The fostering of flexible mental calculation in an inclusive mathematics classroom during Mutual Learning

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TU Dortmund University, Germany; laura.korten@math.tu-dortmund.de

The topic of teaching mathematics in an inclusive classroom provides – by the increased heterogeneity range – a big challenge between individualizing and mutual learning. (How) Can we make sure that all children work and progress on their individual level but at the same time learn with and from each other? Based on this question the aims of this project are the development of, and the research on a teaching-learning arrangement for the inclusive mathematics classroom to foster flexible mental calculation. The approach is a Design Research approach to face research interests on the level of design (consideration for use) and on the level of research (quest for fundamental understanding). This contribution focuses on the research level: first insights about mutual learning processes of elementary students with and without cognitive learning disabilities concerning flexible mental calculation will be presented.

Keywords: Inclusive education, heterogeneity, cooperative learning, flexible mental calculation.

Theoretical framework

Developing flexible mental calculation has been considered as a 'central goal' for more than a decade, not only for middle and high achievers, but also for less advanced children. However, empirical insights about teaching and learning processes of flexible mental calculation in inclusive classrooms do not exist. Although, *inclusive education* is a current international discussion, based on the UN-Convention on the 'Rights of Persons with Disabilities'. *Supporting everyone's learning process* and at the same time *encourage cooperative learning with and from each other* are the two central matters of inclusive education, which imply more than sharing a room. But, in school practice, teachers emphasize the difficulty of learning with and from each other in arithmetic (Korff, 2015). Building on the two matters of inclusive education, this study focuses on the *goal-differentiated fostering of flexible mental calculation* in an inclusive classroom during *Mutual Learning* takes place.

The *goal-differentiated* fostering of flexible mental calculation in an inclusive classroom

Developing flexible mental calculation is not only a central goal but also a 'critical point' in everyone's learning process (Heinze, Star, & Verschaffel, 2009), especially for students with cognitive learning disabilities. In this process, error-prone counting strategies should be replaced with more beneficial calculating strategies. Current literature offers different definitions, which commonly include the two aspects of 'flexibility' and 'adaptivity'. In most of the cases flexibility is understood as the ability to switch between different solution tools (Rathgeb-Schnierer & Green, 2013), while adaptivity is more emphasizing on the selection of the most appropriate strategy. In this project 'adaptivity' is related to the recognision of problem characteristics, number patterns and numerical relations. Consequently, *flexible mental calculating* is a situation-dependent and individual response to specific number and task characteristics and the corresponding construction of a solution process using strategic tools (ibid.).

The fostering of flexible mental calculation competences is influenced by the outlined general assumptions. If flexible calculation is related to number and task characteristics and relations, activities have to be chosen, which support children to focus on these. Thus, the crucial aim is to develop the competence to recognize problem characteristics, number patterns and numerical relations, and to use them for solving problems. Rechtsteiner-Merz and Rathgeb-Schnierer (2016) call this "Zahlenblick" and found out that it is a good vehicle for developing flexible calculation.

Today it is proven that also less advanced students can develop flexible mental calculation (Verschaffel, Luwel, Torbeyns, & van Dooren, 2009) and that the focus on developing "Zahlenblick" especially supports less advanced students (Rechtsteiner-Merz & Rathgeb-Schnierer, 2016). Schröder (2007) points out their problems in the usage of flexible strategies: Even if they know strategies, they very often cannot adapt and use those. Reflecting on characteristics and relations is especially essential for children with cognitive learning disabilities (ibid.) and at the same time supportive and preventive for everyone's learning process, because generally all children show little task-adequate action (Selter, 2000). Further, the content provides opportunities for high-performing students to establish mathematical structures and to generalize. Consequently, flexible mental calculating meets the requirements for a *common content* for an inclusive classroom to encounter the diversity of abilities and skills and to make goal-differentiated learning possible.

Mutual Learning in an inclusive mathematics classroom

The expression of *Mutual Learning* as it is used here combines the two central matters of inclusive education, which were mentioned above: individualizing as well as interacting and cooperating. Mutual Learning means to consciously induce learning situations *as often as possible* in which all children work and learn at a common content, in cooperation with each other, on their individual level, and by use of their current individual skills (Feuser, 1997). This definition is based on a wide sense of inclusion, acknowledging the diversity of all children and counteracting all forms of discrimination and special learning needs. Nonetheless this research project focuses on learning and interaction processes of children with and without *cognitive learning disabilities*. (This distinction is not used to label deficits, but rather with regard to make research and communication possible.)

In consideration of the two central matters of individualized learning and at the same time learning with and from each other, first supportive principles for successful Mutual Learning can be derived: '*content variability'*, *'goal-differentiated learning process -'*, and '*interaction orientation'*. As already outlined, the content of flexible mental calculation meets the requirements of the first two. Having regard to the principle of '*interaction orientation'*, the construction of mathematical knowledge is understood as an active, social and explorative process; also for children with cognitive learning disabilities as today several studies show. Gaidoschik (2009) points out, children with problems need more time and more support to learn arithmetic but they don't need something different. The exploration, understanding and use of arithmetical patterns and particularly the communication about number and task characteristics and strategic tools is especially important for children with cognitive learning disabilities (ibid.; Schröder, 2007). In social-communicative processes, individual mathematical learning develops through 'irritations', 'contradictions' and 're-interpretations' on the basis of individual interpretation processes (e.g. Steinbring, 2005). Therefore, this very *individual processes of learning flexible mental calculation in the context of interaction processes,* needs to be fostered on different cognitive levels for successful Mutual Learning.

Research question and methodological design

For this purpose, it seems important to research when and how successful Mutual Learning occurs and which support means can be reconstructed. Therefore, it is necessary to reconstruct the *individual learning processes* concerning flexible mental calculation as well as the *interaction processes* to be able to evaluate weather Mutual Learning with its two central matters took place.

To meet the aspiration of designing a teaching-learning-arrangement to foster flexible mental calculation on the one side, and to investigate learning and interaction processes on the other, a *Design Research* approach is used (Prediger & Zwetzschler, 2013). This requires research questions on the level of design and research. Nevertheless, this contribution focuses only on the level of research and the investigation of individual learning processes. The following question will be addressed: *How do individual learning processes of elementary students with and without cognitive learning disabilities concerning flexible mental calculation develop during the cooperative-interactive phase of Mutual Learning?*

Iterative design research cycles as an approach for answering the question

To investigate this, a teaching-learning arrangement was designed, tested and refined by conducting *design experiments* in three iterative cycles. Within each cycle the *individual learning processes* as well as the *interaction processes* were reconstructed to be able to evaluate weather Mutual Learning took place. To, in a next step, reconstruct support means for successful Mutual Learning.

Theoretical sample: The design experiments were conducted in classes two and three (7-9 years old), at three different German primary schools. Laboratory situations with couples of learners allowed to learn more about their thinking, their individual learning and interaction processes. Each design experiment consisted of three phases (Figure 1) and took place in a pair setting with one child tested and "termed" with and one child without learning disabilities. The participants were selected with the help of the class teacher and the special needs teacher in order to find pairs of children who like each other to have a positive basis of communication. In the design experiments, the learners processed the learning activities largely by themselves. The researcher, on the one hand, acts as a teacher, in order to give the learner a stimulus or help, and on the other hand as a researcher, who wishes to learn more about the thinking processes and the ways of proceeding by means of observation and targeted inquiry.

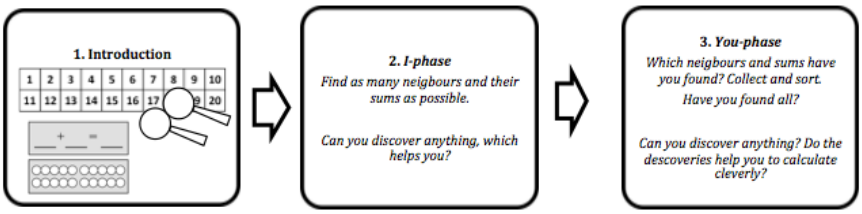


Figure 1: The structure of a design experiment

The teaching-learning arrangement - "We explore neighboring sums": After a mutual introduction (Figure 1), the children individually explore neighboring numbers on a 20frame (*I-/individual-phase*). The focus on neighbors – which are *next to*, *under* or *crosswise* to each other – and their sums enables them to discover number and problem characteristics and relations, as well as to develop mental calculating strategies based on individual abilities, arithmetic-, and context-characteristics. In the following, two children – one with and one without learning disabilities –

work together (*You-/cooperative-interactive-phase*), which enables them to communicate, use, reflect, refine, and/or improve their discoveries and strategic tools. In this way, singular accesses and comprehensions can evolve, through communicative exchange, to new comprehension and understanding. Due to the focus on neighboring sums the arithmetical patterns stay the same even in higher number ranges. This makes communication possible, even though some children already transfer their discoveries to neighbors on the 100frame or generalize the mathematical structures.

In order to reconstruct and categorize the development of goal-differentiate heterogeneous learning processes concerning flexible mental calculation, a model (Figure 2) has been drafted on the basis of previous research (Rathgeb-Schnierer & Green, 2013; Rechtsteiner-Merz, 2013). As mentioned before, *a 'process of solution'* is a situation-dependent and individual response to specific '*cognitive elements'* (Rathgeb-Schnierer & Green, 354) (e.g. 'characteristics and relations of numbers and problems' or 'automatized procedures') and the corresponding construction of the actual solution process using '*tools for solution'* (Rathgeb-Schnierer & Green, 2013, 355). In this sense, 'cognitive elements' are individual experiences, which comprise background-knowledge and -expertise for the individual process of solution. Referring to the theoretical background, the grey fields in Figure 2 are predictors of flexible calculation. Those will be fostered on different cognitive levels as an aim of the designed teaching-learning arrangement.

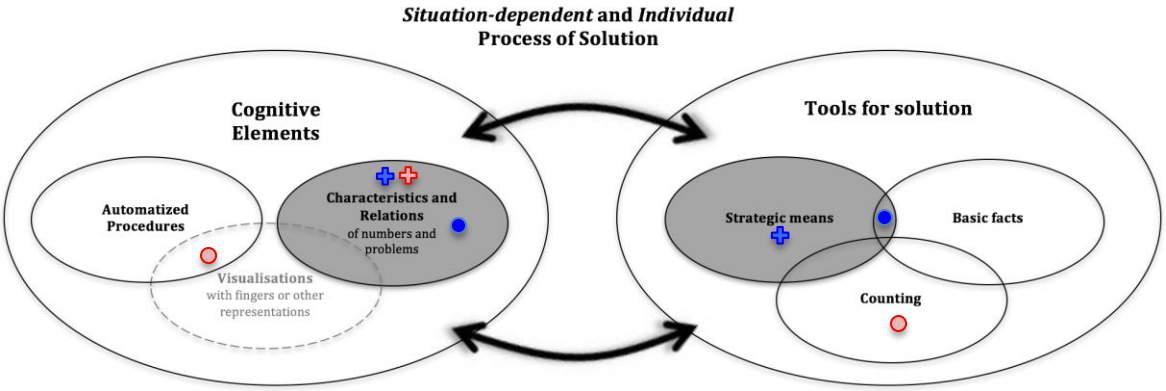


Figure 2: A model to reconstruct children's learning paths (cf. section 'Selected Results')

Each field of this model is described and defined by certain characteristics in order to group children's learning paths (for more information see Korten, i.V.).

Two analytical perspectives for answering the question and for developing local theories

The process of generating local theories gets content-specific theoretically and empirically justified. The data was collected in form of transcribed videos and gets analysed from two perspectives: 1) An *epistemological perspective*, to learn about individual learning processes on different cognitive levels in terms of the common content. 2) An *interactionist perspective*, to learn about the interactive structures during the cooperative-interactive phase of student with and without learning disabilities, and how these interaction processes influence the learning processes.

In order to address the two central matters of successful Mutual Learning, both perspectives are essential to evaluate if the children progress on their individual level and at the same time learn with and from each other. The interpretation of statements and actions, reconstructs interactive knowledge construction. Accordingly, an Interpretatively Epistemological Analysis Approach of Interactive Knowledge Construction (Krummheuer & Naujok, 1999; Steinbring, 2005) gets used.

At the same time this reveals information for the analysis of the teaching-learning arrangement and gives answers weather Mutual Learning in the sense of inclusion is supported or not. Thus, the empirical findings allow elaborating and enhancing the teaching-learning arrangement, as well as local theory building about mutual learning processes. Here the focus will be on the latter.

Selected results

In this section, the described analysis approach is illustrated with a short exemplary cooperative-interactive phase. Afterwards, selected general results concerning the research question addressed in this contribution will be presented. In the exemplary situation, a child with learning disabilities (S1) and a child with average mathematical skills (S2) work together. They explore crosswise neighboring numbers and their sums. Figure 3 shows an example.

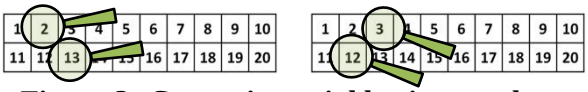
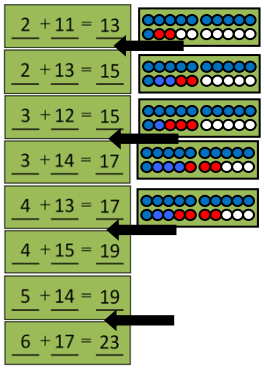


Figure 3: Crosswise neighboring numbers

S1: We need [the sum] 24 in between… 23 (points on 7+16=23) #, 24 (points between 7+16=23 and 7+18=25), 25 (points on 7+18=25)

S2: # No, this is… No... Here is the same. (points on 4+13=17) Also always one. (points on 3+12=15) See, there is 16 missing… here 14 is missing. (points between 2+11=13 and 2+13=15) Here 18. (points between 4+13=17 und 4+15=19) Oh here even (points between 5+14=19 und 6+17=23) two … no, 3…



S1: What? Now I am confused.

S2: Why? Ah! See, …

Interactionist perspective: S1 assumes that the sum 24 is missing and questions the completeness of the sums. This '*incorrect assumption'* (key impulse) leads S2 to exemplify relations between the sums. Her empirical argumentation leads to the hypothesis that the sum 24 does not exist. Both participants communicate with each other about the common content, according to individual assets. A '*balanced cooperation'*, in which both are involved and utterances are linked can be observed. Regarding to Naujok (2000) they are 'collaborating' with the focus on the same topic. Additionally, both develop on their individual level as the reconstruction of learning processes reveals:

Epistemological perspective: The children respond to the same 'incorrect assumption' (Figure 4, sign/symbol) in different ways by referring to *number relations* on the basis of their individual cognitive abilities. Figure 4 and 5 show the progress of the scene from an epistemological

perspective: S1 argues with counting and refers to the number word series (ordinal). S2 uses empirical arguments to prove that the sum of 24 does not exist by referring to arithmetical patterns (relational). Due to S1´s incorrect assumption, S2 discovers, exemplifies and later even generalizes number relations between the addition problems. S1, like this situation shows, is able to see and to question number patterns. This focus of attention on number characteristics and relations only started due to the interaction with S2. In the following, this situation leads S2 even to explore, explain and generalize the *constancy of two sums* (a+b)=(a-10)+(b+10) (Figure 5). From this point on, as a reaction on the interactive situation, she is not only referring to numbers characteristics and relations anymore but to *problem relations*, which she is using later to solve new problems.

Figure 4 & 5: Analysis of interactive learning processes (Steinbring, 2005)

The example shows how the individual learning processes developed during the cooperative-interactive phase of Mutual Learning. Both progressed according to their individual levels, triggered by a key impulse in the interaction, in this case an 'incorrect assumption'. These key impulses – here called '*productive moments*' – seem to be opportunities for fruitful Mutual Learning (Korten, 2017). Previously to the cooperative-interactive phase, **S1** exclusively refers to '*automatized procedures'* and used '*counting'* as her 'tool for solution' with the help of counters as '*visualisations'* (cf. Figure 2, red dots). She only relied on the procedure of counting, which seems to be like a "dead-end road" for developing flexibility in calculating (Rechtsteiner-Merz & Rathgeb-Schnierer, 2016, p. 359). But due to the impulses resulting from the interactive-cooperative phase she starts to look at *number characteristics and relations*. She is able to sort the addition problems according to characteristics and puts them into relation (cf. Figure 2, red cross). This recognition of number patterns and numerical relations is after Rechtsteiner-Merz and Rathgeb-Schnierer (2016) an important skill to overcome counting on the way to flexible calculation. **S2** was also stimulated by the exchange with S1: She was a flexible calculator from the start on. She used '*basic facts'* and adequate '*strategic means'* (e.g. decomposing and composing, using decade analogies, deriving solutions from similar problems such as 2 more and 20 more) by recognizing *relations* (cf. Figure 2, blue dots). Due to the interaction with S1 she was forced to explain and argue, which led her to discover new relations (*constancy of two sums*), which she used to refine her 'strategic means' later (cf. Figure 2, blue cross). It can be concluded that it was rewarding for both children to work in heterogeneous pairs in an arithmetic classroom. A study of Häsel-Weide (2016) about replacing persistent counting strategies with cooperative learning, supports this finding.

These were typical *learning* and *interaction processes*, which took place in all design experiments. In regard to the research question, which is addressed here, five types of individual learning paths

and their development during the cooperative-interactive phase of Mutual Learning could be reconstructed: 1) Children, who did not progress during the cooperative-interactive phase and used none or pre-existing individual insights into number/task characteristics and relations. 2) Children, who gained *new insights* into number/task characteristics and relations due to the task´s context. 3) Children, who gained *new insights* into number/task characteristics and relations due to impulses in the cooperative-interactive phase (e.g. S1). 4) Children, who *refined* their strategic means by taking advantage of new insights into number/task characteristics and relations due to the task´s context. 5) Children, who *refined* their strategic means by reflecting, inquiring and evolving insights into number/task characteristics and relations due to impulses in the cooperative-interactive phase (e.g. S2). In the cases 2)–5) changes could be identified because *new insides* were gained or strategic means were *refined*. Concluding, a development of learning process concerning flexible mental calculation was reconstructed for these situations. With respect to the title of this article and the definition given at the beginning, successful Mutual Learning took place and flexible mental calculation competencies were fostered on different cognitive levels in this inclusive situation.

Outlook

All research cycles demonstrated regularity in the appearance of the 'productive moments' in the interaction, which trigger individual learning as shown in the example. These moments mainly appeared during a ‘balanced cooperation’. Generally, a distinction can be made between ‘*direct-didactical*, *indirect-didactical* and *interactive* productive moments’ (Korten, 2017). In the future, research questions on the level of design will be addressed in order to reconstruct support means for successful Mutual Learning. It will be investigated in more detail how the developed teaching-learning-arrangement can specifically foster these ‘balanced cooperation’ and the 'productive moments'. First analyses show that beneficial and meaningful interaction must be specifically encouraged by an emotional benefit for all participants, which must be created from the outside. This, for example, can be a goal, which they can only reach together and functions as an ‘*extrinsic positive dependence’* (Korten, i.V.). This idea takes up the principle of ‘positive dependency’ from the concept of ‘cooperative learning’ (e.g. Johnson, Johnson, & Holubec, 1994) and advances it for the conditions of an inclusive classroom. Without this ‘extrinsic positive dependence’ a ‘balanced cooperation’ with ‘productive moments’ seems to be impossible in an inclusive setting. With this Topic-specific Design Research Approach the two interests of the goal-differentiated fostering of flexible mental calculation in an inclusive classroom and the general understanding and supporting of Mutual Learning processes could be integrated and first local findings were presented.

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