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# TWG01: Argumentation and proof

## Introduction to the papers of TWG01: Argumentation and proof

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### Introduction

The role and importance assigned to argumentation and proof in the last decades has led to an enormous variety of approaches to research in this area. The 27 papers and 1 poster presented in the Thematic Working Group (TWG) “argumentation and proof” come from 18 countries across 4 different continents, and offer a wide spectrum of perspectives. These contributions intertwine educational issues with explicit references to mathematical, logical, historical, philosophical, epistemological, psychological, curricular, anthropological and sociological issues.

Taking into account this diversity, the contributions were presented and discussed in working sessions organized under the following 7 themes: (1) assessments issues of argumentation and proof; (2) theoretical and philosophical issues of argumentation and proof; (3) argumentation and proof in textbooks; (4) tools for analyzing argumentation and proof; (5) intervention studies on argumentation and proof; (6) argumentation and proof at the university mathematics level; and (7) task design in argumentation and proof. Since the themes are intertwined, each paper could be assigned to multiple themes. Therefore, the assignment of papers to themes was guided by a “best fit” approach as well as practical considerations. We will briefly discuss each theme separately.

### Assessment issues of argumentation and proof

This theme included three papers, related to issues of assessment in the area of argumentation and proof: Kónya and Kovács’ paper focused on development of inductive reasoning of prospective teachers by analyzing their problem-solving processes on a carefully selected problem. Hemmi, Julin and Pörn’s paper investigated teachers’ perspectives on the possibility of using students’ common misconceptions, identified in prior research, as a starting point for activities that develop students’ understanding and skills in proof. Demiray and Bostan’s paper investigated pre-service middle school mathematics teachers’ interpretations of statements regarding proof by contrapositive and the reasons for their incorrect interpretations. The discussion of the three papers in the TWG raised several important issues, such as:

*The influence of sociocultural context* should be considered in assessment findings.

*Alternative variations of task design* should be considered in the interpretation of students’ performances.

*Adopting a more positive model.* Should researchers aim to identify students’ competencies rather than misconceptions?

### **Theoretical and philosophical issues of argumentation and proof**

The two papers in this theme addressed implications of Habermas' rationality theory. Conner's paper discussed how Habermas' rationality can be used to analyse how teachers support argumentation processes in their classrooms. Boero's paper showed the added value of analyzing individual student's thinking processes while attempting to prove a statement. The discussion of the papers raised several points, including the following:

*What is the added value of applying Habermas' rationality to a particular kind of analysis, and what would be lost if it was not used?*

*The difficulty of applying the categories of Habermas' rationality to coding data and, in particular, the difficulty in distinguishing between teleological rationality and epistemological rationality.*

### **Argumentation and proof in textbooks**

The five papers presented in this theme were grouped based on their relation to argumentation and proof in textbooks. Žalská's paper described how different types of arguments enacted in one classroom were influenced by the textbook, the teacher beliefs, and the students. Wong's paper presented an examination of geometry chapters in a prominent Hong Kong textbook series and illustrated the limited opportunities for students to engage in the process of generalizing and providing proofs. Cousin and Miyakawa's paper described the evolution of proof in Japanese geometry textbooks and the role of the specificity of Japanese language on that process. Mesnil's paper described a reference for studying and teaching logic in France, while Bergwall engaged the TWG in analysis and discussion of reasoning-and-proving opportunities in Finish and Swedish textbooks on primitive functions. The discussion addressed several important topics, such as:

*The role of language and linguistics in introducing, teaching, and writing proofs; and how the goal of teaching proof is articulated in a curriculum, represented in textbooks, and enacted in classrooms.*

*The role of mathematical logic in the teaching and learning of proving.*

*Definitions in research frameworks.* Caution is required in the interpretation of the findings from different studies which operationalized certain terms in different ways.

*The need for specialized analytical frameworks when examining argumentation and proof opportunities in textbook tasks versus textbook expositions.*

### **Tools for analyzing argumentation and proof**

The five papers in this theme concerned different tools for analyzing argumentation and proof. Ruwisch's paper concerned a one-dimensional model to rate reasoning competences at the primary level, considering both mathematical reasoning and its linguistic realization. With the same goal to better understand primary students' reasoning characteristics, Koleza, Metaxas and Poli used a simplified model of Toulmin's argumentation, drawing also on argumentation schemes described by Walton. The paper by Mata-Pereira and da Ponte aimed to understand how application of design principles regarding tasks and teacher actions can help provide students with opportunities to justify, and presented a framework that accounts for the level of complexity in students' justifications. In a longitudinal study, Fahse explored secondary school students' ways of argumentation on tasks concerning division by zero. He identified three different types of student argumentation and showed

how these relate to students' age. Focusing on teachers' competencies, Chua's paper presented a theoretical framework that classifies justification tasks by their nature, purpose and the expected element to be provided in the justifications. The discussion of the five papers raised some deep issues, including the following:

*The validity versus utility of theoretical frameworks in argumentation and proof.* The utility of a framework depends on how well it is designed to address a particular goal.

*Multi-dimensional models of proof.* Researchers should acknowledge the complexity of proof and specify the aspect(s) of proving that they are focusing on.

*Language and argumentation.* Investigating relations between language and argumentation requires clarifying what we mean by "mathematical language".

*Classroom culture* should be considered in interpretations of research findings.

### **Intervention studies on argumentation and proof**

The five papers in this theme related to implementing proving activities in school mathematics classrooms. Reid and Vallejo Vargas' paper describes an intervention where 3<sup>rd</sup> graders learn division through "proof-based teaching" by developing a shared toolbox of justification principles. The study showed that 3<sup>rd</sup> graders are capable of reasoning deductively from premises when explaining their thinking. The paper by Soldano and Arzarello described students using game activities in Dynamic Geometry Environments (DGEs) to discover geometric properties of the mutual relationship between two circles. The authors found that games helped students to communicate their claims, formulate and check conjectures, and explain their thinking. Siopi and Koleza's paper focused on students' use of a specific tool, a pantograph, to explore geometrical properties of parallelograms. The paper by Pericleous and Pratt examined how a teacher helped students to foreground mathematical argumentation as they investigated geometrical properties within a DGE. Finally, Buchbinder reported on a study on professional development sessions where teachers became familiar with 'proof-task prototypes', applied them in their teaching, and reflected on this application. These activities helped teachers to involve proof-oriented activities in their ordinary mathematics classrooms. The discussion included the following issues:

*What was the contribution of particular tools to students' learning?*

*Students' investigations within DGEs* and the ambiguity of the expression 'play with the software'.

*The need for structured support for teachers to implement proving activities.*

### **Argumentation and proof at the university mathematics level**

The five papers in this theme were concerned with teaching and learning of proof at the university level. Yan, Mason and Hanna's paper suggests an exploratory teaching style to promote the learning of proof, and describes specific pedagogical strategies. Selden and Selden's paper discusses theoretical perspectives for proof construction and its teaching. They suggest including psychological aspects of proving to these perspectives, and how these aspects should be considered in teaching and future research. Moutsios-Rentzos and Kalozoumi-Paizi's paper also considers psychological aspects by describing affective and cognitive experiences of a mathematics undergraduate student while producing a proof under exam-conditions. The innovative methodology of their study is to examine

students' facially expressed emotions during proving activities, as a way to study and influence students' attitudes towards proof. Gabel and Dreyfus' paper describes an attempt to analyze rhetorical aspects of proof presentation. They use Perelman's "New Rhetoric" as a framework to identify ways to analyze and increase the effectiveness of teachers' argumentation in mathematics classrooms. Azrou's paper suggests that students' lack of meta-knowledge about proof, such as features of mathematical proof and how a proof should be organized, influences their competence to write mathematical proofs. The discussions of these papers raised the following issues:

*Phenomena (as behavioural issues)* that have not been previously considered by psychologists or mathematics educators may play a role in students' difficulties to construct proofs.

*The role of emotions and feelings* in proof construction.

### **Task design in argumentation and proof**

Although many papers touched on issues of task design, this was the main topic of the two papers in this theme. Komatsu and Jones's paper explores how task design can facilitate students' engagement with the mathematical activity of proofs and refutations in the context of a DGE. Hein and Prediger's paper explored the role of task design and scaffolding to foster students' learning of deductive reasoning, making explicit the logical structures and unpacking their verbal representations in geometry. The discussion included the following issues:

*Proving something in a particular case:* how can we help students see the generality of the proof?

*The notion of scaffolding.* How can we make explicit to students the logical structure of proving?

*The special place of geometry* in teaching, learning and researching of argumentation and proving.

### **Conclusions**

We think that the TWG on argumentation and proof has offered the participants the richness of diversity in this research domain and the opportunity of fruitful discussions. At the last session of the TWG, the participants engaged in a discussion to identify areas in which they would like, and hope, to see more research in future CERMEs. The following areas were identified:

*The teaching of proof and argumentation* in both school and university settings, including in teacher education. The study of the classroom implementation of tasks rich in argumentation and proof opportunities, scaffolding and responding to unexpected student responses.

*Issues of language in argumentation and proof.* This also includes representations, structure, oral and written language, rhetoric and logic.

*Aesthetics of proof* and ways in which students of all levels of education can improve their attitudes, emotions, and beliefs about proof.

The identification of these areas is aimed at describing the state of the art of the field, without suggesting prioritizing certain areas of research. The TWG is committed to representing the diversity of perspectives and research areas on argumentation and proof in future CERMEs.