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Supporting classroom implementation of proof-oriented tasks: Lessons from teacher researcher collaboration

Orly Buchbinder

University of New Hampshire, USA; orly.buchbinder@unh.edu

This paper reports on a professional development (PD) which aimed to support secondary teachers in incorporating argumentation and proof-oriented tasks in their classrooms. The teachers interacted with researcher-developed models of proving tasks in a variety of ways, including modifying the tasks to their classrooms contexts, implementing the tasks, sharing and reflecting on the experiences. In the process of modifying proof-oriented tasks by teachers some of the original researcher-intended goals were lost, while other unexpected affordances emerged. This raises important questions regarding modes of teacher-researcher collaborations around proof-oriented classroom interventions, and their potential effectiveness.

Keywords: Reasoning and proof, professional development, instructional activities, classroom interventions.

Introduction

As the body of knowledge on reasoning and proof grows, the focus of mathematics education research has shifted from examining individual students' conceptions of proof and theorizing about potential causes of students' difficulties with proof towards designing classroom interventions that aim to remediate these difficulties and provide instructional support for students and teachers in classrooms (Stylianides & Stylianides, 2016). In this process teachers play a critical role, as they are responsible for establishing learning environments in their classrooms. In line with the wide recognition of the importance of argumentation and proving to students' mathematical experiences (e.g., Reid & Kipping, 2010) teachers are expected to implement tasks that promote reasoning, and have students construct and critique mathematical arguments (CCSS, 2010).

While many teachers agree, in principle, with this vision of mathematics classrooms, they often find them challenging to implement and maintain over time (Brodie, 2010). Moreover, only a limited number of professional development (PD) settings explicitly focus on argumentation and proving in connection to classroom practices (Brodie, 2010). Hence there is a need to expand the theoretical and practical knowledge of successful strategies for supporting teachers in this area.

This paper reports on an experimental model of a PD intended to support teachers in incorporating argumentation and proving in their classrooms. The following sections describe theoretical grounds underlying a special feature of the PD: teachers modifying researcher-designed proof tasks for implication in their classrooms. I illustrate two such modified tasks and analyze them in terms of affordances for students' learning, and their (mis)alignment with the original designer's intentions. I close by discussing some implications for supporting teachers' implementation of proof-oriented classroom activities.

Theoretical framework

Supporting change in teacher practices: the emphasis on argumentation and proving

Research has identified key features of PD settings that have shown to be successful in supporting change in teachers' practices. Among them are: focus on content and pedagogical knowledge, active learning experiences, establishing strong connections to teachers' own classroom contexts, and providing ongoing support for teachers (Copur-Gencturk & Papakonstantinou, 2015). These general features can be adapted to provide targeted support for teaching argumentation and proving, for example, by emphasizing mathematical knowledge for teaching proof (MKT-P).

Building on Stylianides's (2011) notion of "comprehensive knowledge package for teaching proof", Buchbinder et al. (2016) suggest that MKT-P includes 4 types of knowledge. Two types are related to pedagogical content knowledge: (a) knowledge about students' conceptions of proof, and (b) knowledge of pedagogical practices for supporting students' development of correct conceptions of proof. The other two types of MKT-P involve subject matter knowledge: (c) robust knowledge of mathematical content involved in a given task, and (d) meta-mathematical knowledge of proof, such as argument validity, logical connections, types of proof, and the role of examples in proving. These four types of knowledge were addressed in the design of the PD reported in this study. In addition, the PD activities established strong connections to teachers' own classrooms by providing practical tools for teachers to develop and implement proof-oriented instructional tasks in their classrooms.

Task design

Choosing, adapting and designing mathematical tasks is one of the cornerstones of a teacher's work. With textbooks providing only limited opportunities for students to engage in argumentation and proving (Thompson et al., 2012) teachers have been encouraged to treat textbooks' tasks as a starting point for planning instruction: to modify tasks to increase their cognitive demand or develop their own tasks (Stein et al., 2000). Since PD efforts in this area have seldom specifically targeted argumentation and proving tasks, the knowledge on teachers developing and implementing such tasks has been limited. Adding to this concern, Stylianides and Stylianides (2016) argue that it is unrealistic to expect individual teachers to design their own instructional activities that successfully target persisting difficulties with proving. On the other hand, Kim (2016) has found that teachers regularly tend to omit, replace or substitute instructional activities, even when working with reform-based, research informed curricula, which often compromises the original designer intentions.

This dilemma can be addressed by fostering close collaboration between researcher-designer and the teacher (Cobb et al., 2003). While the researcher-designer brings in strong theoretical and empirical knowledge related to proving, the teacher has an intimate knowledge of specific instructional and institutional context. This partnership model was realized in this study by providing teachers with researcher-developed prototypes of proving tasks to modify and implement in their classrooms.

Proof-task prototypes

Six prototypes of proving tasks were developed by the author of this paper in a study of secondary students' conceptions of proof. The tasks, which can be used as diagnostic tools and as instructional activities (Buchbinder & Zaslavsky, 2013), were developed in generic form, so they could be adjusted for a variety of mathematical topics, while maintaining the original structure and goals,

such as recognizing the limitation of examples for proving general claims, or understanding the role of counterexamples. In the context of the PD reported herein, teachers received at least one content specific version of each type of task, and a generic template highlighting task structure. Figure 1 shows an algebraic version of the task *True-or-false*; and Figure 2 shows its generic version¹.

True or False? For each statement below decide whether it is true or false and justify your answer.	
1) Every three numbers a, b, c satisfy the equation: $\frac{a}{b+c} = \frac{a}{b} + c.$	4) There exist four numbers a, b, c, d that satisfy: $\frac{a+c}{b+d} = \frac{a}{b} + \frac{c}{d}.$
2) The (positive) difference between the squares of any two consecutive natural numbers is equal to their sum.	5) There exists a number $a \neq 1$ that satisfies the equation: $a + \left(1 + \frac{1}{a-1}\right) = a \cdot \left(1 + \frac{1}{a-1}\right)$
3) Every two numbers n, m satisfy the equation: $\frac{1}{m} + \frac{1}{n} = \frac{1}{n+m}$	6) There exist three distinct positive integers a, b, c that satisfy $\frac{a+c}{b+c} = \frac{a}{b}$

Figure 1: Algebraic version of the task *True-or-False*

The task *True-or-false* targets multiple aspects of proving and refuting. It requires distinguishing between universal and existential statements, and recognition that the type of statement affects the role of examples in proving or disproving it. To successfully complete the task, students need to construct general proofs, construct appropriate counterexamples to disprove false universal statements, and come up with supporting examples to prove existential statements.

Statement	Type of statement: U / E	Truth value: T / F	“Always-Sometimes-Never”			Type of justification required
			True for all values	True for some values	True for no values	
1)	U	F		✓		Refutation by a counterexample
2)	U	T	✓			General proof
3)	U	F			✓	Refutation by a counterexample
4)	E	T		✓		Proof by a supporting example
5)	E	T	✓			Proof by a supporting example
6)	E	F			✓	General refutation

Figure 2: The structure of the task *True-or-False*

The task *Always-Sometimes-Never*, builds on the task *True-or-false* by asking whether the *propositions* of the statements in the latter task are true for all, some, or no values of relevant variables. This often requires construction of additional arguments, e.g., although statement #3 in Figure 1 can be refuted by a single counterexample, one must construct a general argument to show that no values of variables satisfy the statement. Sequencing these tasks allows to contrast quantified

¹ For complete presentation of all 6 types of task prototypes see Buchbinder & Zaslavsky (2013).

statements, which are either true or false, with non-quantified propositions, which truth-value depends on the value of a particular variable. Creating a combination of statements to address all these aspects of proving is a complex undertaking, which could be supported by using a generic version of the task (Figure 2). The goal of this study was to explore the potential of using generic task prototypes to support the work of mathematics teachers with respect to incorporating argumentation and proving in their classrooms.

Methods

Participants. The study was conducted with 5 secondary teachers, all female, all from different schools in a Northeastern area in the United States. Their teaching experience varied greatly from 5 to over 30 years. Since the PD was advertised as explicitly devoted to classroom implementation of argumentation and proving tasks, all participating teachers were motivated to introduce such tasks in their teaching, but sought to gain practical skills in this area. Hence, the PD aimed to reinforce already existing teachers' motivation, provide ongoing professional support, and foster teachers' sense of self-efficacy as they transformed their practices.

The setting. The PD consisted of 9, two-hour long weekly meetings which took place on the campus of a state university in the Fall of 2015. During the sessions teachers interacted with the 6 types of researcher-developed proof tasks in several ways: they experienced the tasks as learners, examined samples of student work pertaining to these tasks, and analyzed opportunities to learn about argumentation and proving embedded in the tasks. This was done by comparing teachers' own experiences and student work with the generic task prototype to examine the extent that the designer-intended goals have been realized. Throughout the PD teachers were encouraged to try out at least two types of tasks in their classrooms and share their experiences with others.

Modes of Inquiry and Data Sources. All PD sessions were videotaped. Each teacher submitted the tasks they had created or modified for their classrooms, sample student work and a two-page report on the task implementation e.g., the mathematical topic, the number of students, and the modes of work: group, individual, whole class, or combined. Teachers were also asked to describe what kinds of learning opportunities they think their tasks afforded, and what challenges they encountered as they created and implemented the tasks. The teachers also completed a short survey assessing the perceived effect of the PD on their classroom practices.

Results and discussion

Perceived obstacles for classroom implementation

Although all participating teachers expressed their commitment and motivation to incorporate proof-oriented tasks in their teaching, they also frequently shared concerns about feasibility of such shifts in their practices. Their concerns included whether incorporating proof-oriented tasks would compromise curriculum "coverage", or would take out from the time originally allotted to test preparation; whether students would be willing to take social risks associated with sharing mathematical arguments in public, and to critique the arguments of others; and whether students be willing to engage in proof-oriented tasks that vary in form and content from what they are used to. These types of concerns reflect teachers' professional obligations towards the institution of schooling and towards individual students' social and emotional needs (Herbst & Chazan, 2011).

Of the total 11 proof tasks created by the teachers, 2 were of their own design and 9 were modifications of one of the researcher-designed task types: *Is this a coincidence?*, *True-or-false?* and *Always-Sometimes-Never*. The tasks addressed a variety of mathematical topics in algebra, geometry, number and operation, and logical reasoning. The modes of implementation involved: enrichment activities, practice, exam review, or introducing a new topic. In the following I focus on one teacher, Alison (a pseudonym), to illustrate how she had modified two tasks to fit her classroom context. These tasks were chosen because they stood out as one of the most creative modifications to the researcher-designed task prototypes that occurred within this group of teachers.

Alison's modification of the tasks *True-or-false?* and *Always-Sometimes-Never*

Alison has more than 20 years of teaching experience and is well-respected in her school. Similar to other teachers she joined the PD with mixed feelings: committed to provide students with proving experiences but sharing the abovementioned concerns. Alison was inspired to create two proof tasks when her students performed poorly on a particular item on an algebra test: a word problem about money invested and interest earned in two bank accounts. The students found it challenging to set up an equation to represent the total amount of money split between the two accounts, using a single variable. Alison used students' test responses to create a sequence of tasks: *Always-Sometimes-Never* (Figure 3) and a follow-up *True-or-false* task (Figure 4).

Always, Sometimes or Never?

Part 1: Tom invested \$50,000 into two accounts. The first account earns 6% interest and the other account earns 11% interest. How much money did he invest at each rate if his total interest earned from both investments was \$4,900.

The following cards are possible setups to describe the amount of money (x) invested for Tom's investments described above. Indicate whether the setups are:

- true for all of values of x ,
- true for some values of x ,
- or whether there are no values of x which satisfy the set-up of the application.

Explain your reasoning.

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Figure 2: Six out of 8 items from Alison's task *Always- Sometimes-Never*

Part II: True or False?

Given: Tom's investment problem.

- 1) Decide whether each statement is algebraically true or false and *explain* your reasoning.
- 2) Then justify whether the statement could be used to represent and support Tom's investment.

Show algebraic work and / or use complete sentence(s) to support your position.

Tom invested \$50,000 into two accounts. The first account earns 6% interest and the other account earns 11% interest. How much money did he invest at each rate if his total interest earned from both investments was \$4,900.

#1

	Money invested
1st account	x
2 nd account	x
Total	\$50,000

If x represents the amount of money in each account, then $x + x = \$50,000$ for some values of x.

#2

	Money invested
1st account	x
2 nd account	x
Total	\$4,900

If x represents the amount of money in each account, then $x + x = \$4,900$ for all values of x.

Figure 3: Two out of 6 items from Alison's task *True-or-false?*

Alison's goals in developing this sequence of tasks were to confront students with both correct and erroneous charts for setting an equation representing the money split between the two accounts, and have students analyze, validate or critique the equation setups. In the *Always-Sometimes-Never* task students were to determine whether the equation setups are true for all, some, or no values of x, where x is the amount of money in one account. In the *True-or-false* task the same setups were accompanied by conditional statements. Students were to determine whether each equation is algebraically correct, and whether it can be applied to the given word problem. The tasks were implemented with 74 students (4 classes). Students worked in groups of 3 or 4 on the *Always-Sometimes-Never* task in class, and then completed the *True-or-false* task at home.

Opportunities gained and lost through task modification

The design on Alison's tasks reflects the way she balanced her professional obligations. By using students' test responses as a content of the tasks Alison minimized social anxiety associated with presenting and critiquing mathematical arguments. She also addressed her curriculum goals while engaging students in proof-oriented tasks. The mathematical affordances of Alison's tasks encompass many of the original designer intentions. For example, the task *Always-Sometimes-Never* provided students with opportunities to reason through a variety of correct and incorrect equation setups, and evaluate whether they can be true for all, some or no values of the variable. The use of precise mathematical language echoes the goals of the original design. The two tasks build on each other, with *True-or-false* task emphasizing evaluation of conditional (if not quantified) statements. The tasks reflected additional learning goals Alison had for the students: to distinguish between equations that are mathematically correct but are inappropriate in the context of the problem. Further distinctions could be made between equations that do not account for an implicit problem requirement: the investment in either account cannot be \$0 (equations D, E & F); and equations that do not account for the explicit requirement: the total interest earned must be \$4900, meaning that

equal sums of money cannot be invested in two accounts² (equation A). These distinctions came up in students' written responses to the tasks. Alison was very satisfied with students' interactions with the tasks, and indicated that next year she plans to use them to *introduce* the topic of solving word problems, rather than a test review.

Despite the important affordances of Alison's task, many of the original proof-oriented goals of the tasks, such as the limitation of examples for proving general claims, the distinctions between quantified and non-quantified, universal and existential statements, were not realized in the tasks setup. Potentially, Alison's tasks could be used to highlight other issues related to proving, which although not intended by researcher design, arise naturally in the context of her tasks. Justifications for dismissing solutions A, D, E and F (Figure 1) bare resemblance to arguments by contradiction – a proposed equation, assumed as correct, is rejected because it contradicts one of the problem constraints. Such interpretation could pave a way to discuss proof by contradiction in algebra class.

Conclusions

This paper described an exploratory study that tested a PD model which aimed to support secondary teachers' implementation of argumentation and proving tasks in their classrooms. The researcher-designed tasks served as prototypes after which teachers could model their own tasks. The generic versions of the same tasks provided additional support for teachers by outlining the structure of the tasks and highlighting specific proof-related goals. By using researcher-designed tasks as a starting point for creating their own tasks, teachers became critical partners in designing classroom interventions to promote students' engagement with proving. As teachers collaboratively explored, modified and shared experiences of classroom implementation of their tasks, they negotiated a new understanding of what it means to engage students in argumentation and proving. In the post-PD survey, all participating teachers reported increased confidence in their ability to incorporate argumentation and proving tasks in their teaching. One teacher, called here Jenifer, wrote:

The [PD] classes gave me great ideas to take back to my classroom, to look at proofs very differently than what I had always thought of as a proof. Proofs do not need to be the static, two column proofs from my school experience. They can take a couple of minutes or they could be something to wrestle with for a majority of the block. I liked that the activities were easily manipulated to fit a specific time frame or wanted outcome.

The study also revealed challenges associated with supporting teachers in developing proof-oriented tasks. Alison's tasks show that although she created powerful opportunities for students to engage with argumentation, some of the original researcher-intended goals, specifically related to proving, seem to have been lost. The available data sources do not provide sufficient information as to which aspects of proving were explicitly addressed in class, or whether they were completely overshadowed by discussions of the algebraic content. Hence, future studies should involve classroom observations. Finally, the results of this study concur with those suggesting that changing teacher practices is a gradual process which requires structured support (Brodie, 2010) to help teachers to develop a view of proof-oriented classroom activities as means to balance their professional obligations and enhance students' mathematical learning.

² Investing \$25,000 at 6% in one account and \$25,000 at 11% in another account would yield a total interest of \$4,250.

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