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TWG02: Arithmetic and number systems

Introduction to the papers of TWG02: Arithmetic and number systems

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Introduction

Working Group 2 was formed in 2011 in Rzeszów, Poland (CERME 7) as a forum for presenting and discussing theoretical and empirical research on the teaching and learning of arithmetic and number systems. The scope of the working group comprises grades 1-12 and emphasizes research-based specifications of domain-specific goals, analysis of learning processes and learning outcomes in domain-specific learning environments and classroom cultures, new approaches to the design of meaningful and rich learning environments and assessments as well as research on teachers' professional development.

According to the great variety in the field of learning arithmetic and number systems, the group intensively discussed fifteen papers and one poster addressing research for different ages and different approaches¹. The key themes were number sense and structure sense, estimation and estimation tasks, flexibility in mental calculation, derived fact based strategies of multiplication in low-achieving students, understanding of rational numbers and ratio, didactical models as scaffolds for the evolution of mathematical knowledge as well as teachers' knowledge about rational numbers, ratio and place value according to big numbers.

In comparison to former working groups, we had an even greater variety of themes, and therefore the challenges and opportunities to identify common ideas and approaches in the papers presented. Two papers put the emphasis on teaching arithmetic, five on learning arithmetic, five followed a design based research approach with combining teaching and learning, and two papers focused on teachers' professional development.

Teaching arithmetic and number systems

The topics of the two papers focusing teaching were diverse. One paper reflected the role and function of didactical models, the other analyzed and compared textbooks.

Marita Barabash and Raisa Guberman presented a theoretical study on didactical models (DMs) to support students' development of mathematical concepts and ideas. The authors discuss didactical models as a mathematical model (supporting the development of appropriate and consistent

¹ Fourteen papers were resubmitted after the conference and are published in the proceedings.

concepts) as well as a learning tool (providing an experience of using models). In this sense, DMs may help to cope some discontinuity in mathematics teaching across different levels.

With the focus on a foundational number sense, *Anna Löwenhielm Gosia Marschall, Judy Sayers and Paul Andrews* analyzed and compared how English and Swedish textbook tasks facilitate children's learning of those number-related competences that require instruction. Analyses identified both similarities and differences. Swedish pre-school curriculum seems to have prompted a conceptually focused textbook, while the strongly framed English pre-school curriculum seems to have precipitated a procedurally focused textbook.

Learning arithmetic and number systems

The subject area “learning arithmetic and number systems” includes papers on number sense, structure sense, and understanding of rational numbers and ratio.

Students' development of structure sense in arithmetic is the focus of *Andrea Maffia and Maria Alessandra Mariotti*. Structure sense can be mobilized by students to compare and to transform arithmetical expressions, however sometimes it can lead to mathematical inconsistency that pupils might not be aware of. Their paper provides evidence of this type of phenomenon through syntactical transformations.

The assessment of students' ability in number estimation is the topic of a study conducted with Brazilian second and third graders. *Beatriz Vargas Dorneles, Mariana Lima Duro, Nohemy Marcela Bedoya Rios, Camila Peres Nogues and Clarissa dos Santos Pereira* compare a Number Line Estimation Task and a Numeroisty Task. Results show that the Number Line Estimation Task is more accurate in assessing students' performance in estimation.

Luciana Corso and Beatriz Vargas Dorneles put the emphasis on number sense and investigate the relation between three domains: working memory (especially the central executive and the phonological component), number sense and arithmetical performance. Based on different valid instruments, data was collected regarding each single component. The analyses reveal a significant correlation between the central executive component of working memory and number sense.

Ayşenur Yılmaz and Mine Işıksal-Bostan examine to what extent middle-grade students agree on statements about the ordering of two negative integers given within a real-life context, and what kind of procedural and conceptual strategies do middle-grade students generate to order those numbers. The results reveal that students did not explain the concept of ordering in daily life considering their conceptual meanings, and have problems in their procedural knowledge repertoire.

Ozur Soyak examines students' difficulties in proportional reasoning in rate and ratio problems. Students' difficulties are caused by the confusion of unit rate identification and by algorithmically based mistakes. A lack of understanding in the unit of measurement, the difference between additive and multiplicative reasoning, and some errors in the computation process seems to underlie such difficulties. The study suggests emphasizing proportional reasoning in the learning process.

Teaching and learning arithmetic and number systems

This time we had a considerable amount of papers following a design based research approach. All studies were characterized by designing a teaching and learning arrangement and investigating students learning and understanding according to this specific context.

With the focus on students' development, *Laura Korton* explores a teaching-learning arrangement for the inclusive mathematics classroom to foster flexible mental calculation. The approach considered both the level of design (consideration for use) and the level of research (quest for fundamental understanding). Initial findings appear to suggest positive outcomes when Mutual Learning processes are integrated.

Michael Gaidoschik, Kora Maria Deweis and Silvia Guggenbichler exhibit results of an ongoing design research project. Based on a specific instructional design that emphasizes conceptual understanding and derived fact strategies, the study investigates the exhibited strategies in basic multiplication of lower- achieving students. Within the analyses three different strategy types could be developed. Results also show the influence of the instructional context.

Cristina Moraes and Lurdes Serrazinha describe an approach to develop conceptual understanding of decimal numbers by using and adapting the hundred square model. Within a teaching experiment 3rd and 4th grade students worked with three different models focusing on part-whole meaning. Results indicate that models promote students understanding of rational numbers, and suggest the decimat as an important part-whole model.

Helena Gil Guerreiro and Lurdes Serrazina discuss an approach to focus students' conceptual understanding of rational numbers based on teaching percentages in elementary school. Within a teaching experiment, the authors collected data by logbook, audio and video-recording, and analyzed qualitatively. Results suggest that this specific approach supports to understand multiplicative relations and rational numbers.

The pilot study of *Carlos Valenzuela García, Olimpia Figueras, David Arnau and Juan Gutierrez-Soto* contributes to the development of better mental objects for fractions. Using a Theoretical Model for fractions, they designed and developed a seven stage teaching sequence based on the use of applets created with GeoGebra and the number line as a conceptual and didactical resource. Results of the first two stages suggest students' preferences to represent fractions as proper fractions (unit segment). The majority of the participants paid more attention to the graphical aspects of the applet.

Teachers' professional development

The two papers in this field focused on teachers' knowledge and its impact on students' learning.

Frédérick Tempier and Christine Chambris paper aims to reveal teachers understanding of place value related to larger numbers, and its impact on teaching and students understanding. Based on the "Theory of Didactic Transposition" the authors investigate the relations between all three aspects. The qualitative analyses of a teachers' lesson compared with the results of a questionnaire of students' knowledge provide interesting insights in teaching large numbers.

Gulseren Karagoz Akar conducted a single case study on teachers reasoning about ratio. Using a theoretical framework based on the concepts *within (state) ratios* and *between (state) ratios*, the teachers' conceptions of ratio are discussed. The results suggest different levels of reasoning in between-state ratios (as an operator and a combination of two extensive quantities), but some lack understanding about in between-state ratios as a single intensive quantity.

Summary and outcomes

The great variety of papers provided a fruitful base for interesting and often animated discussions in a supportive quizzical environment. We agreed that we were able to broaden our own perspectives in terms of new perspectives on different mathematics educational research domains, and different uses of terms, concepts and theoretical frameworks. In our discussions, we went far beyond the specific themes of the single papers and covered general aspects that influence our work as international and interdisciplinary group of researchers. Those were

- the synergy of cognitive science and mathematics education,
- the similarity and differences in research paradigms and approaches,
- the intercultural differences,
- the different terminologies and the relevance of creating common terminologies and
- the necessity to clarify concepts and theoretical frameworks.

This spectrum of research focus in the papers reflect the number of open questions that still persist in the teaching and learning of arithmetic and number systems, and in need of further research. The intense group discussions raised further awareness amongst the group and questioned some positions that were taken for granted in both the field and in specific contexts. With the commonly perused aim improving mathematics education in the field of arithmetic and number systems in all academic levels, we perceived common ideas as a base for further research and discussion: (1) The importance of number sense regarding different age levels, number systems and perspectives form different domains. (2) The necessity to develop models for fostering flexible relational thinking about numbers. (3) The requirement to enhance our notion of number literacy as interaction of various components.

For our further work, the group agreed on enhancing the exchange of research on arithmetic and number systems from cognitive psychologists and mathematics education researchers, and work on specifying terms for both fields to agree on.