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Do lower-achieving children profit from derived facts-based teaching of basic multiplication? Findings from a design research study

Michael Gaidoschik¹, Kora Maria Deweis² and Silvia Guggenbichler³

¹Free University of Bolzano, South Tyrol, Italy; Michael.Gaidoschik@unibz.it

²Alpen-Adria University Klagenfurt, Austria; KoraMaria.Deweis@aau.at

³Univ. College of Teacher Education Carinthia, Austria; silvia.guggenbichler@ph-kaernten.ac.at

We present findings made during the first cycle of an ongoing design research study on the working out of basic multiplication in 8 Austrian classes. Their teachers had tried to implement an instructional design that put conceptual understanding and derived facts strategies centre stage. Focusing on the degree of fact mastery reached at the end of grade 3, we present a typology of strategy use within a sample of 48 students. We take a closer look at lower-achieving students, in particular those 8 students who had little if any success in mastering basic multiplication. While 6 of them used derived facts strategies quite often, their deficiencies either in adding and subtracting or with regard to the conceptual basis of derived facts strategies seem to have hindered them from mastering more facts. We discuss implications for the planned second cycle of the study.

Keywords: Basic multiplication, derived facts strategies, lower-achieving students, design research.

Introduction

Sherin and Fuson (2005), in an overview of prior work on teaching and learning basic multiplication, refer to 4 different though interlinked threads of research. Like them, in this paper we focus on only one of these, namely the development of computational strategies. We agree with Sherin and Fuson's assertion that strategy development must be examined with close reference to the ways multiplication is taught. Hence, after looking over different ways learners solve multiplication tasks, we summarize contemporary approaches as to how to work out basic multiplication in primary grades. Against this backdrop, we focus on the multiplication learning of lower-achieving students. We contribute to that issue some findings of a design research study on 8 classes whose teachers had tried to base the learning of multiplication on the targeted working out and practicing of derived facts strategies.

Empirical framework and research questions

A taxonomy of strategies used for basic multiplication

Sherin and Fuson propose that the “most important changes” in the development of strategies for basic multiplication are primarily “driven by relatively incremental changes to number-specific computational resources” (Sherin & Fuson, 2005, pp. 353–354). So a child might solve, e.g., 3×4 initially by drawing 3 groups of 4 circles each and “counting all” of them; later by “rhythmic counting” (“one, two, three, *four*, five, six, seven, *eight*, nine, ten, eleven, *twelve*”); then by “repeated addition” of $4+4+4$ or maybe by saying the “count-by sequence” (“four, eight, *twelve*”). Finally, the child may “retrieve from memory” that “ $3 \times 4 = 12$ ”. These strategies as well as “hybrids” such as applying a “derived facts strategy” (“ $2 \times 3 = 6$, then 4×3 must be twice as much”) form part of the taxonomy of strategies the authors devise drawing on prior research. But even if the strategies as listed follow a progression, this is not to be seen as the consequence of an increasingly sophisticated

understanding of multiplication, but mainly driven by a child's growing abilities to, e.g., add, subtract, double and halve, and thereby compute products with increasing ease (Sherin & Fuson, 2005).

This is why, for a long time during the learning process or even permanently, a single individual will presumably use a variety of strategies for solving different basic multiplication tasks, depending on the value of the operands and his or her computational resources. What is more, “because the learning of number-specific resources is very sensitive to instructional emphasis” (Sherin & Fuson, 2005, p. 379), strategy development may differ significantly across classrooms. Such differences are sufficiently documented. For instance, Sherin and Fuson (2005) report that rhythmic counting, while being ascribed an important role by Anghileri (1989), was not observed at all during the interviews they carried out with students whose teachers had not promoted that strategy. Steel and Funnell found no evidence for the use of repeated addition within a sample of 241 children in grades 3 to 7, but a large amount of children using count-by sequences. Their teachers stated they had not encouraged repeated addition, whereas writing down sequences had been used as a method (Steel & Funnell, 2001). This leads to the question of how basic multiplication *should* be taught at the beginning.

The need of further design research on the teaching and learning of basic multiplication

There seems to be an international consensus nowadays that students should *both* acquire a sound conceptual understanding of multiplication *and* eventually solve all basic tasks accurately and effortlessly (cf., e.g., Padberg & Benz, 2011; CCSSI, 2016). It is also widely agreed that fact mastery should not be pursued by rote learning of multiplication tables. As an alternative, children should first learn how to solve harder problems by deriving them from those that are fairly easy to remember, i.e. the problems with 2, 5 and 10 as the multiplier. Only later should they move on to retrieving more and more facts directly from memory (cf. Gasteiger & Paluka-Graham, 2013; Van de Walle, 2007).

However, when it comes to “details of instruction” that might be relevant for teaching success (Sherin & Fuson, 2005), there remain a lot of unresolved issues. One of them is whether or not to make children work within separate multiplication tables (e.g., the “table of 6” from 1×6 to 10×6). A specific answer quite commonly given in Austrian textbooks is “short tables”: Children are supposed to learn the whole body of basic facts by consecutively learning the facts of one table at a time before moving to the next table. Within each table, they are supposed first to automate 1 times, 2 times, 5 times, and 10 times the respective number and to derive from these core tasks the other tasks of that table. In the second step, they should practice all the tasks of the table with the objective of automation.

In contrast, Gaidoschik (2014) advocates what he calls a “consistent” approach to derived facts strategies. According to this, strategies should be worked out in a targeted manner without any consideration of separate tables. For example, as soon as children have learned that they can easily derive a 9-times fact from a 10-times fact, they should be encouraged to do so with *any* 9-times fact. The same applies to any other strategy like halving 10-times to derive 5-times facts or doubling 2-times to solve 4-times facts. If the commutative property is emphasized from the very beginning, there is little need for strategies that demand computations that are more difficult. For instance, 7×9 can be done more easily by thinking of 9×7 , which is $10 \times 7 - 7$, than by adding $2 \times 9 + 5 \times 9$. Therefore, to abandon activities that are restricted to single tables is supposed to reduce the overall workload. Secondly, it should help get a better understanding of any single strategy since it is applied to a wide range of numbers as soon as it has been established, and not just within a single table. Last but not

least, this approach emphasizes the wide reaching power of these strategies, which is supposed to contribute to the children's willingness to acquire them (Gaidoschik, 2014).

Evidence as to whether and how such differing approaches to the teaching of basic multiplication indeed have an impact on children's learning is rather scarce and fragmentary. Cook and Dossey (1982), comparing teaching the tables with a derived facts strategy approach, find empirical support for the latter, but remain vague about the specifics of either approach. Woodward (2006) gives more details about an integrated approach combining explicit teaching of derived facts strategies with timed practice drill which yielded significantly better results than drill only. However, he reports not on regular classroom activities, but on a remedial programme applied when multiplication had already been worked out. So do most other studies in this field (e.g., Kroesbergen, Van Luit, & Maas, 2004).

One of the few studies known to the authors that deal with the initial workout of basic multiplication in a regular classroom and try to deliver a "rich description of the way the design works" (Swan, 2014, p. 151) is Selter's (1994) report on a teaching experiment with one grade-2 class in Germany. The teaching in this class favoured derived facts strategies throughout the second half of the school year, while it deliberately downsized drill. The study indicates that this concept was quite successful with regard to its conceptual targets. However, the account of whether it worked out equally well regarding the development of fact mastery is less satisfying. The study is rather sketchy in that respect. Selter (1994, p. 106) rates it as the "preliminary ending to a research project" to be followed by more detailed inquiries into single issues, such as "learning processes of underachieving pupils" (p. 281).

Questions addressed in the study presented in this paper

Teaching basic multiplication with a focus on derived facts strategies is still not at all common in Austria, where there is a long tradition of basically drilling tables with little, if any, consideration of derived facts strategies. Teachers particularly tend to be sceptical as to whether lower-achieving students would a) understand derived-facts strategies at all and b) reach fact mastery without rote-learning of the tables (Gaidoschik, 2014).

Against that backdrop and in consideration of the empirical framework outlined above, we started a design research project on the teaching and learning of multiplication in grades 2 to 3. In this paper, we present some findings collected during the first cycle of the project that lasted from September 2014 to June 2016. Out of the numerous issues we address in that project, in this paper we have to restrict ourselves to the following:

- 1) If multiplication is taught with a clear focus on derived facts strategies in the "consistent" way suggested by Gaidoschik (2014) and shortly outlined above, what types of strategy performance can be identified at the end of grade 3 with respect to the target of fact mastery?
- 2) To what extent, when taught like this, do children who have been identified by their teachers as mathematically lower achieving actually use derived facts strategies and reach fact mastery?

Method

We report on 48 students from 8 classes in Carinthia, Austria. Their teachers had volunteered to participate in a design research study aiming at evaluating and refining the concept of teaching multiplication in grades 2 to 3 as formulated in Gaidoschik (2014), with the following main ideas:

- 1) In the first half of the second school year, arithmetic lessons should have a clear focus on what children need to understand and be able to compute fluently in order to get comfortable with the strategies that are useful to derive multiplication facts. That is, they should be able to double and halve two-digit numbers effortlessly and to add and subtract fluently up to 100.
- 2) Subsequently, instruction should concentrate on the conceptual understanding of multiplication and its properties, particularly commutativity and distributivity. At the end of this stage, children should be able to translate smoothly terms such as 3×4 into actions, visual representations (identical groups as well as arrays), word problems, and vice versa.
- 3) In the next step, teachers should secure that all children know how to double and decuple any number at least up to 10 with ease, then learn how to derive the 5-times facts from the 10-times facts by halving and do so more and more effortlessly.
- 4) On that basis, a *guided discovery-learning* approach should be complemented with *direct instruction when needed* by single children to convey derived fact strategies as a convenient way to solve multiplication tasks. To this end, single lessons should be devoted to groups of facts as defined through the multiplier, for instance 9-times facts, 6-times facts, and so on. Children should be encouraged to find an easy way for themselves to solve tasks of such a group by deriving the solution from facts they already know, using representations such as arrays of dots or equal groups of interlocking cubes. Strategies found by the children should be discussed and compared in the classroom. Different strategies for the same task are welcome as long as they are mathematically correct. However, children who constantly fall into cumbersome ways to derive a task (such as, e.g., computing 6×9 as $10 \times 9 - 9 - 9 - 9 - 9$ instead of $10 \times 6 - 6$ or $5 \times 9 + 9$) or even resort to repeated addition or counting strategies should receive direct instruction to develop understanding for one derived facts strategy after the other, including the knowledge for which tasks that strategy fits well.
- 5) Subsequent practice should comprise substantial tasks such as explorations of mathematical patterns as well as timely restricted “strategy drill” (Van de Walle, 2007), e.g. using flash cards with the objective of performing a certain strategy with growing ease and speed.

To convey and discuss this concept in detail, the researchers met the participating teachers for 8 working sessions (3 hours each) once a month during the second and 4 follow-up sessions once every 2 months during the third school year of their classes. During each session, the researchers would give theoretical inputs and make concrete proposals for daily classroom activities. Each teacher was visited in the classroom 3 times by one of the researchers to receive feedback on his or her teaching practices. The teachers were interviewed individually 4 times during the cycle to cover as comprehensively as possible if and how closely they had followed the researchers’ recommendations.

To assess the children’s development, we selected 6 children out of each class to be interviewed a total of 7 times, from October 2014, at the beginning of their second school year, till the end of April 2016, before the classes started to move on to multi-digit multiplication. Always 2 of the 6 children had been rated as being above average, average, and below average with respect to their arithmetical performance as perceived by their teacher at the beginning of grade 2. The students were interviewed by the researchers during school time in some quiet extra rooms of the school. The first interviews were centred on addition and subtraction up to 20, which had been the main contents of arithmetic instruction till then. The semi-structured qualitative interviews to follow each reflected what had been the major classroom topics since the previous interview, from the base-10-system in January 2015 to

a focus on multiplication in the later interviews. In accordance with the instructional design, the interviews were restricted to the conceptual understanding of multiplication in March 2015, but starting with May 2015 encompassed both understanding and computation of multiplication tasks.

In the computation part of the interviews the children were presented always with the same 15 tasks, each of them written on a DIN A7 card, 7 being core tasks (10×7 , 2×8 , 4×10 , 9×2 , 5×7 , 8×5 , 5×5), and 8 harder tasks (in the order of the interview: 9×4 , 7×7 , 6×4 , 6×9 , 7×8 , 6×7 , 8×8 , 4×7). The children were requested to solve each task mentally the way they usually would, and to state the result verbally as soon as they knew it. Immediately thereafter they were invited to explain or show how they had arrived at the solution. Strategies were evaluated on the basis of video recordings. “Fact mastery” was assigned to any solution that was accurately produced within 3 seconds (cf. Van de Walle, 2007).

Apart from computing, we invited the children *inter alia* to explain verbally to a fictitious first-grader the meaning of a task such as 3×5 and demonstrate that meaning with different materials (wooden cubes, arrays of dots). Moreover, we asked them to clarify whether and how an easier multiplication task could be of help to solve the not so easy tasks 9×7 and 4×8 , respectively.

Findings

Types of strategy use in solving basic multiplication tasks

Based on the multiplication strategies exhibited by the children in April 2016, we performed an empirically grounded construction of types and distinguished 3 main-types of strategy performance at the end of grade 3:

- A) “Masters”: These students solved all tasks accurately either by retrieval or effortless derivation within about 3 seconds or, in most cases, instantly. Only in single cases, if at all, they would give a wrong answer or take slightly longer to produce a correct one. 19 out of the 48 children go smoothly with this type, and 5 children fall somewhere in between Type A and B (see below).
- B) “Experienced users of derived facts strategies with limited fact mastery”: These children, while exhibiting mastery of the core tasks, relied on derived facts for at least 3 of the 8 harder tasks. As a rule, these tasks were solved with rather little effort in 6 seconds or less. However, up to 2 of the 8 harder tasks still posed quite a challenge to these children, either taking more than 9 seconds or being answered incorrectly. 15 to 20 (see above) of the 48 children rank among this type.
- C) “Users of derived facts with limited mastery and some trouble in deriving”: These children typically solved core tasks by retrieval and harder ones by using derived facts strategies. At least 3 tasks caused them perceivable trouble, with solution times in excess of 9 seconds and/or resulting in incorrect solutions. 6 of the 48 children quite clearly fulfil this description.

These 3 types cover 45 of the sample’s 48 children fairly adequately. Note that fact retrieval and derived facts strategies were the only strategies to be found within these types at the end of grade 3, with no single child relying on strategies like counting all, counting by or repeated addition.

3 children do not fit into this typology. One of them, whom we refer to by the fake name Leo, was the only child in the sample that relied on count-by sequences. He did so when solving 9×4 , 6×4 , and 4×7 , all of them within 3 or 4 seconds. All the other facts he solved by retrieval, except 6×7 , which

he derived from 6×6 . Leo's teacher reported that Leo had consistently been getting "a great deal of support" by his mother. The teacher had "not been able to convince her of not drilling the tables".

Whereas Leo had reached a high degree of fact mastery, this clearly is not true for 2 other children. One of them, we call her Mia, had only mastered 4×10 , 10×7 , 2×8 , 5×7 , and 5×5 . Whereas she retrieved these facts accurately, all the other tasks she either refused to try at all, or admittedly guessed upon. Only 4×7 did she solve correctly by a rhythmic count-all supported by her fingers. In no case did she use a derived facts strategy. The other child, whom we name Lara, solved all the core tasks as well as 7×7 and 8×8 by retrieval. On 7×8 , she used a hybrid strategy by counting down 8 and another 8 from $9 \times 8 = 72$, which she had retrieved from memory. She tried to derive 6×4 as well, this time incorrectly by computing $5 \times 4 + 5$. 9×4 she rated as her "battleground task"; she eventually solved it by drawing 4 rows of 9 circles each and counting them all, which took her about 90 seconds.

Out of the 16 children who had initially been selected as below average, 3 were assigned to Type A at the end of their third year, 6 more to Type B, and one child between A and B. Out of the other 6 students who started as below average, 4 finally belonged to Type C, whereas the remaining 2 of this group have been introduced as Mia and Lara. There were 2 other children who had been rated as average by their teachers at the beginning of the second year but were finally assigned to Type C.

Interrelations with the performances on other tasks

As set forth, 8 children within our sample demonstrated only very moderate, if any, success in having mastered basic fact multiplication by the end of their third school year. To help better understand this, we refer to some of the findings we made aside from the computation part of the interviews. First, the tasks that were conceived to test the conceptual understanding of multiplication unsurprisingly revealed considerable differences, in particular with regard to the verbal competences of the children. However, all 48 children, including the 8 lowest achieving, could without any exception give a comprehensible and adequate verbal explication of what meaning could be ascribed to a term like 3×5 , and support this by laying equal groups or arrays of wooden cubes.

What is more, all these 8 children seemed to have at least some clue of how to derive facts from other facts. This even applies for Mia, albeit in a very restricted way. She was the only child of this subgroup who had not used one single derived facts strategy during the computation part of the interview. Subsequently, when questioned whether there was an easier task that could help her solve 9×7 , she spontaneously answered: "No idea". However, when directly asked whether 10×7 could be helpful, she said without any hesitation: "Yes, you just have to take away 7." Significantly, though, when asked whether knowing 5×8 could help a child solve 4×8 , she stated: "Yes, you have to take away 4." The same mistake was made by Lara. Lara also erred in computing 6×4 as $5 \times 4 + 5$ in the computation part (see above), but she, too, stated that 9×7 could be derived as "10 times 7, then take away 7".

Only 2 other of the 8 lowest achieving children made such mistakes with regard to the *logic* of a derived facts strategy (as distinguished from *calculation errors* in executing a mathematically correct strategy; such errors were made by other children, too). For instance, one child said that to get 6×9 you have to compute $60 - 9$, and the other that $10 \times 8 - 2 \times 7$ equals 7×8 .

The other 4 children of Type C not only used a variety of derived facts strategies in a mathematically correct way during the computation part, but also gave comprehensible explanations of how to derive 9×7 and 4×8 . It seems to be noteworthy that these 4 children as well as Mia and Lara and 2 children

of Type B were the only ones in the whole sample who had considerable trouble with the addition and even more with the subtraction tasks they had to solve in a separate section of the interviews.

Discussion and final remarks

We are well aware of the limitations of our paper. As stated by Swan (2014, p. 151), “writing up design research is problematic”. Due to space restrictions, the design is presented rather sketchily, as is the account of its implementation and the ways different children performed. Having conceded this, we still hope that some of our findings are of use and interest for other researchers in this field.

First, our results seem to corroborate the view of Sherin and Fuson (2005) that the development of strategies for solving multiplication tasks is to a high degree dependent on instruction. We recorded the tedious use of count-by sequences known to be rather typical of low-achievers in higher grades for only one out of 48 students at the end of grade 3. In that case, we have clear indications of parental drill. We may add that this strategy had been equally rare in the preceding interviews. On the other hand, all but one student used derived facts strategies autonomously for those facts that they had not yet automatized. Both occurrences correspond with the applied instructional approach that, as outlined, deliberately neglects working within single multiplication tables, thus aiming to prevent children from using count-by sequences as a solution strategy. As far as can be judged from teacher interviews, classroom visits and the examination of working sheets used in the classrooms, all teachers basically adhered to that concept.

This leads to a second finding: Teaching multiplication with a clear focus on derived facts strategies yielded what might be seen as quite satisfying results even for lower-achieving students. Of course, the study was not conceived to prove the superiority of one design over another, but to examine qualitatively whether and how certain measures may contribute to children’s learning. With regard to lower-achieving students, it can be stated that the chosen combination of discovery-learning and direct instruction of single strategies followed by strategy drill seems to have helped almost all of them *use* these strategies *correctly* (as can be judged from the computation tasks) on the basis of a sufficient conceptual *understanding* (as can be judged from the additional tasks described above).

Thirdly, 8 students in the sample show severe problems with basic multiplication at the end of grade 3. 6 of them, constituting the Type C, still use derivations and do so quite frequently. But they do it in a way which indicates what may contribute to their struggling with multiplication: 4 of them, while apparently knowing how to use known facts as a basis for deriving unknown ones, are not able to add and subtract efficiently. That is why they often need considerable time to solve a task and sometimes miscalculate. We assume that out of the same reason their repeated use of derived facts strategies on harder tasks has not resulted in the mental linking of these tasks and the respective solutions (cf. Van de Walle, 2007) and therefore not contributed to the automating of basic facts to the same extent as it has done for their more successful peers. In this regard, their learning difficulties differ *qualitatively* from those of the other 2 children within Type C who are indeed quite proficient at adding and subtracting, but then again seem to have a limited conceptual basis of their strategy use.

All in all, these 6 children and so much the more Lara and Mia, who were least successful within this sample, have severe problems not only in multiplication but also in areas of elementary arithmetic that form a prerequisite for learning multiplication on a conceptual basis (Gaidoschik, 2014). That is why in a second cycle of this design research project we will focus on how to better foster children

who lag behind in adding and subtracting at the start of grade 2 before and while working on multiplication. From where we stand in our analysis, we assume such measures will have to include individual support for some children additional to what a single teacher can manage in the classroom.

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