



HAL
open science

Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, Harold Vincent Poor

► **To cite this version:**

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, Harold Vincent Poor. Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback. ITW 2018 - IEEE Information Theory Workshop, Nov 2018, Guangzhou, China. pp.1-5. hal-01876880

HAL Id: hal-01876880

<https://hal.science/hal-01876880>

Submitted on 19 Sep 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Approximate Nash Region of the Gaussian Interference Channel with Noisy Output Feedback

Victor Quintero, Samir M. Perlaza, Jean-Marie Gorce, and H. Vincent Poor

Abstract—In this paper, an achievable η -Nash equilibrium (η -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback is presented for all $\eta \geq 1$. This result is obtained in the scenario in which each transmitter-receiver pair chooses its own transmit-receive configuration in order to maximize its own individual information transmission rate. At an η -NE, any unilateral deviation by either of the pairs does not increase the corresponding individual rate by more than η bits per channel use.

Index Terms—Gaussian Interference Channel, Noisy channel-output feedback, η -Nash equilibrium region.

I. INTRODUCTION

The interference channel (IC) is one of the simplest yet insightful multi-user channels in network information theory. An important class of ICs is the two-user Gaussian interference channel (GIC) in which there exist two point-to-point links subject to mutual interference and independent Gaussian noise sources. In this model, each output signal is a noisy version of the sum of the two transmitted signals affected by the corresponding channel gains. The analysis of this channel can be made considering two general scenarios: (1) a centralized scenario in which the entire network is controlled by a central entity that configures both transmitter-receiver pairs; and (2) a decentralized scenario in which each transmitter-receiver pair autonomously configures its transmission-reception parameters. In the former, the fundamental limits are characterized by the capacity region, which is approximated to within a fixed number of bits in [1] for the case without feedback; in [2] for the case with perfect channel-output feedback; and in [3] and [4] for the case with noisy channel-output feedback. In the latter, the fundamental limits are characterized by the η -Nash equilibrium (η -NE) region. The η -NE of the GIC is approximated in the cases without feedback and with perfect channel-output feedback in [5] and [6], respectively.

Victor Quintero is with the Department of Telecommunications, Universidad del Cauca, 19003, Popayán, Cauca, Colombia. (vflorez@unicauca.edu.co).

Samir M. Perlaza and Jean-Marie Gorce are with the laboratoire CITI (a joint laboratory between the Université de Lyon, INRIA, and INSA de Lyon), 6 Avenue des Arts, F-69621, Villeurbanne, France. ({samir.perlaza, jean-marie.gorce}@inria.fr).

H. Vincent Poor and Samir M. Perlaza are with the Department of Electrical Engineering at Princeton University, Princeton, NJ 08544 USA. (poor@princeton.edu).

This research was supported in part by the European Commission under Marie Skłodowska-Curie Individual Fellowship No. 659316; in part by the INSA Lyon - SPIE ICS chair on the Internet of Things; in part by the Administrative Department of Science, Technology, and Innovation of Colombia (Colciencias), fellowship No. 617-2013; and in part by the U. S. National Science Foundation under Grants CNS-1702808 and ECCS-1647198.

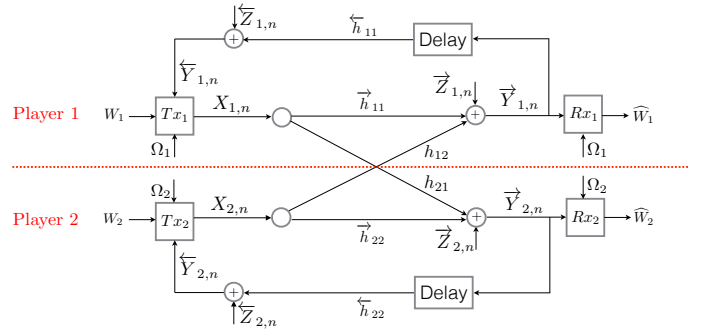


Fig. 1. Two-User Decentralized Gaussian interference channel with noisy channel-output feedback at channel use n .

In this paper the η -NE region of the GIC is studied assuming that there exists a noisy feedback link from each receiver to its corresponding transmitter. The η -NE region is approximated by two regions for all $\eta \geq 1$: a region for which an equilibrium transmit-receive configuration is presented for each of the information rate pairs (an achievable region); and a region for which any information rate pair that is outside of this region cannot be an η -NE (impossibility region). The focus of this paper is on the achievable region.

The results presented in this paper are a generalization of the results presented in [5] and [6], and they are obtained thanks to the analysis of linear deterministic approximations in [7] and [8].

II. DECENTRALIZED GAUSSIAN INTERFERENCE CHANNELS WITH NOISY CHANNEL-OUTPUT FEEDBACK

Consider the two-user decentralized Gaussian interference channel with noisy channel-output feedback (D-GIC-NOF) depicted in Figure 1. Transmitter i , with $i \in \{1, 2\}$, communicates with receiver i subject to the interference produced by transmitter j , with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \dots, \lfloor 2^{N_i R_i} \rfloor\}$, where N_i denotes the fixed block-length in channel uses and R_i the information transmission rate in bits per channel use. At each block, transmitter i sends the codeword $\mathbf{X}_i = (X_{i,1}, X_{i,2}, \dots, X_{i,N_i})^T \in \mathcal{C}_i \subseteq \mathcal{R}^{N_i}$, where \mathcal{C}_i is the codebook of transmitter i . The channel coefficient from transmitter j to receiver i is denoted by h_{ij} ; the channel coefficient from transmitter i to receiver i is denoted by \overrightarrow{h}_{ii} ; and the channel coefficient from channel-output i to transmitter i is denoted by \overleftarrow{h}_{ii} . All channel coefficients are assumed to be non-negative real numbers. At a given channel

use $n \in \{1, 2, \dots, N\}$, with

$$N = \max(N_1, N_2), \quad (1)$$

the channel output at receiver i is denoted by $\vec{Y}_{i,n}$. During channel use n , the input-output relation of the channel model is given by

$$\vec{Y}_{i,n} = \vec{h}_{ii} X_{i,n} + h_{ij} X_{j,n} + \vec{Z}_{i,n}, \quad (2)$$

where $X_{i,n} = 0$ for all n such that $N \geq n > N_i$ and $\vec{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise at the input of receiver i . Let $d > 0$ be the finite feedback delay measured in channel uses. At the end of channel use n , transmitter i observes $\overleftarrow{Y}_{i,n}$, which consists of a scaled and noisy version of $\vec{Y}_{i,n-d}$. More specifically,

$$\overleftarrow{Y}_{i,n} = \begin{cases} \overleftarrow{Z}_{i,n} & \text{for } n \in \{1, 2, \dots, d\} \\ \overleftarrow{h}_{ii} \vec{Y}_{i,n-d} + \overleftarrow{Z}_{i,n} & \text{for } n \in \{d+1, d+2, \dots, N\}, \end{cases} \quad (3)$$

where $\overleftarrow{Z}_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise in the feedback link of transmitter-receiver pair i . The random variables $\vec{Z}_{i,n}$ and $\overleftarrow{Z}_{i,n}$ are assumed to be independent. In the following, without loss of generality, the feedback delay is assumed to be one channel use, i.e., $d = 1$. The encoder of transmitter i is defined by a set of deterministic functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$, with $f_{i,1}^{(N)} : \mathcal{W}_i \times \mathbb{N} \rightarrow \mathcal{X}_i$ and for all $n \in \{2, 3, \dots, N_i\}$, $f_{i,n}^{(N)} : \mathcal{W}_i \times \mathbb{N} \times \mathbb{R}^{n-1} \rightarrow \mathcal{X}_i$, such that

$$X_{i,1} = f_{i,1}^{(N)}(W_i, \Omega_i), \quad \text{and} \quad (4a)$$

$$X_{i,n} = f_{i,n}^{(N)}(W_i, \Omega_i, \overleftarrow{Y}_{i,1}, \overleftarrow{Y}_{i,2}, \dots, \overleftarrow{Y}_{i,n-1}), \quad (4b)$$

where Ω_i is an additional index randomly generated. The index Ω_i is assumed to be known by both transmitter i and receiver i , while unknown by transmitter j and receiver j .

The components of the input vector \mathbf{X}_i are real numbers subject to an average power constraint

$$\frac{1}{N_i} \sum_{n=1}^{N_i} \mathbb{E}_{X_{i,n}} [X_{i,n}^2] \leq 1. \quad (5)$$

The decoder of receiver i is defined by a deterministic function $\psi_i^{(N)} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathcal{W}_i$. At the end of the communication, receiver i uses the vector $(\vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N})$ and the index Ω_i to obtain an estimate

$$\widehat{W}_i = \psi_i^{(N)}(\Omega_i, \vec{Y}_{i,1}, \vec{Y}_{i,2}, \dots, \vec{Y}_{i,N}), \quad (6)$$

of the message index W_i . A *transmit-receive configuration* for transmitter-receiver pair i , denoted by s_i , can be described in terms of the block-length N_i , the rate R_i , the codebook \mathcal{C}_i , the encoding functions $f_{i,1}^{(N)}, f_{i,2}^{(N)}, \dots, f_{i,N_i}^{(N)}$, and the decoding function $\psi_i^{(N)}$, etc. The average error probability at decoder i given the configurations s_1 and s_2 , denoted by $p_i(s_1, s_2)$, is given by

$$p_i(s_1, s_2) = \Pr [W_i \neq \widehat{W}_i]. \quad (7)$$

Within this context, a rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is said to be achievable if it complies with the following definition.

Definition 1 (Achievable Rate Pairs): A rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is achievable if there exists at least one pair of configurations (s_1, s_2) such that the decoding bit error probabilities $p_1(s_1, s_2)$ and $p_2(s_1, s_2)$ can be made arbitrarily small by letting the block-lengths N_1 and N_2 grow to infinity.

The aim of transmitter i is to autonomously choose its transmit-receive configuration s_i in order to maximize its achievable rate R_i . Note that the rate achieved by transmitter-receiver i depends on both configurations s_1 and s_2 due to mutual interference. This reveals the competitive interaction between both links in the decentralized interference channel. The fundamental limits of the two-user D-GIC-NOF in Figure 1 can be described by six parameters: $\overrightarrow{\text{SNR}}_i$, $\overleftarrow{\text{SNR}}_i$, and INR_{ij} , with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$, which are defined as follows:

$$\overrightarrow{\text{SNR}}_i \triangleq \vec{h}_{ii}^2, \quad (8)$$

$$\text{INR}_{ij} \triangleq h_{ij}^2 \quad \text{and} \quad (9)$$

$$\overleftarrow{\text{SNR}}_i \triangleq \overleftarrow{h}_{ii}^2 (\vec{h}_{ii}^2 + 2\vec{h}_{ii}h_{ij} + h_{ij}^2 + 1). \quad (10)$$

The analysis presented in this paper focuses exclusively on the case in which $\text{INR}_{ij} > 1$ for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. The reason for exclusively considering this case follows from the fact that when $\text{INR}_{ij} \leq 1$, the transmitter-receiver pair i is impaired mainly by noise instead of interference. In this case, feedback does not bring a significant rate improvement. Denote by \mathcal{C} the capacity region of the two-user GIC-NOF with fixed parameters $\overrightarrow{\text{SNR}}_1, \overrightarrow{\text{SNR}}_2, \text{INR}_{12}, \text{INR}_{21}, \overleftarrow{\text{SNR}}_1$, and $\overleftarrow{\text{SNR}}_2$. The achievable region $\underline{\mathcal{C}}$ in [4, Theorem 2] and the converse region $\overline{\mathcal{C}}$ in [4, Theorem 3] approximate the capacity region \mathcal{C} to within 4.4 bits [4].

III. GAME FORMULATION

The competitive interaction between the two transmitter-receiver pairs in the interference channel can be modeled by the following game in normal-form:

$$\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}). \quad (11)$$

The set $\mathcal{K} = \{1, 2\}$ is the set of players, that is, the set of transmitter-receiver pairs. The sets \mathcal{A}_1 and \mathcal{A}_2 are the sets of actions of players 1 and 2, respectively. An action of a player $i \in \mathcal{K}$, which is denoted by $s_i \in \mathcal{A}_i$, is basically its transmit-receive configuration as described above. The utility function of player i is $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$ and it is defined as the information rate of transmitter i ,

$$u_i(s_1, s_2) = \begin{cases} R_i, & \text{if } p_i(s_1, s_2) < \epsilon \\ 0, & \text{otherwise,} \end{cases} \quad (12)$$

where $\epsilon > 0$ is an arbitrarily small number. This game formulation for the case without feedback was first proposed in [9] and [10].

A class of transmit-receive configurations that are particularly important in the analysis of this game is referred to as

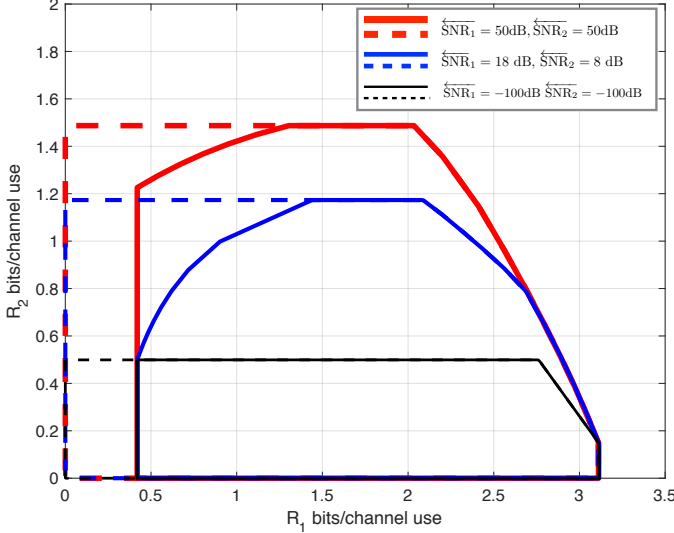


Fig. 2. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overline{\text{SNR}}_1 = 24$ dB, $\overline{\text{SNR}}_2 = 3$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 9$ dB, $\overline{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overline{\text{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$.

the set of η -Nash equilibria (η -NE), with $\eta > 0$. This type of configurations satisfy the following definition.

Definition 2 (η -Nash equilibrium): Given a positive real η , an action profile (s_1^*, s_2^*) is an η -Nash equilibrium (NE) in the game $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$, if for all $i \in \mathcal{K}$ and for all $s_i \in \mathcal{A}_i$, it follows that

$$u_i(s_i, s_j^*) \leq u_i(s_i^*, s_j^*) + \eta. \quad (13)$$

Let (s_1^*, s_2^*) be an η -Nash equilibrium action profile. Then, none of the transmitters can increase its own transmission rate more than η bits per channel use by changing its own transmit-receive configuration and keeping the average bit error probability arbitrarily close to zero. Note that for η sufficiently large, from Definition 2, any pair of configurations can be an η -NE. Alternatively, for $\eta = 0$, the definition of Nash equilibrium is obtained [11]. In this case, if a pair of configurations is a Nash equilibrium ($\eta = 0$), then each individual configuration is optimal with respect to each other. Hence, the interest is to describe the set of all possible η -NE rate pairs (R_1, R_2) of the game in (11) with the smallest η for which there exists at least one equilibrium configuration pair.

The set of rate pairs that can be achieved at an η -NE is known as the η -Nash equilibrium (η -NE) region.

Definition 3 (η -NE Region): Let $\eta > 0$ be fixed. An achievable rate pair (R_1, R_2) is said to be in the η -NE region of the game $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ if there exists a pair $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that is an η -NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \quad \text{and} \quad u_2(s_1^*, s_2^*) = R_2. \quad (14)$$

The η -NE regions of the two-user GIC with and without perfect channel-output feedback have been approximated to within a constant number of bits in [5] and [6], respectively. The next section introduces a generalization of these results.

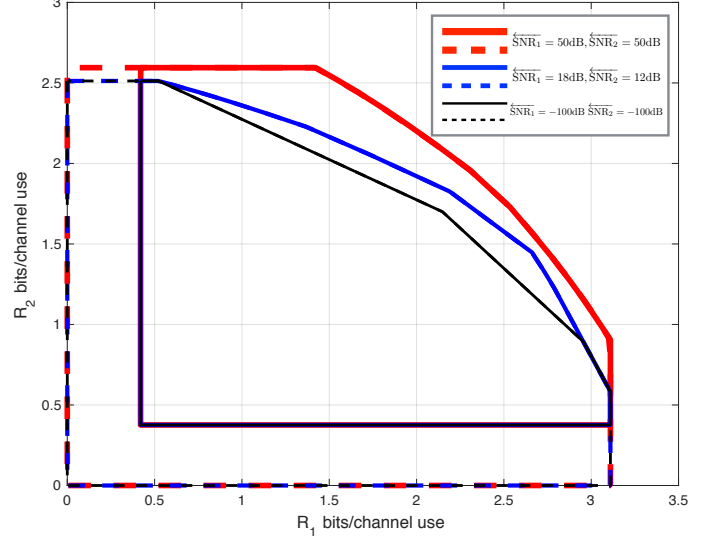


Fig. 3. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overline{\text{SNR}}_1 = 24$ dB, $\overline{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 10$ dB, $\overline{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overline{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

IV. MAIN RESULTS

A. Achievable η -Nash Equilibrium Region

Let the η -NE region (Definition 3) of the D-GIC-NOF be denoted by \mathcal{N}_η . This section introduces a region $\mathcal{N}_\eta \subseteq \mathcal{N}_\eta$ that is achievable using a coding scheme that combines rate splitting [12], common randomness [5], [6], block Markov superposition coding [13] and backward decoding [14]. In the following, this coding scheme is referred to as randomized Han-Kobayashi scheme with noisy channel-output feedback (RHK-NOF). This coding scheme is presented in [8] and uses the same techniques of the schemes in [5] and [6]. Therefore, the focus of this section is on the results rather than the description of the scheme. A motivated reader is referred to [15]. The RHK-NOF is proved to be an η -NE action profile with $\eta \geq 1$. That is, any unilateral deviation from the RHK-NOF by any of the transmitter-receiver pairs might lead to an individual rate improvement which is at most one bit per channel use. The description of the achievable η -Nash region \mathcal{N}_η is presented using the constants $a_{1,i}$; the functions $a_{2,i} : [0, 1] \rightarrow \mathbb{R}_+$, $a_{1,i} : [0, 1]^2 \rightarrow \mathbb{R}_+$, with $l \in \{3, \dots, 6\}$; and $a_{7,i} : [0, 1]^3 \rightarrow \mathbb{R}_+$, which are defined as follows, for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$a_{1,i} = \frac{1}{2} \log \left(2 + \frac{\overline{\text{SNR}}_i}{\text{INR}_{ji}} \right) - \frac{1}{2}, \quad (15a)$$

$$a_{2,i}(\rho) = \frac{1}{2} \log \left(b_{1,i}(\rho) + 1 \right) - \frac{1}{2}, \quad (15b)$$

$$a_{3,i}(\rho, \mu) = \frac{1}{2} \log \left(\frac{\overline{\text{SNR}}_i (b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1}{\overline{\text{SNR}}_i ((1-\mu)b_{2,i}(\rho) + 2) + b_{1,i}(1) + 1} \right), \quad (15c)$$

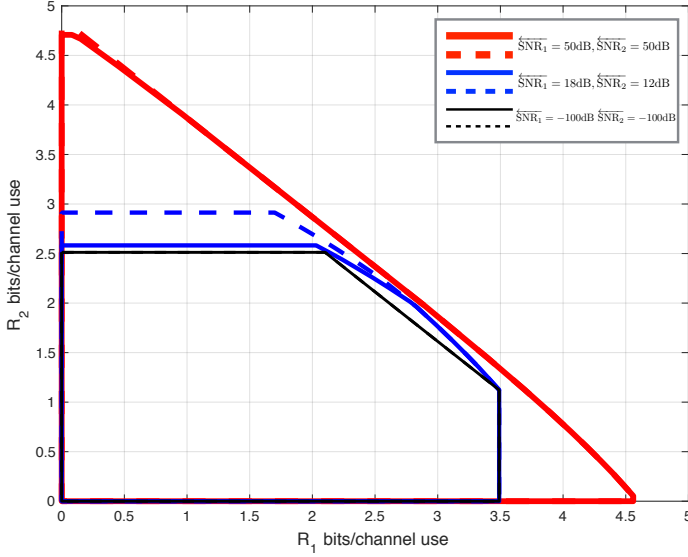


Fig. 4. Achievable capacity regions \mathcal{C} (dashed-lines) in [4, Theorem 2] and achievable η -NE regions \mathcal{N}_η (solid lines) in Theorem 1 of the two-user GIC-NOF and two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 48$ dB, $\text{INR}_{21} = 30$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$.

$$a_{4,i}(\rho, \mu) = \frac{1}{2} \log \left((1 - \mu) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15d)$$

$$a_{5,i}(\rho, \mu) = \frac{1}{2} \log \left(2 + \frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} + (1 - \mu) b_{2,i}(\rho) \right) - \frac{1}{2}, \quad (15e)$$

$$a_{6,i}(\rho, \mu) = \frac{1}{2} \log \left(\frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left((1 - \mu) b_{2,j}(\rho) + 1 \right) + 2 \right) - \frac{1}{2},$$

$$a_{7,i}(\rho, \mu_1, \mu_2) = \frac{1}{2} \log \left(\frac{\overrightarrow{\text{SNR}}_i}{\text{INR}_{ji}} \left((1 - \mu_i) b_{2,j}(\rho) + 1 \right) + (1 - \mu_j) b_{2,i}(\rho) + 2 \right) - \frac{1}{2}, \quad (15f)$$

where the functions $b_{l,i} : [0, 1] \rightarrow \mathbb{R}_+$, with $l \in \{1, 2\}$ are defined as follows:

$$b_{1,i}(\rho) = \overrightarrow{\text{SNR}}_i + 2\rho \sqrt{\overrightarrow{\text{SNR}}_i \text{INR}_{ij}} + \text{INR}_{ij} \quad \text{and} \quad (16a)$$

$$b_{2,i}(\rho) = (1 - \rho) \text{INR}_{ij} - 1. \quad (16b)$$

Note that the functions in (15) and (16) depend on $\overrightarrow{\text{SNR}}_1$, $\overrightarrow{\text{SNR}}_2$, INR_{12} , INR_{21} , $\overrightarrow{\text{SNR}}_1$, and $\overrightarrow{\text{SNR}}_2$, however as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, the achievable η -NE region is presented by Theorem 1 on the next page. The proof of Theorem 1 is presented in [8]. The inequalities in (17) are additional conditions to those defining the region \mathcal{C} in [4, Theorem 2]. More specifically, the η -NE region is described by the intersection of the achievable region \mathcal{C} and the set of rate pairs (R_1, R_2) satisfying (17).

Figure 2 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with

parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 3$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 9$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 8, 50\}$ dB and $\eta = 1$. Note that in this case, the feedback parameter $\overrightarrow{\text{SNR}}_2$ does not have an effect on the achievable η -NE region \mathcal{N}_η and the achievable capacity region \mathcal{C} ([4, Theorem 2]). This is due to the fact that when one transmitter-receiver pair is in low interference regime (LIR) and the other transmitter-receiver pair is in high interference regime (HIR), feedback is useless on the transmitter-receiver pair in HIR [15], [16].

Figure 3 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 16$ dB, $\text{INR}_{21} = 10$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. Figure 4 shows the achievable region \mathcal{C} in [4, Theorem 2] of a two-user centralized GIC-NOF and the achievable η -NE region \mathcal{N}_η in Theorem 1 of a two-user D-GIC-NOF with parameters $\overrightarrow{\text{SNR}}_1 = 24$ dB, $\overrightarrow{\text{SNR}}_2 = 18$ dB, $\text{INR}_{12} = 48$ dB, $\text{INR}_{21} = 30$ dB, $\overrightarrow{\text{SNR}}_1 \in \{-100, 18, 50\}$ dB, $\overrightarrow{\text{SNR}}_2 \in \{-100, 12, 50\}$ dB and $\eta = 1$. In this case, the achievable η -NE region \mathcal{N}_η in Theorem 1 and achievable region \mathcal{C} on the capacity region [4, Theorem 2] are almost identical, which implies that in the cases in which $\overrightarrow{\text{SNR}}_i < \text{INR}_{ij}$, for both $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$, the achievable η -NE region is almost the same as the achievable capacity region in the centralized case studied in [4]. At low values of $\overrightarrow{\text{SNR}}_1$ and $\overrightarrow{\text{SNR}}_2$, the achievable η -NE region approaches the rectangular region reported in [5] for the case of the two-user decentralized GIC (D-GIC). Alternatively, for high values of $\overrightarrow{\text{SNR}}_1$ and $\overrightarrow{\text{SNR}}_2$, the achievable η -NE region approaches the region reported in [6] for the case of the two-user decentralized GIC with perfect channel-output feedback (D-GIC-POF). These observations are formalized by the following corollaries.

Denote by $\mathcal{N}_{\eta\text{PF}}$ the achievable η -NE region of the two-user D-GIC-POF presented in [6]. The region $\mathcal{N}_{\eta\text{PF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 1 (η -NE Region with Perfect Output Feedback): Let $\mathcal{N}_{\eta\text{PF}}$ denote the achievable η -NE region of the two-user D-GIC-POF with fixed parameters $\overrightarrow{\text{SNR}}_i$ and INR_{ij} , with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. Then, the following holds:

$$\mathcal{N}_{\eta\text{PF}} = \lim_{\substack{\overrightarrow{\text{SNR}}_1 \rightarrow \infty \\ \overrightarrow{\text{SNR}}_2 \rightarrow \infty}} \mathcal{N}_\eta. \quad (18)$$

Denote by $\mathcal{N}_{\eta\text{WF}}$ the achievable η -NE region of the two-user D-GIC presented in [5]. The region $\mathcal{N}_{\eta\text{WF}}$ can be obtained as a special case of Theorem 1 as shown by the following corollary.

Corollary 2 (η -NE Region without Output Feedback): Let $\mathcal{N}_{\eta\text{WF}}$ denote the achievable η -NE region of the two-user D-GIC, with fixed parameters $\overrightarrow{\text{SNR}}_i$ and INR_{ij} , with $i \in \{1, 2\}$

Theorem 1: Let $\eta \geq 1$ be fixed. The achievable η -NE region \mathcal{N}_η is given by the closure of all possible achievable rate pairs $(R_1, R_2) \in \underline{\mathcal{C}}$ in [4, Theorem 2] that satisfy, for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$, the following conditions:

$$R_i \geq \left(a_{2,i}(\rho) - a_{3,i}(\rho, \mu_j) - a_{4,i}(\rho, \mu_j) - \eta \right)^+, \quad (17a)$$

$$R_i \leq \min \left(a_{2,i}(\rho) + a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta, \quad (17b)$$

$$a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) - a_{2,j}(\rho) + \eta,$$

$$a_{2,i}(\rho) + a_{3,i}(\rho, \mu_j) + 2a_{3,j}(\rho, \mu_i) + a_{5,j}(\rho, \mu_i) + a_{7,j}(\rho, \mu_1, \mu_2) - 2a_{2,j}(\rho) + 2\eta \Big),$$

$$R_1 + R_2 \leq a_{1,i} + a_{3,i}(\rho, \mu_j) + a_{7,i}(\rho, \mu_1, \mu_2) + a_{2,j}(\rho) + a_{3,j}(\rho, \mu_1) - a_{2,i}(\rho) + \eta, \quad (17c)$$

for all $(\rho, \mu_1, \mu_2) \in \left[0, \left(1 - \max \left(\frac{1}{\text{INR}_{12}}, \frac{1}{\text{INR}_{21}} \right) \right)^+ \right] \times [0, 1] \times [0, 1]$.

and $j \in \{1, 2\} \setminus \{i\}$. Then, the following holds:

$$\mathcal{N}_{\eta\text{WF}} = \lim_{\substack{\frac{\text{SNR}_1 \rightarrow 0}{\frac{\text{SNR}_2 \rightarrow 0}{\rho = 0}}} \mathcal{N}_\eta. \quad (19)$$

B. Impossibility Region

This section introduces an impossibility region, denoted by $\overline{\mathcal{N}}_\eta$. That is, $\overline{\mathcal{N}}_\eta \supseteq \mathcal{N}_\eta$. More specifically, any rate pair $(R_1, R_2) \in \overline{\mathcal{N}}_\eta^c$ is not an η -NE. This region is described in terms of the convex region $\overline{\mathcal{B}}_{\text{G-IC-NOF}}$. Here, for the case of the two-user D-GIC-NOF, the region $\overline{\mathcal{B}}_{\text{G-IC-NOF}}$ is given by the closure of the rate pairs $(R_1, R_2) \in \mathbb{R}_+^2$ that satisfy for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$\overline{\mathcal{B}}_{\text{G-IC-NOF}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : R_i \geq L_i, \right. \\ \left. \text{for all } i \in \mathcal{K} = \{1, 2\} \right\}, \quad (20)$$

where,

$$L_i \triangleq \left(\frac{1}{2} \log \left(1 + \frac{\overrightarrow{\text{SNR}}_i}{1 + \text{INR}_{ij}} \right) - \eta \right)^+. \quad (21)$$

Note that L_i is the rate achieved by the transmitter-receiver pair i when it saturates the power constraint in (5) and treats interference as noise. Following this notation, the impossibility region of the two-user GIC-NOF, i.e., $\overline{\mathcal{N}}_\eta$, can be described as follows.

Theorem 2: Let $\eta \geq 1$ be fixed. The impossibility region $\overline{\mathcal{N}}_\eta$ of the two-user D-GIC-NOF is given by the closure of all possible non-negative rate pairs $(R_1, R_2) \in \overline{\mathcal{C}} \cap \overline{\mathcal{B}}_{\text{G-IC-NOF}}$ for all $\rho \in [0, 1]$.

The impossibility region in Theorem 2 has been first presented in [6] and it is very loose in this case. A better impossibility region is presented in [15].

V. CONCLUSIONS

In this paper, an achievable η -Nash equilibrium (η -NE) region for the two-user Gaussian interference channel with noisy channel-output feedback has been presented for all $\eta \geq 1$. This result generalizes the existing achievable regions of the η -NE for the cases without feedback and with perfect channel-output feedback.

REFERENCES

- [1] R. H. Etkin, D. N. C. Tse, and W. Hua, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.
- [2] C. Suh and D. N. C. Tse, "Feedback capacity of the Gaussian interference channel to within 2 bits," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2667–2685, May. 2011.
- [3] S.-Q. Le, R. Tandon, M. Motani, and H. V. Poor, "Approximate capacity region for the symmetric Gaussian interference channel with noisy feedback," *IEEE Trans. Inf. Theory*, vol. 61, no. 7, pp. 3737–3762, Jul. 2015.
- [4] V. Quintero, S. M. Perlaza, I. Esnaola, and J.-M. Gorce, "Approximate capacity region of the two-user Gaussian interference channel with noisy channel-output feedback," *IEEE Trans. Inf. Theory*, vol. 64, no. 7, pp. 5326–5358, Jul 2018.
- [5] R. A. Berry and D. N. C. Tse, "Shannon meets Nash on the interference channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2821–2836, May. 2011.
- [6] S. M. Perlaza, R. Tandon, H. V. Poor, and Z. Han, "Perfect output feedback in the two-user decentralized interference channel," *IEEE Trans. Inf. Theory*, vol. 61, no. 10, pp. 5441–5462, Oct. 2015.
- [7] V. Quintero, S. M. Perlaza, J.-M. Gorce, and H. V. Poor, "Nash region of the linear deterministic interference channel with noisy output feedback," in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT)*, Aachen, Germany, Jun. 2017.
- [8] —, "Decentralized interference channels with noisy output feedback," INRIA, Lyon, France, Tech. Rep. 9011, Jan. 2017.
- [9] R. D. Yates, D. Tse, and Z. Li, "Secret communication on interference channels," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Toronto, Canada, Jul. 2008.
- [10] R. Berry and D. N. C. Tse, "Information theoretic games on interference channels," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Toronto, Canada, Jul. 2008.
- [11] J. F. Nash, "Equilibrium points in n -person games," *Proc. National Academy of Sciences of the United States of America*, vol. 36, no. 1, pp. 48–49, Jan. 1950.
- [12] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.
- [13] T. M. Cover and C. S. K. Leung, "An achievable rate region for the multiple-access channel with feedback," *IEEE Trans. Inf. Theory*, vol. 27, no. 3, pp. 292–298, May. 1981.
- [14] F. M. J. Willems, "Information theoretical results for multiple access channels," Ph.D. dissertation, Katholieke Universiteit, Leuven, Belgium, Oct. 1982.
- [15] V. Quintero, "Noisy channel-output feedback in the interference channel," Ph.D. dissertation, Université de Lyon, Lyon, France., Dec. 2017.
- [16] V. Quintero, S. M. Perlaza, I. Esnaola, and J.-M. Gorce, "When does output feedback enlarge the capacity of the interference channel?" *IEEE Trans. Commun.*, vol. 66, no. 2, pp. 615–628, Feb. 2018.