralized GIC (D-GIC). Alternatively, for high values of  $\overrightarrow{\text{SNR}}_1$  and  $\overrightarrow{\text{SNR}}_2$ , the achievable  $\eta$ -NE region approaches the region reported in [6] for the case of the two-user decentralized GIC with perfect channel-output feedback (D-GIC-POF). These observations are formalized by the following corollaries.

Denote by  $\mathcal{N}_{\eta\text{PF}}$  the achievable  $\eta$ -NE region of the two-user D-GIC-POF presented in [6]. The region  $\mathcal{N}_{\eta\text{PF}}$  can be obtained as a special case of Theorem 1 as shown by the following corollary.

**Corollary 1 ( $\eta$ -NE Region with Perfect Output Feedback):** Let  $\mathcal{N}_{\eta\text{PF}}$  denote the achievable  $\eta$ -NE region of the two-user D-GIC-POF with fixed parameters  $\overrightarrow{\text{SNR}}_i$  and  $\text{INR}_{ij}$ , with  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ . Then, the following holds:

$$\mathcal{N}_{\eta\text{PF}} = \lim_{\substack{\overrightarrow{\text{SNR}}_1 \rightarrow \infty \\ \overrightarrow{\text{SNR}}_2 \rightarrow \infty}} \mathcal{N}_\eta. \quad (18)$$

Denote by  $\mathcal{N}_{\eta\text{WF}}$  the achievable  $\eta$ -NE region of the two-user D-GIC presented in [5]. The region  $\mathcal{N}_{\eta\text{WF}}$  can be obtained as a special case of Theorem 1 as shown by the following corollary.

**Corollary 2 ( $\eta$ -NE Region without Output Feedback):** Let  $\mathcal{N}_{\eta\text{WF}}$  denote the achievable  $\eta$ -NE region of the two-user D-GIC, with fixed parameters  $\overrightarrow{\text{SNR}}_i$  and  $\text{INR}_{ij}$ , with  $i \in \{1, 2\}$

*Theorem 1:* Let  $\eta \geq 1$  be fixed. The achievable  $\eta$ -NE region  $\mathcal{N}_\eta$  is given by the closure of all possible achievable rate pairs  $(R_1, R_2) \in \mathcal{C}$  in [4, Theorem 2] that satisfy, for all  $i \in \{1, 2\}$  and  $j \in \{1, 2\} \setminus \{i\}$ , the following conditions:

$$R_i \geq \left( a_{2,i}(\rho) - a_{3,i}(\rho, \mu_j) - a_{4,i}(\rho, \mu_j) - \eta \right)^+, \quad (17a)$$

$$R_i \leq \min \left( a_{2,i}(\rho) + a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \right. \quad (17b)$$

$$\left. \begin{aligned} & a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \\ & a_{2,i}(\rho) + a_{3,i}(\rho, \mu_j) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) + a_{7,j}(\rho, \mu_1, \mu_2) - 2a_{2,j}(\rho) + 2\eta \end{aligned} \right),$$

$$R_1 + R_2 \leq a_{1,i} + a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + a_{2,j}(\rho) + a_{3,j}(\rho, \mu_1) - a_{2,i}(\rho) + \eta, \quad (17c)$$

for all  $(\rho, \mu_1, \mu_2) \in \left[0, \left(1 - \max\left(\frac{1}{\text{INR}_{12}}, \frac{1}{\text{INR}_{21}}\right)\right)^+\right] \times [0, 1] \times [0, 1]$ .

and  $j \in \{1, 2\} \setminus \{i\}$ . Then, the following holds:

$$\mathcal{N}_{\eta\text{WF}} = \lim_{\substack{\text{SNR}_1 \rightarrow 0 \\ \text{SNR}_2 \rightarrow 0 \\ \rho = 0}} \mathcal{N}_\eta. \quad (19)$$

### B. Impossibility Region

This section introduces an impossibility region, denoted by  $\bar{\mathcal{N}}_\eta$ . That is,  $\bar{\mathcal{N}}_\eta \supseteq \mathcal{N}_\eta$ . More specifically, any rate pair  $(R_1, R_2) \in \bar{\mathcal{N}}_\eta$  is not an  $\eta$ -NE. This region is described in terms of the convex region  $\bar{\mathcal{B}}_{\text{G-IC-NOF}}$ . Here, for the case of the two-user D-GIC-NOF, the region  $\bar{\mathcal{B}}_{\text{G-IC-NOF}}$  is given by the closure of the rate pairs  $(R_1, R_2) \in \mathbb{R}_+^2$  that satisfy for all  $i \in \{1, 2\}$ , with  $j \in \{1, 2\} \setminus \{i\}$ :

$$\bar{\mathcal{B}}_{\text{G-IC-NOF}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : R_i \geq L_i, \right. \\ \left. \text{for all } i \in \mathcal{K} = \{1, 2\} \right\}, \quad (20)$$

where,

$$L_i \triangleq \left( \frac{1}{2} \log \left( 1 + \frac{\overrightarrow{\text{SNR}}_i}{1 + \text{INR}_{ij}} \right) - \eta \right)^+. \quad (21)$$

Note that  $L_i$  is the rate achieved by the transmitter-receiver pair  $i$  when it saturates the power constraint in (5) and treats interference as noise. Following this notation, the impossibility region of the two-user GIC-NOF, i.e.,  $\bar{\mathcal{N}}_\eta$ , can be described as follows.

*Theorem 2:* Let  $\eta \geq 1$  be fixed. The impossibility region  $\bar{\mathcal{N}}_\eta$  of the two-user D-GIC-NOF is given by the closure of all possible non-negative rate pairs  $(R_1, R_2) \in \bar{\mathcal{C}} \cap \bar{\mathcal{B}}_{\text{G-IC-NOF}}$  for all  $\rho \in [0, 1]$ .

The impossibility region in Theorem 2 has been first presented in [6] and it is very loose in this case. A better impossibility region is presented in [15].

### V. CONCLUSIONS

In this paper, an achievable  $\eta$ -Nash equilibrium ( $\eta$ -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback has been presented for all  $\eta \geq 1$ . This result generalizes the existing achievable regions of the  $\eta$ -NE for the cases without feedback and with perfect channel-output feedback.

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