Robust switching control design for uncertain discrete-time switched affine systems.

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Relaxed periodic switching controllers of high-frequency DC-DC converters using the δ-operator formulation

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***Abstract*— This paper deals with the design of new periodic switching control laws for high frequency DC-DC converters. The contributions are twofolds. On a first hand, the DC-DC converter model is rewritten as a periodic switched affine system thanks to a δ-operator formulation, which represents an efficient framework for the numerical discretization at high frequencies. On a second hand, three different control laws are provided, the first one being the usual Lyapunov-based control law and the two others being relaxed versions of this first solution. The benefits of these two new control laws over the usual Lyapunov-based one are demonstrated on a simple example.**

I. Introduction

Nowadays, there is a relevant interest for DC-DC converters due to their numerous applications in the industry, as for example in computer power supply, cell phones, appliances, automotive, aircraft, etc. These systems can be modeled as switched affine systems (SASs), which represent a particular nonlinear class of switched systems. They correspond to a class of hybrid dynamical systems consisting of several operating modes represented by continuous-time subsystems and a rule that selects between these modes [12]. Compared to the linear case, the affine structure of these systems imposes a set of operating points defined by an averaged dynamic, leading to solutions in the generalized sense of Krasovskii.

Many works found in the literature in continuous time control the SASs by a min-projection strategy [2], [5], [17], even for systems with a general nonlinear form [14], [15]. In these works the provided controllers are good, but may lead to arbitrarily fast switching control. Some solutions to this problem can be found in the literature, as [3], [18], [19], where, the authors aim at ensuring a dwell-time associated with an admissible chattering around the operating point. Nevertheless, [3] does not prove a minimum time associated to the spacial regularization. In [19], a focus on specific electronic architecture related to boost converters is proposed. In addition, the contributions of [18] do not provide a complete stability proof. On the other hand, in [4], the authors present an open-loop stabilization strategy based on dwell-time computation, [1] proposes a minimum dwell-time with a space and time regularization, [10] guarantees a minimum and maximum dwell-time by solving optimization problems. These solutions present a common characteristic: systems are controlled by aperiodic switching.

In many occasions, it is necessary to control this class of systems with periodic switching, due to physical constraints. In order to deal with this issue, a solution consisting in the discretization of the continuous-time model with a fixed periodic sampling time was provided in [6], [11]. The authors of [6] present a controller based on a Lyapunov function synthesized by solving an optimization problem, whose objective is to minimize the area around the equilibrium, where the solutions converge. On the other hand, in [11], the authors design a sampled-data switching control with an upper-bound designed to ensure robustness in continuous-time systems. Through Linear Matrix Inequalities (LMI) based conditions, the upper-bound of the length of the inter-sampling interval can be directly related to the size of the asymptotic stability set around the considered equilibrium. Practical stability is obtained using Lyapunov-Krasovskii functional and the Jensen's inequality. Both solutions do not consider a high-frequency sampling, beside the fact that they are conservative because the controllers are based on a Lyapunov-function.

In some applications when discretizing these systems, if the sample time is very low, several problems may appear to assess stability, because of numerical issues. Several solutions have been considered in the literature of automatic control. Among them, the δ-operator has been introduced in [16]. It consists of a discretization method that becomes sufficiently close to the continuous-time model, ensuring continuity of the conditions for high-frequency samplings. In [7], a comparison is made between the operator q and the operator δ to perform this discretization. Here, it is possible to observe as the operator δ presents as advantage the natural convergence to a continuous system, while avoiding numerical problems when the sampling period is very short. In [20] it is possible to observe that the delta operator is used to perform the sliding mode fuzzy controller of a DC-DC buck converter due to the need for very fast sampling. Therefore, the use of the δ-operator presents a great advantage in the design of controllers with very fast sampling times like the one presented in this paper.

In this paper, we model the DC-DC converter in discrete-time by using the δ-operator and we control the system with a well-known min-projection strategy, based on a Lyapunov function. Then, we propose a periodic-sampling relaxed controller for these systems, allowing to obtain results less conservative, even for high-frequency systems. Moreover, this approach presents a

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trade-off between the sampling period and the size of the chattering effects. Some simulations in Matlab valid our contribution.

The paper is organized as follows: the problem formulation is stated in Section II. Then, a classical controller is presented in Section III. From this, Section IV proposes some relaxed controllers. An optimization of the controllers is given in Section V. Section VI illustrates the potential of this method on a particular DC-DC converter. The paper ends with a conclusion section.

Notation: Throughout the paper $\mathbb{N}$ and $\mathbb{R}$ denote the set of natural and real numbers, respectively. $\mathbb{R}^n$ the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all real $n\times m$ matrices. The set composed by the first N positive integers, namely $\{1,2,...,N\}$, is denoted by $\mathbb{K}_N$. I is the identity matrix of suited dimension. The Euclidean norm of vector $x\in\mathbb{R}^n$ is denoted by $|x|$. For any symmetric matrix M of $\mathbb{R}^{n\times n}$, the notation $M\succ 0$ ($M\prec 0$) means that the eigenvalues of M are strictly positive (negative).

II. Problem formulation

A. System data

Inspired by the work in [6], we focus on the following class of switched affine systems, which is relevant in the context of DC-DC converters

$$\dot{z} = A_\sigma z + a_\sigma, \tag{1}$$

where $z\in\mathbb{R}^n$ is the state and it is accessible, A_σ and a_σ present suited dimensions. The control action is performed through the high frequency switching signal $\sigma\in\mathbb{K}_N := \{1,2,...,N\}$, which may be only modified at sampling instants t_k, with $k\in\mathbb{N}$. In this paper, the length of the sampling interval $t_{k+1}-t_k = T$ is assumed to be constant, known and small enough.

This paper focuses on the design problem of a feedback law for the high frequency periodic switching signal σ, in such a way to ensure suitable practical convergence properties of the plant state z to a operating equilibrium z_e, which is not necessarily an equilibrium for the continuous-time dynamics in (1), but can be obtained as an equilibrium for the switching system with arbitrary switching. A necessary and sufficient condition characterizing this equilibrium is then represented by the following standard assumption (see [5], [13]).

Assumption 1: There exists $\lambda = [\lambda_1,\lambda_2,...,\lambda_N]$ satisfying $\sum_{i\in\mathbb{K}}\lambda_i = 1$, such that the following convex combination holds:

$$\sum_{i\in\mathbb{K}}\lambda_i(A_i z_e + a_i) = A(\lambda)z_e + a(\lambda) = 0. \tag{2}$$

Remark 1: It is emphasized that Assumption 1 is both necessary and sufficient for the existence of a suitable switching signal ensuring forward invariance of the point z_e (namely inducing an equilibrium at z_e) when understanding solutions in the generalized sense of Krasovskii or Filippov. Indeed, under (2), we can conclude that the error equation of (1):

$$\dot{x} = A_\sigma x + B_\sigma, \tag{3}$$

where the error vector is denoted by $x := z - z_e$ and where the matrices B_σ are defined by $B_\sigma := A_\sigma z_e + a_\sigma$ and verify the following convex combination $\sum_{i\in\mathbb{K}}\lambda_i B_i = 0$. The objective is to ensure that the error state x converges to the equilibrium $x=0$ in the Filippov sense. ⌟

In addition, the following property is assumed.

Assumption 2: The matrices A_i, for $i\in\mathbb{K}_N$ are non-singular and $A(\lambda)$ is Hurwitz.

B. δ-operator for high frequency switching function

In this paper, we will propose a discrete-time model based on the δ-operator [16], which is suitable for high switching frequencies. The δ-operator has been widely used in the literature to avoid numerical problems in the computation of discrete-time dynamics. This is based on the continuous ones in the situation where the sampling period T is potentially very small. The definition of the δ-operator is as follows. For any function ξ from $\mathbb{R}^+$ to $\mathbb{R}^n$, the vector $\delta\xi_k$, at any sampling instant $t_k\in\mathbb{R}^+$, is defined as follows

$$\delta\xi_k := \frac{1}{T}(\xi_{k+1}-\xi_k), \qquad \forall k\geq 0,$$

where we used the convention $\xi_k = \xi(t_k)$ and $\delta\xi_k = \delta\xi(t_k)$, for all integer $k\geq 0$. Hence the dynamics of system (1) can be rewritten in the framework of the δ-operator, which yields the following dynamics

$$\delta x_k = E_\sigma x_k + F_\sigma \tag{4}$$

where the matrices that defines the system dynamics are given by

$$E_\sigma = \tfrac{1}{T}(e^{A_\sigma T}-I), \quad F_\sigma = \tfrac{1}{T}\int_0^T e^{A_\sigma(T-s)}ds B_\sigma. \tag{5}$$

The interest of this formulation compared to the usual discrete-time formulation comes from the fact that, when T goes to zero, matrices E_σ and F_σ converge to A_σ and B_σ, respectively. Another important issue is that matrices E_σ and F_σ depend explicitly on the switching period T. Indeed, considering small values of T may lead to several numerical problems when discretizing (3).

Remark 2: Note that if matrix A_σ is non singular, then a simple expression of F_σ is provided by

$$F_\sigma = \frac{e^{A_\sigma T}-I}{T}A_\sigma^{-1}B_\sigma.$$

⌟

It is worth noting that model (4) does not account for the continuous evolution of (1) during the intersampling time. It is however possible to characterize the continuous solution by integrating the solution over a sampling interval, leading, for all $t\in[t_k,\ t_k+T]$ and for all $k\in\mathbb{N}$, to

$$x(t) = e^{A_\sigma(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A_\sigma(\tau-t_k)}d\tau a_\sigma$$

Since t belongs to the bound interval $[t_k,\ t_k+T]$, the solutions to the system are obviously bounded during the inter sampling time.

C. Control objectives

When considering such switching affine systems, asymptotic stability to zero is in general not possible. Therefore one has to relax the control objectives and to consider attractor sets, which are not necessarily reduced to the equilibrium set. In this paper, we will consider an estimation of a set that ensures that outside $\delta V < 0$, which is of the following quadratic form

$$\mathcal{E} := \left\{x \in \mathbb{R}^n, \quad x^\top S x \leq 1\right\}. \tag{6}$$

with S being a symmetric positive definite matrix to be optimized. This formulation is quite usual and has been used in other contexts as in [1], [6], [11].

This paper focuses on the design problem of a feedback law for the high frequency periodic switching signal σ, in such a way to ensure suitable practical convergence properties of the plant state x to 0, which is not necessarily an equilibrium for the continuous-time dynamics in (1), but can be obtained as an equilibrium for the switching system with arbitrary switching. A necessary and sufficient condition characterizing this equilibrium is then represented by the following standard Assumption 2 (see [5], [13]). The problem can be summarized as follows

Problem 1: For any small sampling period T, the problem is to find a switching control law that selects, at each sampling time, the mode or subsystem among all possibilities that stabilizes system (1) with certain performance guarantees at its equilibrium.

III. Lyapunov-based switching control

Looking at the literature on switched affine systems, one can find the well-known min-projection control law for such a class of systems [6], [11], [17]. The underlying idea of this control law is to select the mode of the system which minimizes the decrease of a quadratic Lyapunov function given by

$$V(x) := x^\top P x, \qquad \forall x \in \mathbb{R}^n \tag{7}$$

where $P \succ 0$ is a positive definite matrix of $\mathbb{R}^{n\times n}$. This idea is formalized in the following theorem.

Theorem 1: Consider Assumptions 1 and 2 and matrices $P \succ 0$ and $S \succ 0$ of suited dimension that are solution to the feasibility problem

$$\Gamma^{(1)}_{ij} \prec 0, \quad \forall i, j \in \mathbb{K} \tag{8}$$

for any pair (i, j) in $\mathbb{K}^2$.

$$\begin{aligned}\Gamma^{(1)}_{ij} \quad &= \Psi_i(P) + \mu_i \begin{bmatrix} S & 0 \\ 0 & -1\end{bmatrix} + \gamma_i \left[\Psi_j(P) - \Psi_i(P)\right], \\ \Psi_i(P) &= \begin{bmatrix} PE_i + E_i^\top P + TE_i^\top PE_i & PF_i + TE_i^\top PF_i \\ F_i^\top P + TF_i^\top PE_i & TF_i^\top PF_i\end{bmatrix},\end{aligned} \tag{9}$$

for some given parameters $\gamma_i > 0$ and $\mu_i > 0$. Then the switching control (C1) law defined by

$$\text{(C1)} \quad \sigma(x_k) = \underset{i\in\mathbb{K}_N}{\operatorname{argmin}} \begin{bmatrix} x_k \\ 1\end{bmatrix}^\top \Psi_i(P) \begin{bmatrix} x_k \\ 1\end{bmatrix} \tag{10}$$

guarantees $\delta V{<}0$ outside of set $\mathcal{E}$. □

Proof: Consider the Lyapunov function given in (7), where $P \succ 0 \in \mathbb{R}^{n\times n}$. Let us first compute the expression of δV_k as follows

$$\begin{aligned}\delta V(x_k) &= \frac{1}{T}(V(x_{k+1}) - V(x_k)) \\ &= \frac{1}{T}((x_k + T\delta x_k)^\top P(x_k + T\delta x_k) - x_k^\top P x_k) \\ &= 2\delta x_k^\top P x_k + T\delta x_k^\top P \delta x_k.\end{aligned} \tag{11}$$

Replacing δx_k by its expression given in (4), and using the definition of the matrix $\Psi_i(P)$ provided in (9) yields

$$\delta V(x_k) = \begin{bmatrix} x_k \\ 1\end{bmatrix}^\top \Psi_\sigma(P) \begin{bmatrix} x_k \\ 1\end{bmatrix}.$$

Using (9), the previous expression can be rewritten as follows, for any j in $\mathbb{K}$

$$\delta V(x_k) = \begin{bmatrix} x_k \\ 1\end{bmatrix}^\top \Bigg(\Gamma^{(1)}_{\sigma j} - \mu_\sigma \begin{bmatrix} S & 0 \\ 0 & -1\end{bmatrix} - \gamma_\sigma \left[\Psi_j(P) - \Psi_\sigma(P)\right]\Bigg) \begin{bmatrix} x_k \\ 1\end{bmatrix}$$

Since the matrix inequalities $\Gamma^{(1)}_{i,j} \prec 0$ holds for any pair (i, j) in $\mathbb{K}^2$, we have

$$\begin{aligned}\delta V(x_k) < &\ \mu_\sigma(1 - x_k^\top S x_k) \\ &-\gamma_\sigma \begin{bmatrix} x_k \\ 1\end{bmatrix}^\top \left[\Psi_j(P) - \Psi_\sigma(P)\right] \begin{bmatrix} x_k \\ 1\end{bmatrix}\end{aligned}$$

Note that the switching control law (10) ensures that the last term of the right hand side of the previous inequality is non positive, which guarantees that

$$\delta V(x_k) < \mu_\sigma(1 - x_k^\top S x_k)$$

The previous inequality finally guarantees that for any values of x_k outside of $\mathcal{E}$ (i.e. $1 - x_k^\top S x_k < 0$), the quantity $\delta V(x_k)$ is strictly negative, which concludes the proof. ■

IV. Relaxed switching controller

In the previous section, a Lyapunov-based switching control was presented. This control was clearly inspired from the existing literature on switched affine systems such as [6], [11] but adapted to the δ-operator modelling of DC-DC converters. The motivation of this section is to present a relaxed version of the previous control law. This relaxation considered here is related to the fact that the a priori intuition behind this Lyapunov-based control law might be too restrictive in the sense that the selection of the Lyapunov matrix P is done to verify two distinct purposes, namely, the definition of the Lyapunov function and the construction of the switching signal. Based on this comment, a relaxed control law can be provided by decoupling the two problems of selecting a Lyapunov function and of designing the switching control law.

The relaxed control law is based on the simple idea consisting of keeping the same structure of control law presented in (10). However, instead of using the Lyapunov matrix P, a new unconstrained matrix is introduced to define the switching law. This is formalized in the following theorem.

Theorem 2: Consider Assumptions 1 and 2, matrices $P \succ 0$ and $S \succ 0$ of suited dimension and a new matrix $N \in \mathbb{R}^{n\times n}$ that are solution to the feasibility problem

$$\Gamma_{ij}^{(2)} \prec 0, \quad \forall i,j \in \mathbb{K} \tag{12}$$

where, for any pair (i,j) in $\mathbb{K}^2$,

$$\Gamma_{ij}^{(2)} = \Psi_i(P) + \mu_i \begin{bmatrix} S & 0 \\ 0 & -1 \end{bmatrix} + \gamma_i \left[\Psi_j(N) - \Psi_i(N)\right], \tag{13}$$

where the matrices $\Psi_i(P)$ and $\Psi_i(N)$ are given in (9) and, again, for some given parameters $\gamma_i > 0$ and $\mu_i > 0$. Then the switching control (C2) law defined by

$$\text{(C2)} \quad \sigma(x_k) = \underset{i\in\mathbb{K}_N}{\operatorname{argmin}} \begin{bmatrix} x_k \\ 1 \end{bmatrix}^\top \Psi_i(N) \begin{bmatrix} x_k \\ 1 \end{bmatrix} \tag{14}$$

guarantees $\delta V < 0$ outside of set $\mathcal{E}$. □

Proof: The proof strictly follows the proof of Theorem 1, except that, now, the switching control law is not characterized by the Lyapunov matrix P but by an arbitrary matrix N, which only has to be a solution to the feasibility problem (12). ■

Remark 3: Compared to Theorem 1, there are two main advantages. The first one relies on the fact that matrix N is not required to be symmetric nor positive. The second one consists in the fact that the switching law is now completely decoupled from the definition of the Lyapunov function. Moreover, one can see that selection $N = P$ in Theorem 2 leads to the same statement as in Theorem 1. This ensures that the set of feasible solutions of (12) is greater than the ones of (8). It is then expected to derive relaxed solutions that will be presented in the example section where the optimization procedure presented later on in Section V is included. ⌟

V. Optimisation procedure

The feasibility problems proposed in Theorems 1 and 2, only ensure that there exists a switching control law that stabilizes the system to a bounded region around the equilibrium. Without an optimization process, the resulting regions might be too large to be relevant from the physical point of view. Indeed, considering a too large set $\mathcal{E}$ can possibly increase the chattering effects which are the main phenomena to avoid or limit in the control design of DC-DC converters. This chattering behavior can damage or even break the devices. Therefore, it is necessary to include an optimization procedure in these theorems, whose objective is to minimize the size of the set $\mathcal{E}$. Since this set is fully characterized by the symmetric positive definite matrix S, minimizing the size of $\mathcal{E}$ can be achieved by maximizing the determinant of S. Based on the discussion above, the following proposition of stated dealing with the optimization of the solutions to the conditions of Theorems 1 and 2 is stated.

Proposition 1: For any a priori fixed scalar parameters μ_j and γ_i, the optimisation problem

$$\max_{S,P} \det(S), \quad \text{s.t.} \quad \Gamma_{ij}^{(c)} \prec 0, \quad \forall i,j \in \mathbb{K} \tag{15}$$

for any $c = \{1,2\}$, minimizes the size set (6) guaranteeing $\delta V < 0$ outside of set $\mathcal{E}$.

Remark 4: Note that we do not optimize the chattering region, because it is constrained to an ellipse form and, the controller is also constrained for a given structure. Thus, it is expected that set $\mathcal{E}$ will be relaxed with the control given in Theorem 2, with respect to the control law C1. ⌟

Remark 5: This optimization process strongly depends on the selection of the scalar parameters γ_i and μ_j. Hence it is expected that an iterative procedure to selected the best parameters needs to be included. ⌟

Remark 6: Other optimization objectives can be considered such as the maximization of the eigenvalues of S, which can be done by maximizing a scalar τ such that $\tau I \leq S$. ⌟

VI. Application to a boost converter

The control laws introduced above are evaluated on a classical boost converter system. This converter switches at high frequency between two modes ($N = 2$) corresponding to two affine subsystems. The state variable is defined by $x = [i_L \quad v_c]^\top$, where i_L denotes the inductor current and v_c the capacitor voltage.

We take the parameters given in [5] for comparison with the switched control algorithm presented therein, which switches with arbitrary aperiodic switching in the steady-state. This type of switching tends to be complicated in physical applications. The considered nominal values are: $V_{in} = 100V$, $R = 2\Omega$, $L = 500\mu H$, $C_o = 470\mu F$ and $R_o = 50\Omega$. The switched system state space model (1) is defined by the following matrices for $i = 1,2$:

$$A_i = \begin{bmatrix} -\frac{R}{L} & \frac{(1-i)}{L} \\ \frac{(i-1)}{C_0} & -\frac{1}{R_0 C_0} \end{bmatrix}, \quad a_i = \begin{bmatrix} \frac{V_{IN}}{L} \\ 0 \end{bmatrix}.$$

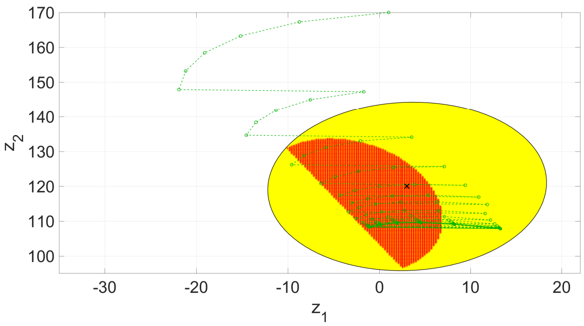
The chosen simulation parameters are given by

$$z_e = \begin{bmatrix} 3 & 120 \end{bmatrix}^\top, \ \lambda = [0.22 \ \ 0.78]$$

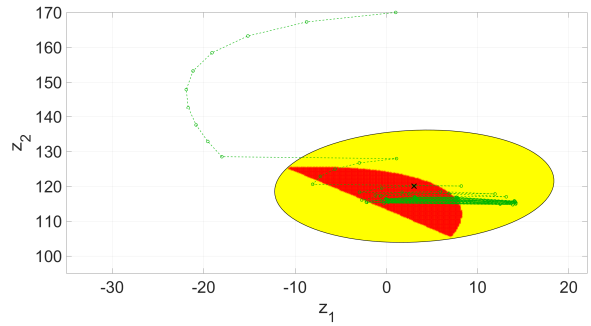
for which simple calculations ensure the satisfaction of Assumption 2. The optimization problems given in Proposition 1 is solved using the CVX solver [8], [9]. The results obtained with this software illustrate our the efficiency of the new control law presented in Theorem 2 with respect to the Lyapunov-based controller employed in the literature, presented in Theorem 1.

As pointed out in Remark 5, the optimization scheme presented in Proposition 1 delivers different results for different values of γ_i and μ_j. Therefore, a random algorithm has been considered to obtain the best tuple of parameters. The values of these parameters μ_1, μ_2, γ_1 and γ_2, obtained for several values of the sampling period T_s are provided in Table I.

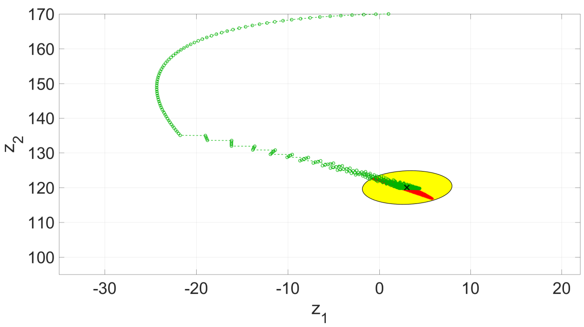
Figure 1 shows the state trajectories, set $\mathcal{E}$ and $\delta V > 0$ surface in the state-plane for the different controllers



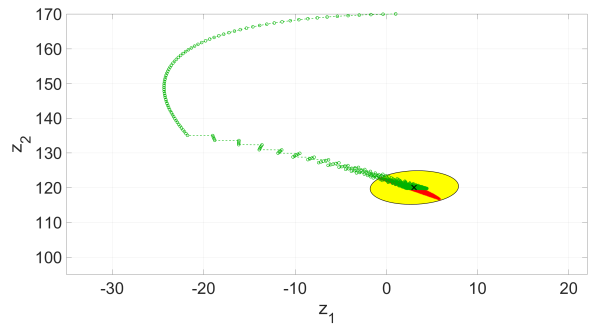
(a) C1 with $T = 10^{-4}$.



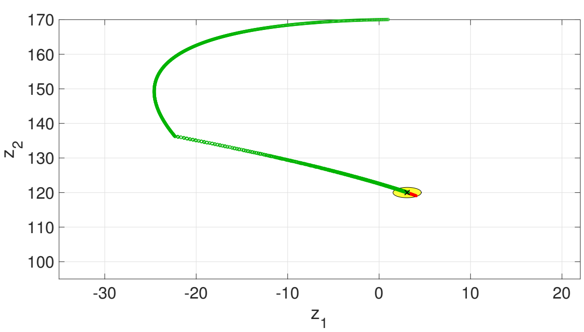
(b) C2 with $T = 10^{-4}$.



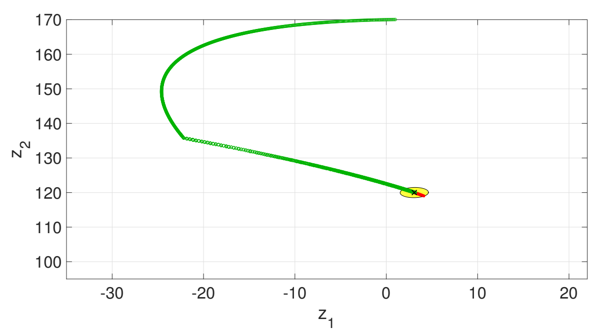
(c) C1 with $T = 10^{-5}$.



(d) C2 with $T = 10^{-5}$.



(e) C1 with $T = 10^{-6}$;



(f) C2 with $T = 10^{-6}$.

Fig. 1: Numerical results of Proposition 1. Time trajectories in green, set $\mathcal{E}$ in yellow and $\delta V > 0$ in red.

Parameters	T_s	μ_1	μ_2	γ_1	γ_2
Th. 1	10^{-4}	9.84	9.86	0.948	0.0515
	10^{-5}	0.16	0.157	0.777	0.223
	10^{-6}	0.907	0.907	0.782	0.218
Th. 2	10^{-4}	3.16	3.16	0.916	0.327
	10^{-5}	1	1	0.349	0.1
	10^{-6}	0.313	0.313	10	2.793

TABLE I: Numerical values of μ_1, μ_2, γ_1 and γ_2.

given in Proposition 1, as well as for different sampling periods. Note that the $\delta V > 0$ surface is in the interior of the set $\mathcal{E}$ as is expected from Theorem 1 and 2. Remark also as $\mathcal{E}$ is reduced, as T_s decreases. We can see that set $\mathcal{E}$ is reduced with control law C2 w.r.t. C1, showing as the control law C2 provide reduced region with respect to C1. This is consistent with Remark 3. Another important remark concerns the fact that the region where $\delta V > 0$ is concentrated in a smaller area in control law C2 with respect to C1.

These simulations demonstrate the advantages of the relaxed control law over the existing Lyapunov-based one. Indeed, our proposed controller C2 allow to control efficiently the DC-DC converters under study even with relatively high sampling period with a notable reduction of the switches of these systems, which ensures an increase of the lifespan and the reduction of the dissipated energy.

VII. Conclusions

In this paper, we have presented two main contributions. The first one deals with the definition of accurate discrete-time model for high frequency sampling DC-DC converters. The second contribution consists in the extension of the usual Lyapunov-based control laws employed in this domain to a less restrictive control law that encompass this first formulation. This new control law delivers notable improvements with respect to the Lyapunov-based control law in terms when comparing the estimate of the set $\mathcal{E}$ of the switched affine system.

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