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## Number estimation in children: An assessment study with number line estimation and numerosity tasks

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*Number estimation is an important skill for both everyday and school situations that involve a group of cognitive abilities. The ability to estimate may provide a feedback tool to check performance in different mathematics learning contents. The most widely used task to assess number estimation is the Number Line Estimation Task (Siegler & Booth, 2004), and some researchers used a kind of Numerosity Task (Luwel, Verschaffel, Onghena & De Corte, 2003). This research compares the students' accuracy in two tasks that assess the ability of Brazilian children (N = 60), attending the 2nd and 3rd grades of a public school, to estimate. The children's success in the Number Line Estimation Task suggests it is more accurate than the Numerosity Estimation Task in assessing children's performance in estimation maybe because of the different cognitive functions required by the two tasks. The study's educational implications are discussed.*

*Key words: Number line estimation, numerical development, spatial representation of number magnitude.*

### Introduction

The decimal number system is used to establish exact quantifications, in contrast to quantity estimation processes. In everyday situations, we often use either exact quantification or number estimation (Feigenson, Libertus, & Halberd, 2013), and sometimes estimation can be easier than exact quantification (Siegler & Booth, 2004). Number estimation is a cognitive process used for quick or approximate answers or, for example, to calculate the duration of a movie or the distance between two places. It can be used as a feedback tool to check performance in different areas of mathematics including those requiring exact quantification. From our point of view, two complementary ideas define number estimation: a non-counting based quantitative answer to represent a set of objects; and a translation (Siegler and Booth, 2004) between two different ways of representing a number. We know that mathematical competence involves a group of abilities and cognitive processes. Number estimation has been considered one such process (Levine, 1982) despite the fact it has been less studied than exact quantification (Piazza, Mechelli, Price & Butterworth, 2006) although its importance has been highlighted (Rousselle & Noel, 2008). This can be explained by the variability of the tasks used to assess estimation skills in children, adolescents and adults as well as the different situations in which we use estimation (Siegler & Booth, 2004). The recognition of small amounts may be related to the ability to represent quantities in a mental number line and this ability would assist in comparing the magnitude between two numbers (Schneider, Grabner, & Paetsch, 2009). The estimation performance can be necessary in solving some mathematical tasks and the development of estimation is also considered a good predictor of later symbolic math skills (Park & Brannon, 2013). Despite the importance of its use, number estimation is not part of the school curriculum in many countries, including Brazil. Changes designed to improve mathematical achievement, including the introduction of number estimation in the curriculum, are currently being introduced in Brazil. For teachers, assessing the ability to estimate using different tasks might be a good starting point to analyze the

importance of the ability as well as to highlight the topic's importance in mathematics education. One of the most consistent conclusions reached by studies about the development of estimation is that children are not very able estimators, even when estimation is used in various daily applications. However, some researchers have hypothesized that children's estimations reflect their internal representation of numbers (Siegler & Opfer, 2003).

Moreover, there is some evidence to suggest estimation is related to mathematical competence in general and arithmetical performance in particular (Siegler & Booth, 2004; Booth & Siegler, 2006; Schneider et al., 2009; Mazzocco, Feigenson, & Halberda, 2011; Laski & Siegler, 2007) and can be improved (Park and Brannon, 2013). Recent research has highlighted the importance of estimation for mathematical development (Link, Nuerk, & Moeller, 2014; Laski & Yu, 2014). This research indicated that the better the students' accuracy in mental number line is, the better their performance in other numerical and arithmetic tasks (Link, Nuerk, & Moeller, 2014). Hence, it is important to understand the estimation process, the abilities involved, how to assess them and its role in mathematical performance, especially regarding the proposed changes to the curriculum in Brazil. It can be said that even though the estimation processes have been studied for the past twenty years, there is no consensus regarding the assumption that the estimations assessed by different tasks reflect a pure mental representation of numbers as proposed by Siegler and Booth (2004) and some new data indicate that it is affected by the limited knowledge of numbers (Ebersbach, Luwel & Verschaffel, 2015), as well as by visuospatial abilities (Crollen & Noël, 2014).

In the current scientific debate, among the explanations for the development of number estimation in children, two stand out. The first, the multiple representations of numbers model (Siegler & Booth, 2004), assumes that children initially represent numbers in a less accurate algorithmic way and develop a more accurate linear representation with age and experience. That is, in children the mental number line is compressed and they tend to maximize the distance between the magnitudes of numbers at the low end of the range and minimize the distance between the magnitudes of numbers in the middle and upper ends of the range. This tendency was named logarithmic representation. Gradually, children develop a linear representation, which maintains the same distance between the numerical magnitudes. Empirical evidence to support this logarithmic-to-linear shift model has largely come from the Number Line Estimation Task (NLET) proposed by Siegler and Booth (2004) in which children or adults must estimate the magnitude of a number by marking its proper position on a number line. However, the logarithmic-to-linear-shift hypothesis has been questioned by researchers studying number-line estimation (Barth & Palladino, 2011; Ebersbach, Luwel & Verschaffel, 2015). Some of the issues raised led to the development of a second explanation, the proportion-judgement strategies model (Barth & Palladino, 2011), which suggests that in the NLET children estimate the size of a part, the numerical magnitude of a specific number, relative to the size of the whole, thus making a judgement about the proportion of the size of the former. Hence, according to the numerical range, the more reference points (landmarks) made available, the more accurate the estimation will be, especially close to the landmarks. In other words, estimation performance reflects the strategies chosen to solve the tasks. Recently, a third model, the Two-Linear Account has been proposed as a plausible alternative (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008) and explains the developmental changes in number estimation as a result of children's familiarity with numbers. In this model, the mental numerical representation can be alternatively described as a combination of two linear patterns with different slopes, depending on number familiarity. In other words, this linear representation of numbers changes according to the age and numerical range known by children; the unknown numbers have

a slower linear representation than the known numbers. A recent paper (Dackermann, Huber, Bahnmüller, Nuerk & Moeller, 2015) proposes the integration of these accounts, which is a line of reasoning that we support as they introduce the idea that aspects of all three accounts may complement each other and facilitate a more comprehensive understanding of children's development of number line estimation.

Among the tasks most widely used to assess the ability to estimate are the NLET, described above, and the Numerosity Estimation Task (NET), which requires the subject to estimate the quantity of objects in a set (Luwel, Verschaffel, Onghena & De Corte, 2003; Barth, Starr & Sullivan, 2009). To the best of our knowledge, there is no research that indicates which of these tasks best assesses students' accuracy in estimation and therefore which would be best for application in research and in schools. Thus, the purpose of the study was to compare the accuracy in the numerical estimation of 60 children from the 2<sup>nd</sup> and 3<sup>rd</sup> grades of a public school in the city of Porto Alegre (Brazil) in order to determine which task (NLET or NET) best assesses the students' accuracy in number estimation. As the children in both grades were used to manipulating objects and completing tasks similar to the NET and admittedly had no contact with the number line before, we assumed the students would perform better in the NET than in the NLET. We chose the two tasks because the NLET is the most frequently used in estimation research and is widely used to examine how the human mind represents numbers (Barth & Palladino, 2011; Ebersbach, Luwel & Verschaffel, 2015), while the NET is similar to another task often used by some research groups that have a slightly different theoretical viewpoint regarding estimation, for example, Barth, Starr & Sullivan (2009). Both tasks are assumed to assess the same numerical estimation ability.

## Method

Using two different tasks, we compared performance in number estimation within a group of sixty children (mean age = 8.4, SD = .69, age range from 7.4 to 11.2 years) who were recruited from one public school, 37 boys and 23 girls: 28 from 2<sup>nd</sup> grade, (M = 7.8, SD = .29) and 32 from 3<sup>rd</sup> grade (M = 8.9, SD = .50). Two tasks were used: the NLET (Siegler & Booth, 2004) and the NET, adapted from Luwel, Verschaffel, Onghena & De Corte (2003). We used only one criterion to determine the students' accuracy in the presented tasks: the measure proposed by Siegler and Booth (2004), described below. The task was applied collectively in the classroom, the workplace and the school activities were affected as little as possible. The time to perform the activities in each class was about 40 minutes per task. The solution to both tasks involved the use of pen and paper.

The NLET requires the subjects to mark points corresponding to specific numbers along a number line bounded by 0 and 100. Children were asked to mark the place they considered most suitable for the position of the number to be estimated. Before each item, the experimenter said, "This number line goes from 0 at this end to 100 at this end. If this is 0 and this is 100, where would you put n?" (n being the number specified in the trial). 29 number estimations were required, one at a time. Each number was presented twice. Each child received a booklet with a number line drawn on each sheet to mark their answers. The difference between the two estimations of the same number provided a measure of the variability of the estimations. The 29 numbers comprised the 24 proposed by Siegler and Booth (2004), plus another 5 numbers that were also used in the second task. The 29 numbers presented were 3, 4, 6, 7, 8, 9, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 49, 52, 57, 61, 64, 72, 78, 79, 81, 84, 90, 95, 96. They were presented in random order and then repeated in the same random order. Children had no pre-determined time to finish the task, each one could complete the task in the time they wanted.

The NET requires the children to estimate the amount of dots distributed in a checkered 10x10 grid. Before starting the task, the students were told the empty grid contained 0 dots and the full grid contained 100 dots. To reduce the possibility of verbal counting, the stimuli were presented quickly (1 second for each group of ten dots presented) and immediately followed by a white screen. Students were asked to perform a numerical estimation of the amount observed, writing them down in a notebook. They could not use any additional tool to solve the task. In the task, eight numbers from 0 to 100 were randomly matched (4, 7, 9, 17, 25, 49, 78, 95) in two different ways. In the first, the dots were presented in clusters and, in the second, they were presented dispersed. Both tasks were carried out collectively on different days. There was no feedback for correct or wrong answers.

## Results

To calculate the accuracy of the estimations given, the calculation of absolute percentage error of each child was used, adapted from Siegler and Booth (2004), and represented by the formula:

$$\frac{\text{Mean Estimation} - \text{Estimated Quantity}}{\text{Scale of Estimations (100)}}$$

To illustrate how this measure works, if a child was asked to estimate the location corresponding to the number 60 (or quantities of dots) in a number line from 0 to 100 (or 10X10 grid) and his/her answers were 65 in the first estimation and 75 in the second, we calculated the mean between the estimated values in the two attempts (in this example,  $(65+75)/2=70$ ). The absolute percentage error would be 10%, corresponding to the result of  $(70 - 60)/100$ , according to the above formula. We used this calculation because previous analysis showed that the difference between two answers for each number in both tasks was not significant ( $p=.24$ ) for NLET or ( $p=.06$ ) for NET.

After that, a descriptive analysis of the accuracy of each child was conducted to identify the general standard of performance for each task. These analyzes show the students' accuracy is higher in the NLET. The reported performance tends to be more cohesive in the NLET (Table 1).

	Mean	SD	Significance
Number Line Estimation Task	.089	(.050)	$p = .018^*$
Numerosity Estimation Task	.158	(.250)	

\* $p < .05$

**Table 1: Comparison of the Mean of Percentage Error in each task**

To determine the correlation between the two tasks, Pearson's correlation coefficient was carried out ( $r = .67, p < .01$ ), and indicated a positive correlation between both tasks, suggesting that either demand similar cognitive functions or both are related to other skills that were not measured.

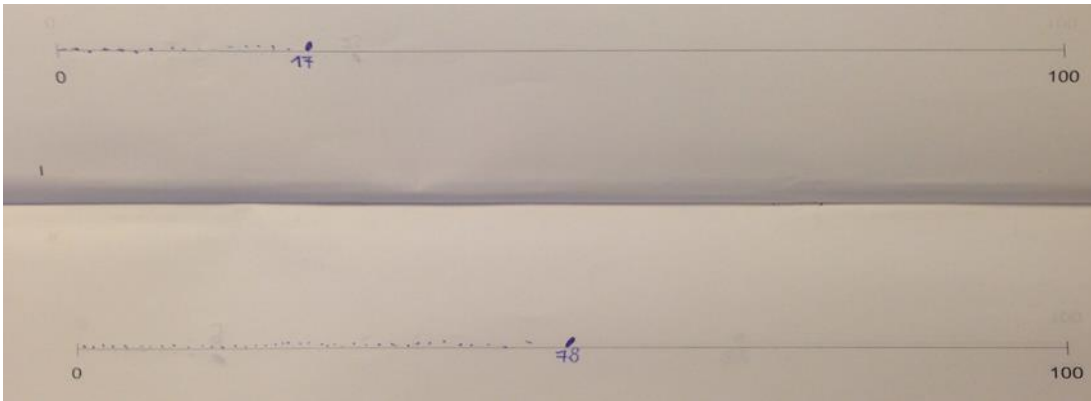
To test for differences in the children's estimation accuracy when required to estimate smaller and larger amounts, we considered the same numbers estimated in both tasks. A paired-samples t-test only indicated difference when estimating the same number in each task in larger quantities (Table 2).

Number to be estimated	Number Line Estimation Task		Numerosity Estimation Task		Sig.
	Mean	SD	Mean	SD	
4	.034	(.043)	.086	(.333)	<i>ns</i>
7	.063	(.086)	.084	(.397)	<i>ns</i>
9	.081	(.101)	.083	(.229)	<i>ns</i>
17	.109	(.112)	.188	(.495)	<i>ns</i>
25	.117	(.087)	.151	(.184)	<i>ns</i>
49	.085	(.062)	.204	(.145)	.000*
78	.118	(.064)	.249	(.224)	.000*
95	.091	(.080)	.219	(.441)	.022*

\* $p < 0.5$

**Table 2: Comparison of estimations between the same numbers**

This analysis showed that the students' estimations were all more accurate in the NLET, however, these differences were only statistically significant with the numbers 49, 78 and 95 (Table 2), while there is greater variability in the children's estimation in the NET. This may suggest important differences between the tasks performed. Maybe the three numbers (49, 78 and 95) are closer to "quarters" (50, 75 and 100), which would help children to identify the position of the numbers in the number line. Hypotheses to explain these variations will be discussed later. In the NLET, one of the most common strategies used by students was to fill the number line with marks that represent the numbers before or after the number proposed, as illustrated in Figure 1. In the NET, some students tried counting groups of dots and imagining how many similar groups could be in the whole. Progressions related to the speed with which the children performed the estimations were not tested in this analysis, considering that the time for execution was the same for all participants.



**Figure 1. Example of marking strategy in two numbers of NLET**

## Discussion and conclusions

As we described above, number line estimation is related to both basic and complex arithmetical abilities. Moreover, there is evidence to suggest number estimation is related to mathematical achievement. Despite this, number estimation is rarely taught in Brazil and other countries. In our research, we observed children more accurately estimate numbers on a number line than dots on a grid. Although the two tasks measure the ability to estimate, as indicated by the general correlation between the students' accuracy in the two tasks, they may be linked to different cognitive functions, as suggested by the differential performance when the tasks involve numbers over 25. Although speculative in nature, some ideas help us to understand the results. The NLET requires transposition from numerical knowledge to a position on a line, whereas the NET requires transposition from a perceptual estimation (linked to quantities) to numerical knowledge. Both demand translation between different representations. However, unlike the NET, the NLET allows children to try a discrete quantity representation, sometimes marking the number line with lines or dots from the beginning to the point that could represent the required number. This marking strategy was the most widely used by the children and helped them identify an almost correct answer. Children used different strategies or representations when estimating. The linear distance of numbers along the number line seems to be an important support for the estimations, as Siegler and Booth (2004) have described. Additionally, children used decision-based strategies considering the proximity of the extremities, for example, to represent number 78, they made marks from 100 to 78 in descending order. Alternatively, some decided to begin from 50 and made marks from 50 to 78 in an ascending sequence. Another strategy was to mark the quarters (e.g. 25, 50, 75) as landmarks. All these strategies have been identified as means to improve the way children estimate (Siegler & Opfer, 2003; Siegler & Booth, 2004). The use of these strategies suggests the children did not estimate the numbers by chance, but instead coordinated mathematical knowledge and spatial skills to assess the place to mark. This tactic fits very well with the proportion-judgement strategies model (Barth & Palladino, 2011). The central number (50) was understood by some students as a reference mark used to estimate the other numbers.

The difference found in the tasks with the numbers 49, 78 and 95 can be explained by the fact the three numbers are close to “quarters”, which would help children to identify the position of the numbers on the number line. Also, we must remember that, generally, younger children tend to overestimate small numbers and compress large numbers toward the end of the scale, whereas older children, using the same number range, tend to estimate more accurately (Siegler & Booth, 2004). Moreover, in our research, it may be the

case that due to lack of familiarity with the number line task, the older children continued to overestimate small numbers, as described in the logarithmic model. Another possible factor influencing the results was the time. In the NLET, children have time to think about the relation between the number and the place on the line, while in the NET they have only a few seconds to observe the quantities and more time to estimate the quantity represented. The NLET requires the ability to coordinate the knowledge of number systems with a kind of spatial graphic representation on a line. Maybe the effort made by children to coordinate these two cognitive demands helps them to estimate more accurately, despite having little experience with number line tasks. Moreover, the NET may not help students to access their knowledge of the number system, since it provides no clues that would allow students to adopt some proportion-judgement strategies coordinated with their number system knowledge. We suggest the NET requires more “guessing” than the NLET because there is no opportunity to mark “quarters” or anything like that to help them to estimate. One limitation of the research is that only eight numbers were repeated in both tasks. For future research, we suggest amplifying the analysis of the cognitive processes involved in both tasks. We did not control the number system knowledge of the subjects. We do not discuss whether or not the number line task reflects an internal mental number line, since this was not the subject of the paper. However, considering the results of our research, we can say that the NLET is a relevant measure even if it does not reflect an internal mental number line. We support this idea because, although the children were not familiar with the number line estimation task and were more used to NET-like activities, they performed better in the NLET. This surprising result provides the opportunity to introduce the discussion about estimation in the Brazilian curriculum. Furthermore, considering that the ability to estimate is correlated to many aspects of mathematics, for example, number comparison, addition, and subtraction, it is important to take in consideration the assessment of this ability in mathematics education as well as the ways to improve it.

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