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# How middle-grade students explain ordering statements within real life situation? An example of temperature context

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*The purpose of the study is to examine the extent to which middle-grade students agree on statements about the ordering of two negative integers given within a real-life context, and what kind of procedural and conceptual strategies they generate to order those numbers. Data is collected through a questionnaire including two statements about ordering integers from fifty-seven middle-grade level students in a public school. The results show that even though students agreed with both of the statements, they did not explain the concept of ordering in daily life with regard to conceptual meanings and that they have problems in their procedural knowledge repertoire.*

*Keywords: Integers, contextual problems, negativity.*

## Introduction

Integers is one of the main topics that students have to understand to be successful in later contents such as algebra, geometry, or data analysis. While integers are required to solve algebraic expressions, sometimes it is required to understand the number system (Levenson, 2012). In particular, negative integers are hard for students to understand because of difficulty in representing those numbers physically (Davidson, 1992), locating them on the number line, performing four operations (e.g.: Ojose, 2015), or ordering them (e.g. Schindler & Hußmann, 2013). Ball (1993) exemplifies the dilemma of the case of teaching negative numbers regarding the ordering of two integers. Furthermore, according to Ball (1993), the transition among direction and magnitude aspects of negative integers are the heart of understanding negative integers.

## Students' solution strategies

Linchevski and Williams' (1999) study reports on a disco game supported by an abacus model that helps students make sense of net-change, zero situations, and develop strategies such as compensation and cancellation within the everyday life situations requiring operations of integers. In addition to this, students have solution strategies of four operations such as the tendency of using the first addend's sign, ignorance of the second addend's sign, or generalizing the statement by rote as 'two negatives make a positive' and so on (Ashlock, 2010). Considering the literature, it is seen that many studies focus on student solution strategies about operations on integers. However, in the available literature, there are limited studies that focus on students' strategies about ordering integers (Schindler & Hußmann, 2013; Ojose, 2015). Furthermore, what kind of pieces of information students have regarding ordering integers, is rare.

## Conceptual and procedural knowledge

According to Hiebert and Lefevre (1986), pieces of information might be separately located or are in closely related within the knowledge network in students' minds. Once students start learning, they can make links among those pieces or students keep in mind the necessary steps to solve problems and they apply the steps to solve the problems without questioning. The pieces of knowledge can be

constructed with appropriate links to make meaningful understanding and that can be accomplished by creating relationships among the separately existing pieces of knowledge. Hiebert and Lefevre (1986) define conceptual knowledge as ‘... knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network’ (Hiebert & Lefevre, 1986, pp. 3-4). The other kind of knowledge which is used by students in learning mathematics while solving mathematical problems is procedural knowledge. Hiebert and Lefevre (1986) describe procedural knowledge through including parts as ‘one part is composed of the formal language, or symbol representation system of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks’ (Hiebert & Lefevre, 1986, p.6). Both procedural and conceptual knowledge is necessary for students in order to finish the process with a correct answer and to understand the relationships among concepts. Students’ backgrounds include procedural and conceptual knowledge and both of them are required for mathematical proficiency (Kilpatrick, Swafford, & Findell, 2002). They are constantly interacting with each other and in light of instructions given to students, the kinds of knowledge can be revealed. In this regard, this study will examine students’ explaining strategies while ordering two negative integers through conceptual and procedural knowledge descriptions of Hiebert and Lefevre (1986). It gives information about how middle school students conceptualize ordering of integers, how they give meaning to integers in real life contexts, and what kind of procedural and conceptual strategies they have while ordering integers. With this study, possible explanations of the reasons behind middle-grade students’ difficulties and their understandings in integers, specifically about ordering integers are revealed. Thus, the aim of this study is to explore answer for the following questions: (1) To what extent do middle-grade students agree on statements about the ordering of two negative integers? (2) What kind of strategies do middle-grade students generate to order two negative integers given within a real-life context? Related with the second research question, this study examines how those strategies can be classified as procedural or conceptual.

### **Context of the study**

The context of the study might be understood better via curricular guidelines of ordering integers in Turkish middle school mathematics curriculum. The topic starts at the beginning of middle-grade levels. Objectives related to this topic are about interpreting integers, the meaning of absolute value, operations of integers, the meanings of the operations, comparing and ordering integers, solving integer-related problems, and relate exponential numbers to integers. Specifically, in learning to order integers, the objective says that: "Students should be able to compare and order integers" (MoNE, 2013, p.14). The curriculum advises teachers that the largest number is located to the right side in reference to the small number on the number line while ordering numbers. Therefore, it is seen that teachers are supported to use the link between number line location and integers’ ordering. In other words, the curriculum supports ‘direction’ aspects of negative integers while the magnitude aspect of negative integers within real life situations (e.g.: temperature, asset and debt, elevator and so on.) are an application of objective presented in the middle school curriculum (MoNE, 2013). In this sense, in their instruction, students are given real-life contexts to understand how integers are represented within real life contexts in a meaningful way (MoNE, 2013). For ordering integers, the location of numbers on a number line is the basis of teaching the content (MoNE, 2013). In this respect, ordering

integers is not interpreted on the basis of real life examples. Rather than that, interpretation of the ordering integers mostly depends on the location of numbers on the number line.

## Method

In this study, qualitative research method, specifically single case study approach, is used in order to reveal students' strategies and investigate students' understanding of contextual statements. The research method enables researchers analyzing data through creating a theme and codes (Creswell, 2005).

## Sampling

Fifty-seven middle-grade students participated in the study. These students are selected under the purposeful sampling and they are studying at the middle-grade level in a public school in Ankara, Turkey. The school could be regarded as successful on the basis that it accepts students with having higher grades in nationwide held examinations. Before conducting the main study, a pilot study was conducted in two classrooms together with another fifty-four middle-grade students at the same grade level as the school the main study was conducted. The purpose of the pilot study is to make the statements understandable to the participants and to minimize the misunderstandings derived from the format of the questionnaire. The objectives which are aimed to be evaluated in the study require middle school students interpret integers within real life situations and to compare and order integers. Within this context, the content validity of the questionnaire is validated with mathematics teachers and one mathematics education instructor in the university.

## Data collection tools

In this study to collect data the questionnaire given in Figure 1 is used. In order to understand middle grade students' general tendency of ideas about the different nature of ordering statements, students were asked whether they agree with the two ordering statements given in real life context. In addition, they were required to give details about the reason of agreement status in order to understand their reasoning in ordering two negative integers of strategies for explanation. As it is seen, there are two ordering statements given in the context of temperature. In this regard, questions were generated based on the context which middle grade students are familiar with and use the knowledge of ordering two negative integers by relating the direction and magnitude (quantity) aspects. The questionnaire has two statements pointing out two kinds of relations about ordering two negative integers. The first statement says that "As -10 degrees are less hot than -5, -10 is smaller than -5." The statement allows middle grade students to order two negative integers considering temperature as a quantity and direction. In other words, they can order two negative integers considering the mentioned degrees as similar to ordering two positive quantities. In line with this, -5 is more hot (more quantity of hot) than -10. In addition, temperature context enables students ordering integers considering direction integers. The second statement says that "As -10 degrees is situated on thermometer, lower than -5, -10 is smaller than -5. The statement allows students to interpret ordering of two negative integers based on their locations on the thermometer and their distance from zero. In this regard, whether they are using any conceptual or procedural strategies for using the nature of 'direction' and 'magnitude' meaning of two negative integers is examined.

"As -10 degrees are less hot than -5, -10 is smaller than -5. "	AGREE	NOT AGREE	NEUTRAL
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Why do you think so?			
"As -10 degrees situated on thermometer lower than -5, -10 is smaller than -5.	AGREE	NOT AGREE	NEUTRAL
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Why do you think so?			

**Figure 1: Questionnaire items of the study**

### Data analysis

Similar and different categories of answers are grouped through content analysis. Explanation strategies are revealed by examining words or group of words students use in their answers (manifest content) and following this the underlying meaning of those wordings are revealed by investigating their explanation strategies deeply (latent content) (Fraenkel, Wallen, & Hyun, 2011, pp.483-484). Answers of the questions are analyzed for the agreement status, students' explanation strategies of the agreement status regarding conceptual and procedural strategies while explaining their responses. Data of the study are analyzed within Hiebert and Lefevre (1986)'s conceptual and procedural knowledge framework.

### Findings

#### Agreement status of students about the ordering statements

Based on the analysis, it is seen in Table 1 that participants are agree with both of the statements.

Temperature Context	A	NA	N	NM	Total
1.statement	75	16	7	2	100
2.statement	66	14	16	4	100

\*A: Agree, NA: Not Agree, N: neutral, NM: No marking

**Table 1: The percentage of agreements to the ordering statements**

The analysis of the results show that middle grade students agree with the idea that ordering two negative integers can be explained by using 'hot' concept together with the word 'less'. In this regard, they agree on the idea that two negative integers can be ordered by using the words 'more' and 'less' which express the quantity of something and came to the conclusion that 'hotter' is bigger than 'less hot'. Thus, for the first statement "As -10 degrees is less hot than -5 degree, -10 is smaller than -5", most of the students (75%) selected the choice *agree*. Parallel with this, for the second statement most of the students (67%) also agree on the idea that two negative integers can be ordered based on the location of the integers on the thermometer. In other words, the idea accepted by most of the participants is that the number which is below the other is much smaller.

### The explanation strategies of middle grade students

Table 2 shows that the most preferred explanation strategies for the given statements are related to the network of hot and cold (37%), and rule-based explanations in reference to zero or positive numbers (18%). The strategy of *network of hot and cold* represents a relationship between the concepts of temperature, coldness and integer in the minds of students. In line with this, students make transitions among those concepts. For example, students form a link between negative integers and coldness saying if negative numbers increase the weather gets cold. The strategy of rule-based explanation in reference to zero or positive numbers is about rules with which students are familiar and which are created using zero and positive numbers like negative numbers are [ordered as] opposite to positive numbers. This table also depicts that students do not tend to use *rule-based explanations* in both of the statements. Put differently, *rule-based explanations* are not a dominant strategy for explaining those two ordering statements. They are used in the second statement, which mentions the location of numbers on the thermometer, compared to the first statement which is about interpreting the coldness, hotness, and their relationship. Another unexpected result is that although not many, some students did not consider this sort of order and they agree with the idea that "the number -10 is more than -5". In other words, they reject the order and think that -10 is bigger than -5 or -10 is hotter than -5. Similar to this, some students criticize the statements saying 'they are illogical' or 'they [both of the statements] are the same' and so on. Besides, some students used *the copy of the statement* strategy which is related to writing the same statements given in the questionnaire. In this regard, a substantial portion of students (44% for the first statement; 62% for the second one) do not employ any kind of explanation strategies for ordering integers in a given context. Students who copy the statement, use no strategies (e.g.: I don't know), use unclear statements (e.g.: I don't know why I am saying that I agree with the statements), and left the answer blank did not give reasons in writing as if they could not interpret the situation.

### Conceptual and procedural strategies

Examination of students' written responses showed that students used conceptual and procedural strategies while explaining their agreement. Whether the strategy a student is used is related to procedures or concepts is determined considering how the concepts are interpreted in the statements, how transitions are made among the statements, and the content of the definition of Hiebert and Lefevre (1986). Students' responses of conceptual strategy were analyzed based on conceptual knowledge definition of Hiebert and Lefevre (1986). In this regard, *network of hot and cold* category was created when students interpret hot and cold concepts and make transitions between them saying that 'more hot' means 'less cold' etc. In addition to this, as Hiebert and Lefevre's definition for conceptual knowledge supports the relationship between pieces of information, *the network of hot and cold* was appropriate for this category. On the other hand, procedural strategy category was created based on Hiebert and Lefevre's (1986) procedural knowledge definition which supports the repertoire of basic factual knowledge and symbolic representation without interpretation of those facts and representations. In line with this, *rule-based explanation reference to zero or positive numbers* was categorized as procedural knowledge. *Rule-based explanation reference to zero or positive numbers* was related to the facts which are presented as context independent relations including facts of ordering two negative integers regarding their magnitude and direction.

Explanation Strategies		Example of students ' statements	Statements (%)	
			1.	2.
Conceptual strategy	Connected knowledge of zero	Closer to zero is connected to being hotter	2	6
	Network of hot and cold	The big number (-10) is colder than the smaller number (-5)	21	4
		As numbers increases hotness increases	5	2
		If negative numbers increase the weather gets cold	4	-
		-5 has more quantity of hot than -10	7	2
		Quantity of quicksilver is smaller at -10 degree	-	2
<i>Total</i>			39	16
Procedural strategy	Rule-based explanation reference to zero or positive numbers	Bigger number is closer to zero (and the reverse of the statement)	9	14
		Numbers get smaller if you move to the left side of zero	4	4
		Negative numbers are ordered as opposite to positive numbers	5	-
		Numbers above the number line are bigger than the numbers below the number line	-	2
<i>Total</i>			18	20
Other statements	Criticism of the content of the statement	No relationship between the given statements, they are contradictory to each other	5	9
	Rejecting the order	-10 is not less than -5	8	9
	Copying the statement	Student write the same statements given to them	7	14
	Unclear statements	For the reason that this is appropriate	7	1
		Drawings help ordering two negative integers	2	9
	Blank	-	5	4
No strategy	I don't know	12	18	
<i>Total</i>			46*	64
<i>Total answers*</i>			100*	100

\*One student suggested more than two ways for explanation

**Table 2: Explanation strategies and conceptual and procedural strategies of students**

Table 2 indicates that for the first statement, students used more conceptual strategies than the second one, and students used more procedural strategies for explaining the second statement than the first one. This might be derived from the nature of the statements which allow students to focus on the conceptual nature of the word 'less hot' and of allocating numbers on a number line. However, it seems that a considerable number of students used the procedural and conceptual strategies (20% and 16% for the first and second statement, respectively) regardless of the nature of the problem.

Most of the students used the *network of hot and cold* strategy with the conceptual strategy of ordering two negative integers using coldness (e.g.: -10 is colder than -5). In other words, most of the students made a transition from the word 'less hot' to the word 'colder'. As opposed to the expected interpretation of students, they explained ordering two negative integers considering the quantity of hotness concept (e.g.: -5 have more hotness than -10). Students used *connected knowledge of zero* strategy, for instance, saying that 'being closer to zero is connected to being hotter'. Similarly, in the first statement, the procedural strategy of 'bigger number is closer to zero' is used (e.g.: -5 is closer

to zero and so it is bigger and vice versa). Parallel with this, for the second statement, the same procedural strategy was mostly used by the participants.

Most of the participants supported the idea that two negative integers can be ordered by using *rule-based explanation in reference to zero or positive numbers*. In this regard, it might be said that students have pieces of knowledge about the nature of magnitude and direction of integers. Participants agreed that 1) the number which is located below the other on the thermometer is the smaller (direction) or (2) the number which is closer to zero is bigger than the other negative integer, which is less close to zero (magnitude).

## Discussion and conclusion

In this study, the two statements about ordering allow students to see how they interpret those statements and what kind of strategies they use. Moreover, this study helps reveal the difficulties students encounter in their learning process. As seen in Table 1, students agreed with the given statements; however, Table 2 showed that middle grade students have variety of conceptual and procedural strategies that might not support their agreement. The results of the study show that their dominant strategies are spread over the sample of the students and are rarely used. Table 1 shows that most of the participants supported the idea that negative integers can be ordered by the nature of the amount of hotness that each integer is assigned. In other words, each negative integer is assigned to the concept of being hotter or being less hot. A possible explanation might be that students conceptualize ordering with the help of quantity or cardinal conception of numbers (Davidson, 1992). It means that students agree that ordering negative integers can be thought as similar to ordering positive integers when giving meaning to hotness concept. In this regard, for the first statement, students changed the word *hotness* to the word *coldness* and explained the statement based on coldness. Most of the students interpreted the situation by transferring hotness to the cold. While comparing two negative integers, they used the ‘the hotter is less cold’ or ‘less hot is colder’ relationships. Thus, these kinds of explanations indicate students’ lack of interpretation of ordering as an amount of hotness because they might have a potential for interpreting the smaller number (-10) as a bigger number while ordering the concept of coldness a quantity. However, the relationship which indicates what being less cold or being colder means needs to be established within ordering context. Otherwise, it causes misconceptions or errors about misinterpreting what the integer statements or symbols mean (Ashlock, 2010). Moreover, this finding supports the idea that it is not easy to infer order relations from context-related statements, but teachers can integrate those strategies to classroom activities to establish a relationship between real life situations and negative numbers (Schindler & Hußmann, 2013). It is worth emphasizing that the procedural strategies illustrated in Table 2 might create faulty decision while comparing two negative integers. Students’ understanding of negative numbers might be achieved by using the procedural strategies carefully being aware of the overgeneralization. For instance, the procedural strategy of ‘bigger number is closer to zero’ might be problematic when the number is a whole number.

Taken together, students have a variety of conceptual or procedural strategies that can be used for interpreting ordering in real life situations. Those strategies are important to make instruction better and to facilitate student understanding. In future studies, the meaning of ordering negative integers within real life contexts and in mathematics can be examined to explain some students’ lack of interpretations of the given statements.

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